# Non-abelian Anyons : From Majoranas to parafermions

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Correlations in Topological Quantum Matter
Lammi Biological Station
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# Plan of the talk

- I - Introduction to anyons:
  exchange statistics, braiding, non-abelian anyons and Topological Quantum Computing
- II - Majorana modes :
  Where are they? Have they been found ? Our recent work
- III - Parafermions :
  Why are they of interest ? Where to find them?
  Our recent work

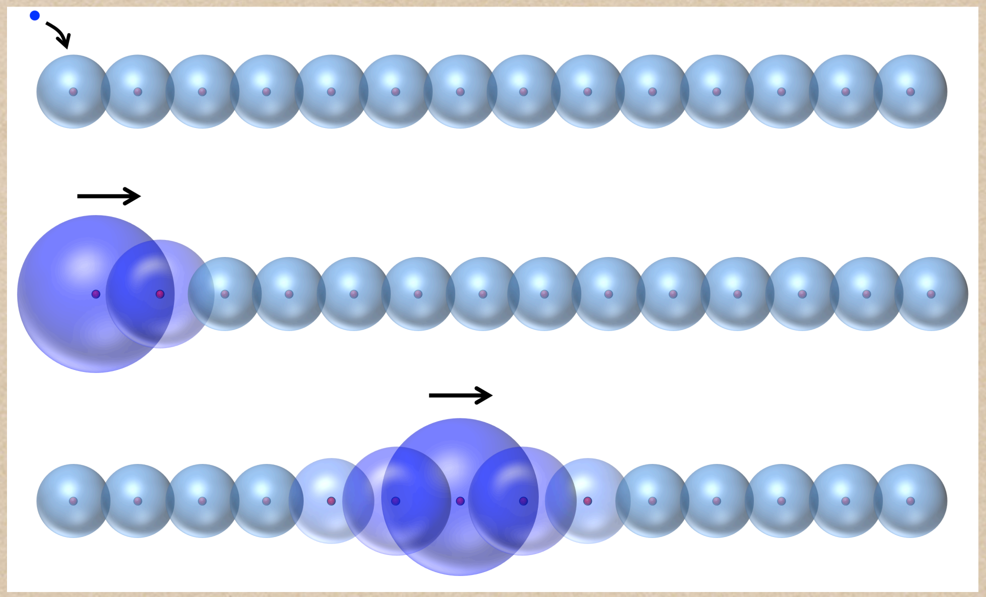
# Introduction

# Indistinguishable particles

- All elementary particles are either fermions or bosons. When identical particles are exchanged $\psi(\mathbf{r}_1, \mathbf{r}_2) = \pm\, \psi(\mathbf{r}_2, \mathbf{r}_1)$ with positive sign for bosons and negative sign for fermions

- But for emergent particles or quasiparticles in condensed matter systems , this is not necessary

# When we think of electrons flowing in a wire



- In terms of probability densities, injecting electron implies increasing probability density at one end and having that increase propagate

- MIRACLE - can think of the moving object as a particle - a new kind of `electron' - a quasiparticle or collective excitation which is made up of all the electrons.

- Still called `electron' because it behaves like a particle - it has a mass $m^*$ (different from original electron mass), but same spin and the same electric charge - behave as a single particle - Landau Fermi liquid theory

# Quasiparticles

- So anyons, parafermions,  can emerge as quasiparticles in condensed  matter systems
- Emergent excitations - from a soup of particles

# Abelian Anyons

- Unlike bosons and fermions, emergent particles can have complex phases under exchange

$$\psi(\mathbf{r}_1, \mathbf{r}_2) = e^{\pm i\theta}\psi(\mathbf{r}_2, \mathbf{r}_1)$$

- Anyons obey braid group statistics - different from permutation group because how one exchanges particles also important

