Dipolar optical plasmon in thin-film Weyl semimetals

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Abstract

We investigate the collective charge oscillations in topological surface states such as Fermi arcs for Weyl or Dirac semimetals. We show that, Weyl semimetal thin-film host a single $\omega \propto \sqrt{q}$ plasmon mode, that results from collective, anti-symmetric charge oscillations between the two surfaces, in stark contrast to conventional 2D bilayers as well as Dirac semimetals with Fermi arcs, which support antisymmetric acoustic modes along with a symmetric optical mode.

Model

The minimal model of a WSM has two Weyl nodes at the Fermi energy. The low-energy Hamiltonian can be written using a two-band model. In general, the model Hamiltonian for n pairs of Weyl nodes consists of n blocks of two band systems. The basic Hamiltonian block for the semimetal may be written as,

$$H_\eta = (\sigma_y q_x - \sigma_x q_z) + \sigma_z M_\eta(k_y), \tag{1}$$

The mass is given by $M_\eta(k_y) = \eta\,(m - \cos(k_y))$. The two Weyl nodes are at $\mathbf{k} = (0, \pm k_0, 0)$ with $k_0 = \cos^{-1}(m)$.

Slab Geometry

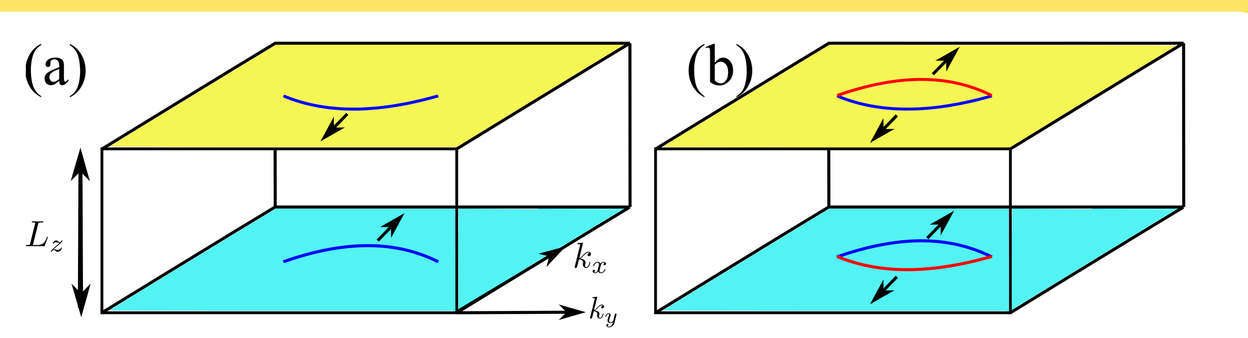

Fig. 1: Schematic picture of a Weyl Semimetal (*a*) and a Dirac semimetal (*b*) slab. For Weyl semimetal, the Fermi arcs are shown in blue colour. The Fermi arcs disperse in opposite directions on top and bottom surfaces. For Dirac semimetal, there are double Fermi arcs on each surface.