1977

Leinaas, Myrrheim, Wilczek

# Braid group statistics

- Easiest to see pictorially as world lines in a space-time diagram

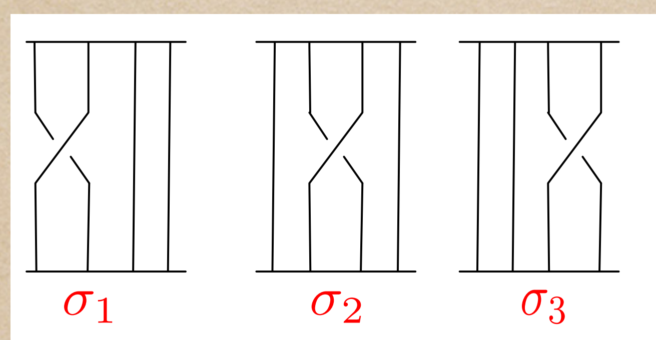



FIG. 7: The three generators of the braid group $B_4$

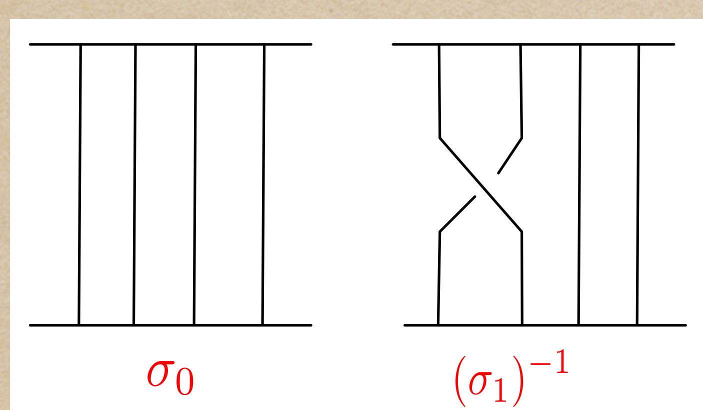
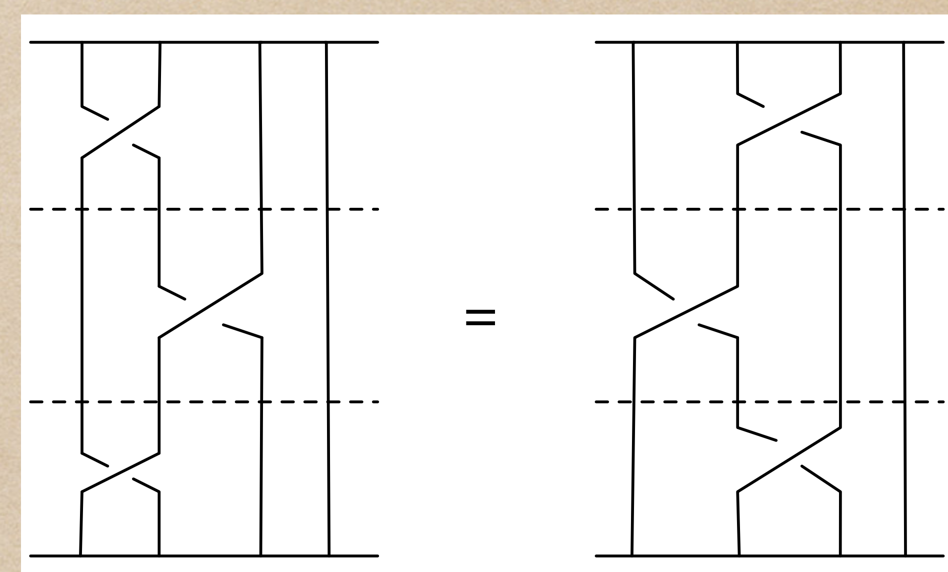
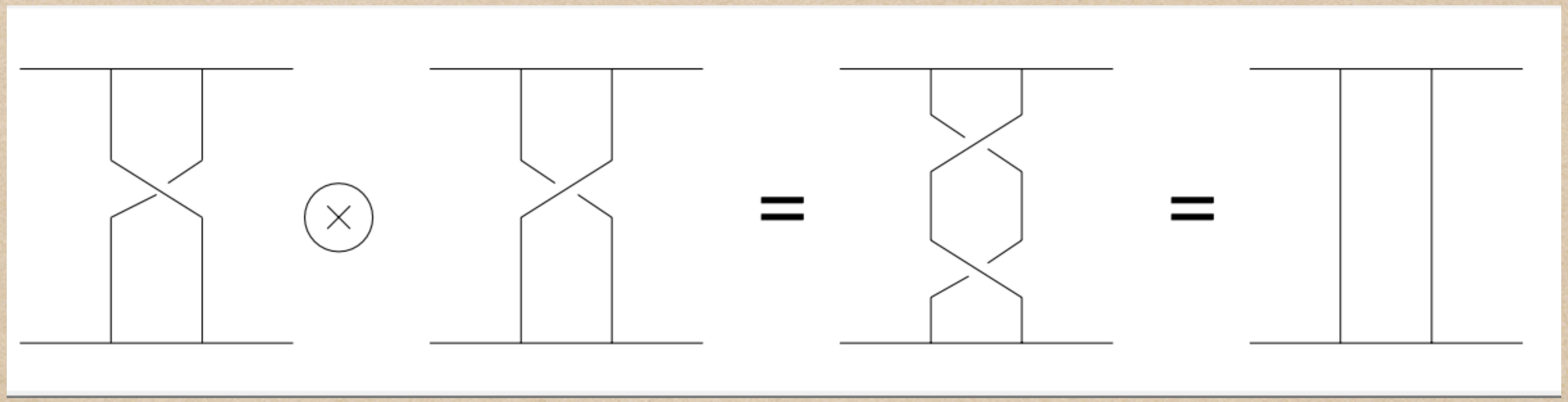
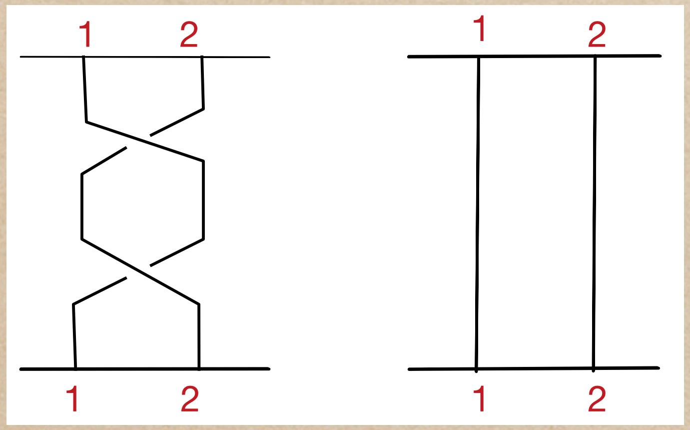



FIG. 8: The identity and the inverse of the generator $\sigma_1$.



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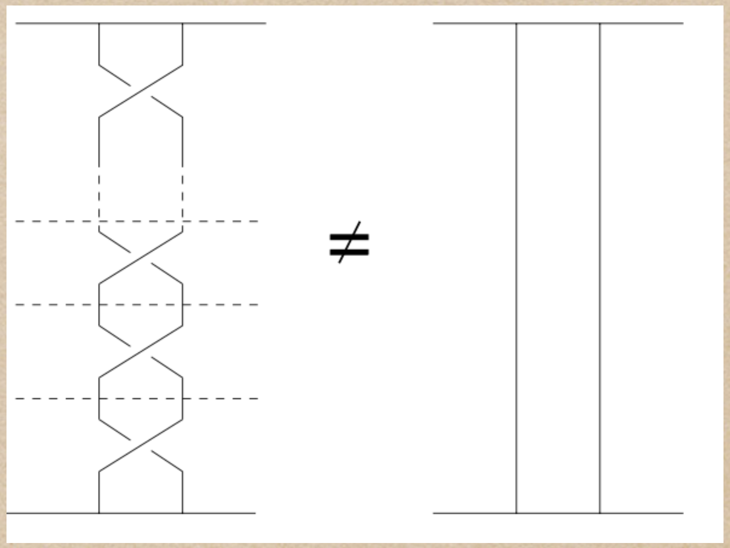


- Main point is that quantum statistics obeyed by anyons is very different from bosons or fermions
- Cannot just deal with symmetrized or anti-symmetrized wave functions for many particles
- Entire history is important - reason for why even a system of free anyons is very hard to solve

# Non-abelian anyons

- Anyons which acquire just a phase under exchange are abelian anyons
- Quasiparticles can also transform as non-abelian representations of the braid group
- $\psi_a \rightarrow U_{ab}\psi_b$ Frohlich, 1988

- Typically happens when there are degeneracies in the ground state - multiple states have same configuration of identical particles
- Prepare system in one ground state - exchange two quasi-particles - transformed by unitary transformation to another state in the ground state manifold
- Why  are such quasi-particles important?