Low-energy band structure of WSM slab

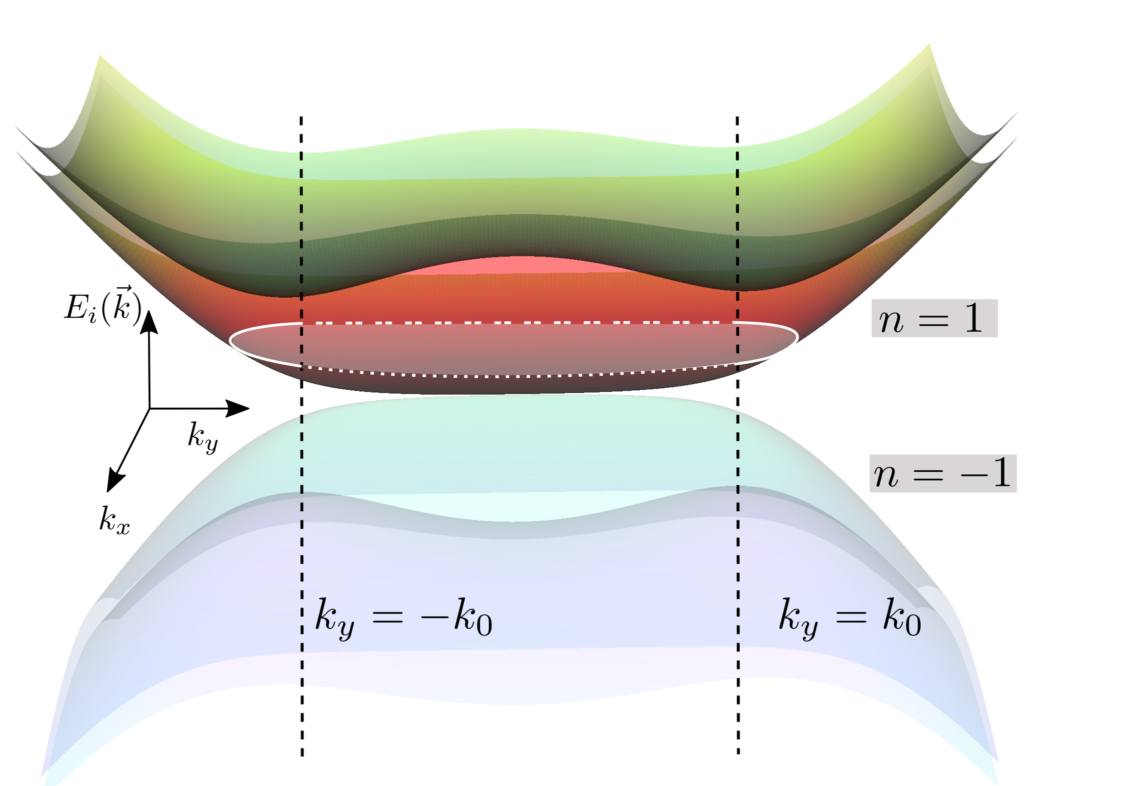

Fig. 2: Low-energy bands of the WSM slab. The $n=\pm1$ bands contain all the FAs as well as bulk states. A Fermi surface at low energy is marked with states at the dotted side and is supported by one surface and the states at the dashed-dotted side is supported by the other surface, whereas the states at the solid sides have support dominantly in the bulk

Heuristic explanation

The system broadly falls in the category of two-layer 2D electronic system, separated by dielectric medium.

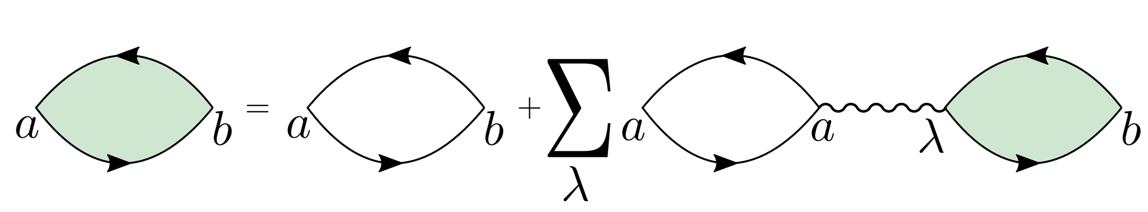

Fig. 3: The self-consistent equations for the interacting response functions $\tilde{\chi}_{ab}$ (filled), where $a, b, \lambda = \pm1$ are layer indices, written at RPA approximation with single curly lines being the interaction $V_{a\lambda}$ and the unfilled loop being the non-interacting response function $\chi_{ab}\delta_{ab}$.

The interacting response function can be written as,

$$\begin{pmatrix} 1 - V_{11}\chi_1 & -V_{12}\chi_1 \\ -V_{21}\chi_2 & 1 - V_{22}\chi_2 \end{pmatrix} \begin{pmatrix} \tilde{\chi}_{11} \\ \tilde{\chi}_{21} \end{pmatrix} = \begin{pmatrix} \chi_1 \\ 0 \end{pmatrix}, \tag{2}$$

The conditions for self-sustaining collective modes are found from :

$$\begin{aligned} 1 - V(\mathbf{q})\big(\chi_1(\mathbf{q},\omega) + \chi_2(\mathbf{q},\omega)\big) \\ + V(\mathbf{q})^2(1 - e^{-2qL})\chi_1(\mathbf{q},\omega)\chi_2(\mathbf{q},\omega) = 0. \end{aligned} \tag{3}$$

$V_{11} = V_{22} = \alpha_c/q$ and $V_{12} = V_{21} = \alpha_c e^{-qL}/q$ are the bare intra- and inter-layer Coulomb interactions, where L is the separation between the layers and $\alpha_c = 2\pi e^2/\varepsilon$.$\chi_1(\mathbf{q},\omega)$ and $\chi_2(\mathbf{q},\omega)$ are the non interacting response function for the top and bottom surfaces.