# Relevance to quantum computing

- States are the qubits and the unitary transformations are the quantum gates that act on the qubits
- Intrinsically decoherence free because information is encoded non-locally

# Majorana modes

# $Z_2$ anyons /Ising anyons

# What are Majoranas?

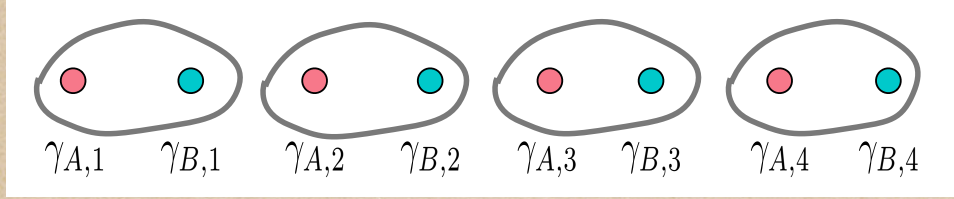


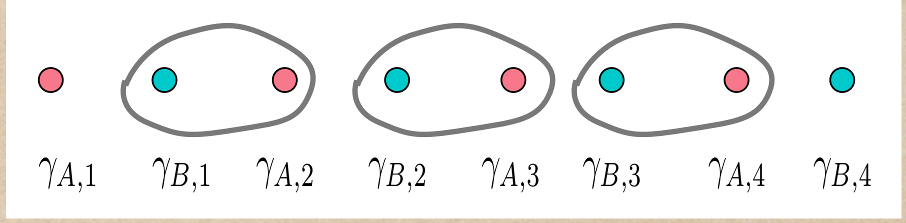





- Majorana modes or self-conjugate modes can exist as quasi-particles in condensed matter systems
- These are not fermions, but instead behave as non-abelian anyons
- Expect to be able to use them to make qubits for quantum computation

# Kitaev model 2001




- Essential idea is to think of the fermions on a site as being made up of 2 \`Majorana modes' and changing parameters in the Hamiltonian so that these two modes get separated.

$$H = -\mu \sum_{x=1}^{N} c_x^\dagger c_x - \frac{1}{2} \sum_{x=1}^{N} (t c_x^\dagger c_{x+1} + \Delta c_x c_{x+1} + h.c.)$$

- Spinless fermions with $\mu$ = chemical potential, t = hopping and $\Delta$ = pairing potential
- Can rewrite in terms of Majorana modes $\gamma_a, \gamma_b$

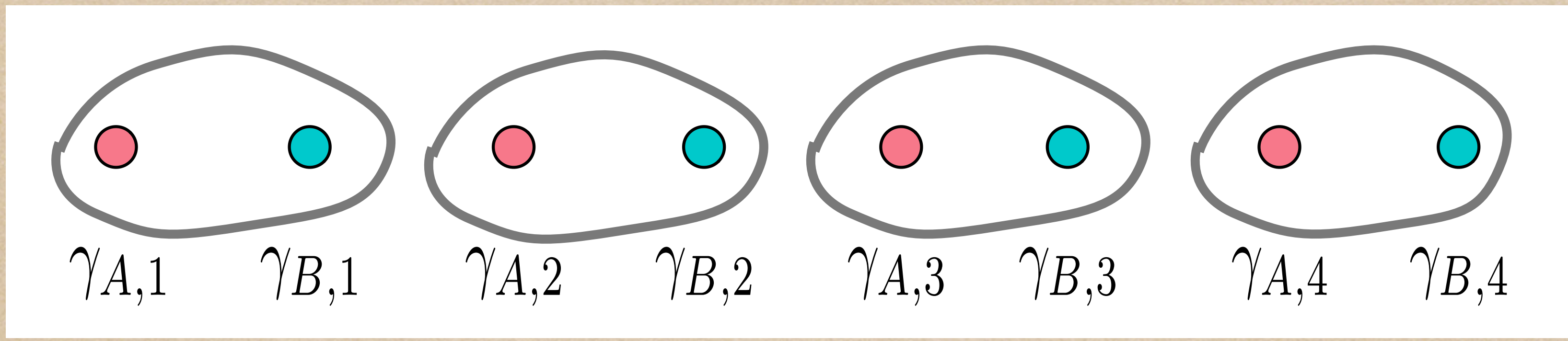
$$c_x = \frac{1}{2}(\gamma_{x,A} + i\gamma_{x,B}), \quad c_x^\dagger = \frac{1}{2}(\gamma_{x,A} - i\gamma_{x,B})$$

$$\{\gamma_a, \gamma_b\} = \delta_{ab}, \quad \gamma_a^2 = 1, \gamma_b^2 = 1 \qquad \gamma^\dagger = \gamma$$

$$\gamma_{x,A} = (c_x + c_x^\dagger) \qquad \gamma_{x,B} = -i(c_x - c_x^\dagger)$$

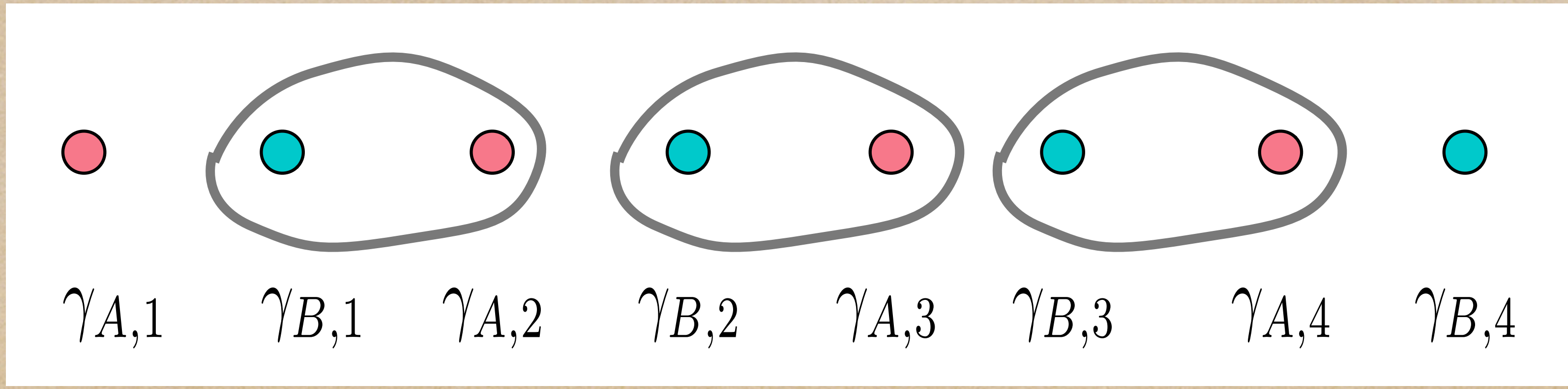
- When $\mu < 0$ and $t = \Delta = 0$, only bonds between Majoranas at same site

$$H = \frac{-\mu}{2} \sum_{x=1}^{N} (1 + i\gamma_{x,B}\gamma_{x,A})$$




- When $\mu = 0,\ t = \Delta$, unpaired Majoranas at the two ends

$$H = -\frac{it}{2} \sum_{x=1}^{N-1} \gamma_{x,B}\gamma_{x+1,A}$$

- 

- Essential point, Hamiltonian has no dependence on end Majoranas and is independent of whether or not the non-local fermion formed from them is occupied or not occupied - ground state is not unique, it is doubly degenerate

- Range of parameters for the topological phase, not just the points where we have solved it

- Kitaev's trick was to fractionalize the fermion and put different pieces of them at the two ends of the chain so that they behave as independent quasi-particles - Majorana modes
- No energy is required to occupy this non-local fermion state made of the Majorana modes - so occupied and unoccupied states are degenerate
- Occupation of state cannot be changed by local fluctuations  at one end of chain - so decoherence free

# Relevance to quantum computing

- Kitaev's proposal - degenerate ground state ( non-local fermion state filled or unfilled) acts as topological memory - cannot be easily disturbed by local errors
- Exchanging Majoranas implemented by unitary rotations or gates

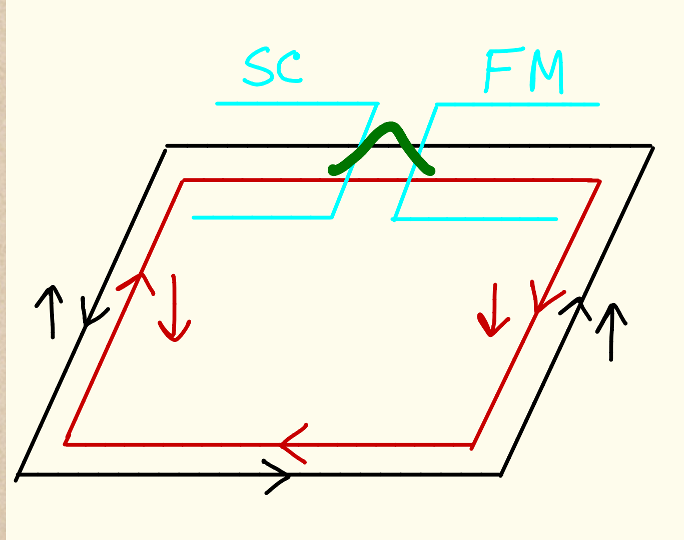
$$\begin{pmatrix} \gamma_i \\ \gamma_j \end{pmatrix} = U \begin{pmatrix} \gamma_j \\ \gamma_i \end{pmatrix}$$