In general there are two resulting mode [2, 3]. A $\omega \propto q$ (short range) plasmon, resulting from out-of-phase oscillation of the layers. A $\omega \propto \sqrt{q}$(long range) plasmon, resulting from in-phase oscillation of the layers.

Result

Plasmon

Plasmons: poles of density-density correlation in presence of coulomb interaction.

$$\chi(\mathbf{r}, \mathbf{r'}, t) = -i\theta(t)\langle[\rho(\mathbf{r}, t), \rho(\mathbf{r'})]\rangle \tag{4}$$

In matrix form,

$$\chi(\mathbf{q}, \omega) = (1 - B)^{-1}\chi_0(\mathbf{q}, \omega) \tag{5}$$

The poles of the response function can then be found by solving $\det[\mathrm{I} - \mathrm{B}(\mathbf{q}, \omega)] = 0$.

We found a single $\omega \propto \sqrt{q}$ plasmon mode for WSM whereas in DSM there are two plasmon mode $\omega \propto \sqrt{q}$ as well as $\omega \propto q$.

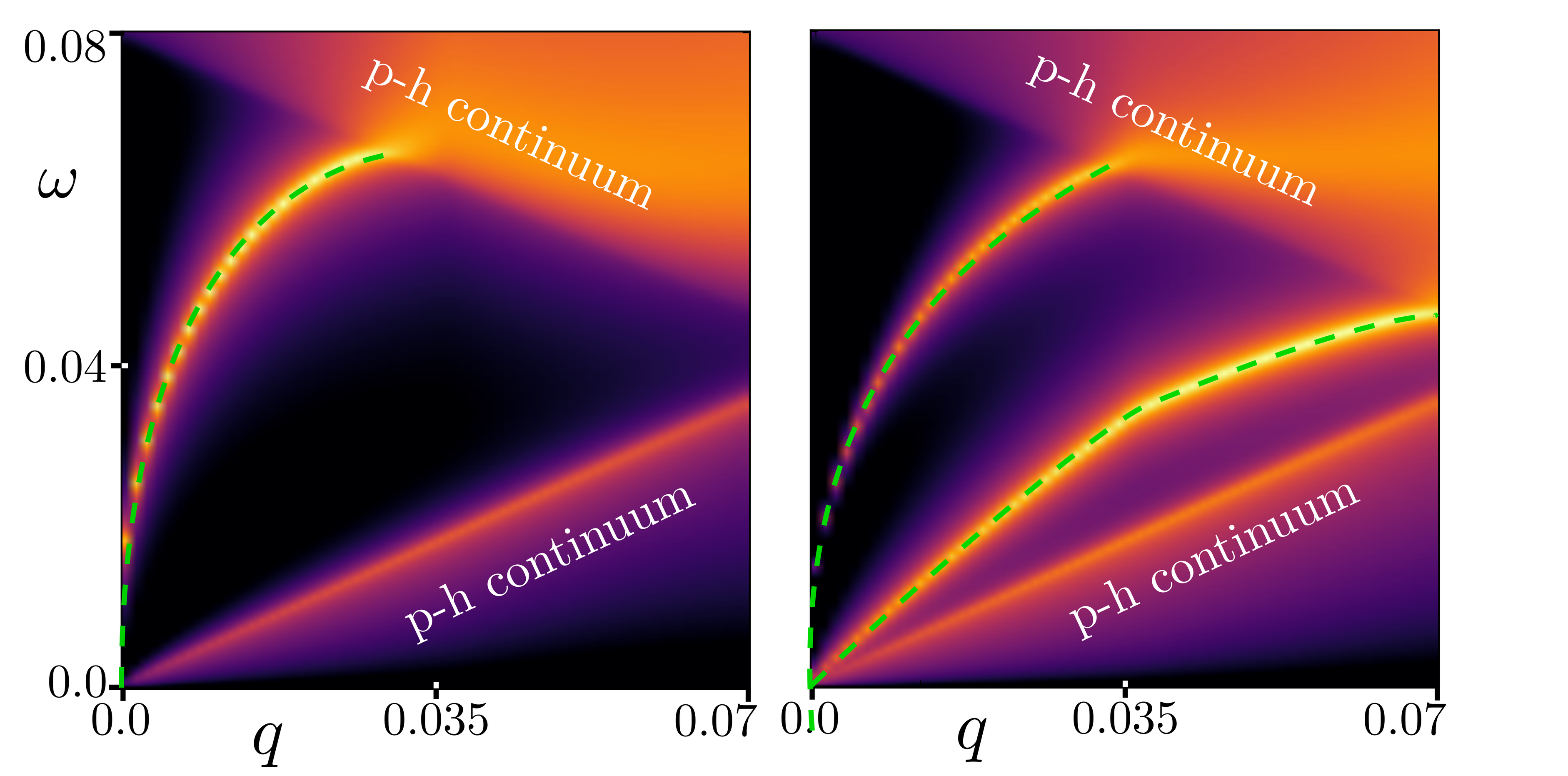

Fig. 4: Density plot of the loss function, defined as $-\mathrm{Im}\epsilon^{-1}(\mathbf{q},\omega) = \frac{\mathrm{V}(\mathbf{q})\mathrm{Im}[\chi^0]}{(1-\mathrm{V}(\mathbf{q})\mathrm{Re}[\chi^0])^2+(\mathrm{V}(\mathbf{q})\mathrm{Im}[\chi^0])^2}$. Particle-hole continuum (lighter area) and the sharp plasmonic modes (marked) as a function of q along the direction of $\tan^{-1}(q_y/q_x) = 60^0$. The left and the right plots are for the WSM and DSM thin films, respectively.

charge oscilattion

We construct the charge fluctuations associated with the collective modes using the eigenvectors of the density response matrix $\chi(\mathbf{q}, \omega)$.

The charge fluctuations which are antisymmetric across surfaces appear in a $\sqrt{q}$ mode for the WSM, whereas in the DSM – as in conventional semiconductor bilayers – this behavior is found in an acoustic mode.

The low-**q** plasmons are undamped, *highly non-local* and supported by both the surfaces.