- Only braiding properties of Majoranas important and not local nature of paths
- Hence topologically protected from decoherence and noise - TOPOLOGICAL quantum computation

# How to find Majoranas

- Majorana does not conserve particle number - so a combination of electron and hole
- Need superconductivity, but usual superconductors have electron and hole states with opposite spin
- Need to get rid of one spin species - need effectively spinless or p-wave superconductivity

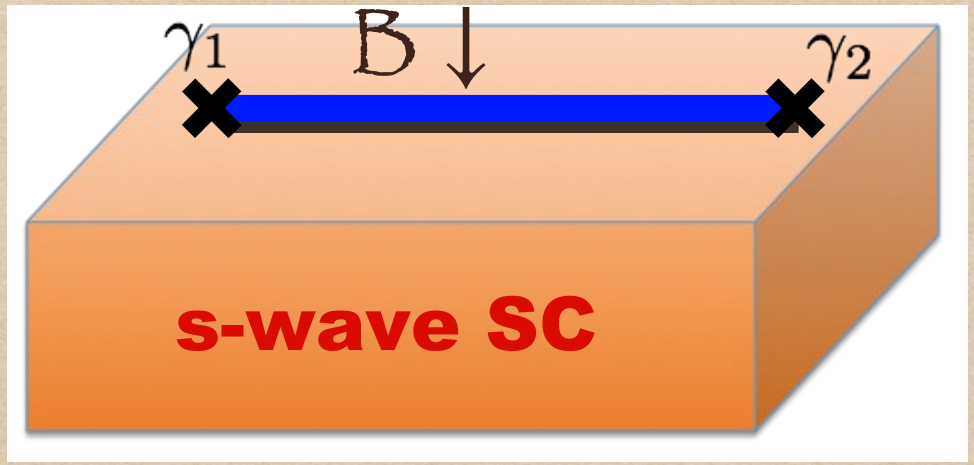
# Engineering Majoranas




- Fu and Kane (quantum spin Hall insulator edges) - essential point is that spin is coupled to the momentum. So effectively spinless fermions
- To localise the end Majoranas, need another gap provided by ferromagnet insulator