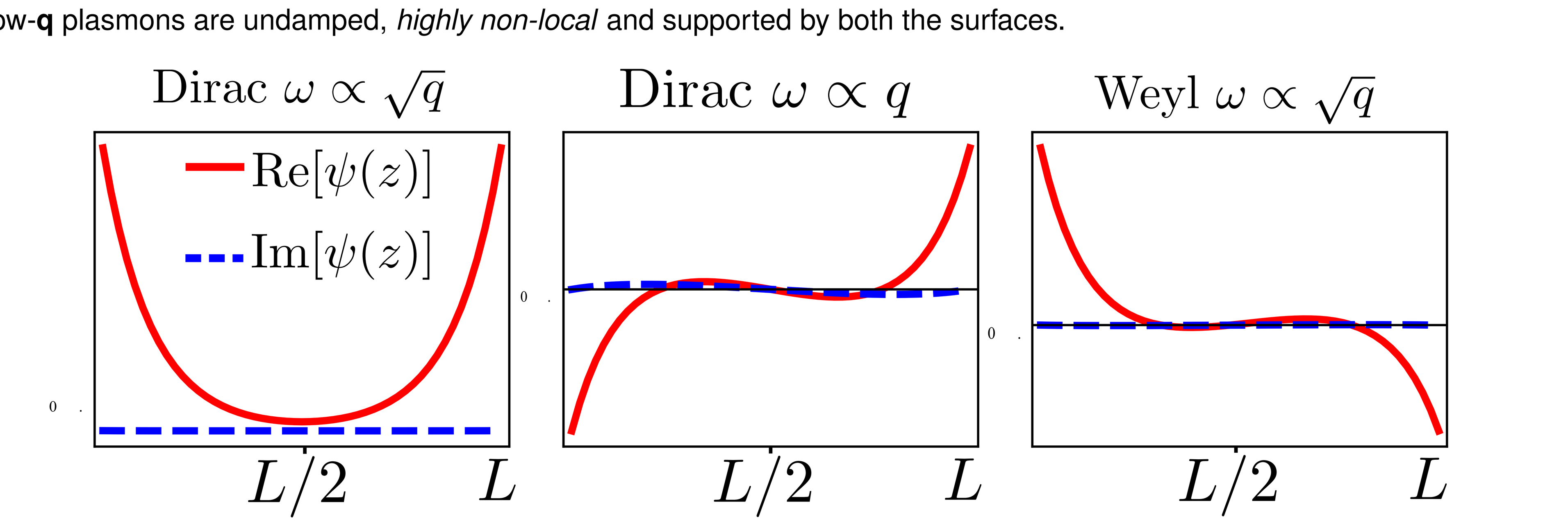

Fig. 5: At the condition of the plasmonic mode, one of the eigenvalues of the matrix vanishes. We plot the corresponding (normalized) eigenvector $\psi(z)$, showing the $\omega \propto \sqrt{q}$ mode in the Weyl (right most) is indeed an antisymmetric mode, which is contrary to the DSM.

Heuristic explanation

In the cases of WSM's and DSM's these surface states may be modeled as a collection of helical states dispersing linearly in the $\hat{x}$-direction,

Dirac Fermi arcs: $E_s^{(\pm)}(\mathbf{k}) = (\pm)s\hbar v_F k_x, \quad k_0 < k_y < k_0, \quad s = 1$ for top surface and $s = -1$ for bottom surface.

The noninteracting polarizability function is, $\chi_T^{\mathrm{DSM}}(\mathbf{q},\omega) = \chi_B^{\mathrm{DSM}}(\mathbf{q},\omega) = \frac{2\beta q_x^2}{\omega^2 - q_x^2}$, where $\beta = \frac{k_F k_0}{2\pi^2\hbar v_F}$ Solving Eq.(3) one find,

$$\omega_{\mathrm{D}}^{(1)} = v_F\sqrt{1 + 2\alpha_c\beta L}\cos\theta\; q, \quad \omega_{\mathrm{D}}^{(2)} = v_F\sqrt{4\alpha_c\beta}\cos\theta\;\sqrt{q}. \tag{6}$$

Weyl Fermi arcs: $E^{T/B}(\mathbf{k}) = (\pm)\hbar v_F k_x, \quad k_0 < k_y < k_0$. For WSM the non-interacting polarizability is given by, $\chi_T^{\mathrm{WSM}}(\mathbf{q},\omega) = \frac{\beta q_x}{\omega - q_x}$ and $\chi_B^{\mathrm{WSM}}(\mathbf{q},\omega) = -\frac{\beta q_x}{\omega + q_x}$. In the limit $qL << 1$, one find,

$$\omega_{\mathrm{W}} = v_F\sqrt{2\alpha_c\beta}\sqrt{1 + \alpha_c\beta L}\cos\theta\sqrt{q}. \tag{7}$$

The net charge fluctuations on the two surfaces $\delta\rho_i(\mathbf{q},\omega)$ $(i = 1, 2)$ are written as, $\frac{\delta\rho_1}{\delta\rho_2} = \frac{\chi_1 V_{12}}{1 - V_{11}\chi_1}$. For small q and for the $\omega \propto \sqrt{q}$ mode, $\chi_1 \propto \sqrt{q}$ whereas $V_{11}, V_{12} \propto 1/q$. This implies $\left.\frac{\delta\rho_1}{\delta\rho_2}\right|_{\mathrm{WSM}} \approx -\frac{V_{12}}{V_{11}} \approx -1$, i.e. anti symmetric charge oscillation.

In the case of DSM, for the $\omega \propto \sqrt{q}$ mode, $\left.\frac{\delta\rho_1}{\delta\rho_2}\right|_{\mathrm{Dirac}} \approx 1$, this implies a symmetric charge oscillation.

References

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