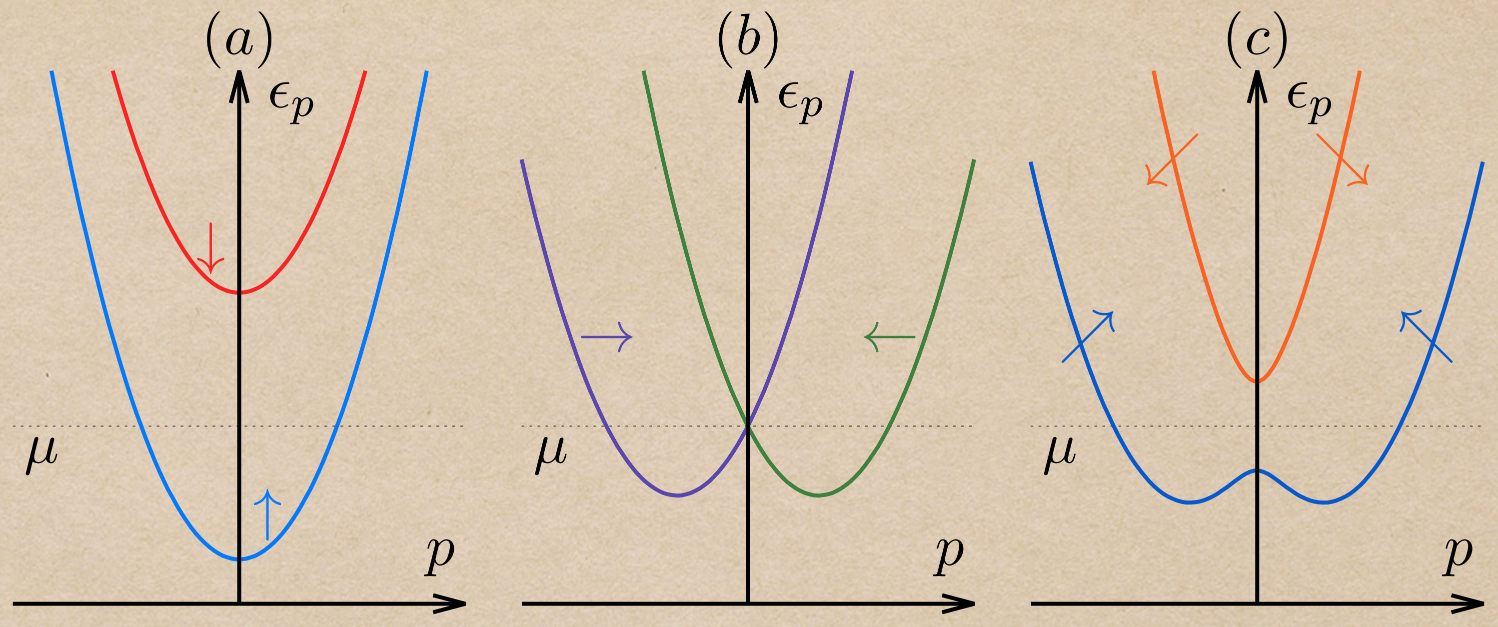
Fu, Kane, 2008

# Semiconductor wires




- Semiconductor wires with spin orbit coupling and $B$ and coupled to s-wave superconductor

$$H = \int dx \psi^\dagger [-\frac{\partial_x^2}{2m} - \mu - ui\hbar\partial_x \sigma_y - \frac{g\mu_b B}{2}\sigma_z]\psi + \Delta[\psi_\uparrow \psi_\downarrow + h.c.]$$




- Engineered to mimic the Kitaev model, so expect to have a Majorana bound state at the edge of the topological superconductor

Oreg,Refeal and von Oppen, 2010

Lutchyn,Sau and Das Sarma, 2010

# Quest for Majoranas

- Many experiments have tried to look for signals of the MBS

**Zero-bias peaks and splitting in an Al-InAs nanowire topological superconductor as a signature of Majorana fermions**

Anindya Das[†], Yuval Ronen[†], Yonatan Most, Yuval Oreg, Moty Heiblum[★] and Hadas Shtrikman

**Signatures of Majorana Fermions in Hybrid Superconductor-Semiconductor Nanowire Devices**

V. Mourik,[1*] K. Zuo,[1*] S. M. Frolov,[1] S. R. Plissard,[2] E. P. A. M. Bakkers,[1,2] L. P. Kouwenhoven[1†]

**Spin-resolved Andreev levels and parity crossings in hybrid superconductor–semiconductor nanostructures**

Eduardo J. H. Lee[1], Xiaocheng Jiang[2], Manuel Houzet[1], Ramón Aguado[3], Charles M. Lieber[2] and Silvano De Franceschi[1★]

**Parity lifetime of bound states in a proximitized semiconductor nanowire**

A. P. Higginbotham[1,2†], S. M. Albrecht[1†], G. Kiršanskas[1], W. Chang[1,2], F. Kuemmeth[1], P. Krogstrup[1], T. S. Jespersen[1], J. Nygård[1,3], K. Flensberg[1] and C. M. Marcus[1★]

**Anomalous Zero-Bias Conductance Peak in a Nb–InSb Nanowire–Nb Hybrid Device**

M. T. Deng,[†] C. L. Yu,[†] G. Y. Huang,[†] M. Larsson,[†] P. Caroff,[‡] and H. Q. Xu[†,§,*]

# Consensus?

## Quantized Majorana conductance

Hao Zhang[1]*, Chun-Xiao Liu[2]*, Sasa Gazibegovic[3]*, Di Xu[1], John A. Logan[4], Guanzhong Wang[1], Nick van Loo[1], Jouri D. S. Bommer[1], Michiel W. A. de Moor[1], Diana Car[3], Roy L. M. Op het Veld[3], Petrus J. van Veldhoven[3], Sebastian Koelling[3], Marcel A. Verheijen[3,5], Mihir Pendharkar[6], Daniel J. Pennachio[4], Borzoyeh Shojaei[4,7], Joon Sue Lee[7], Chris J. Palmstrøm[4,6,7], Erik P. A. M. Bakkers[3], S. Das Sarma[2] & Leo P. Kouwenhoven[1,8]



**Majorana zero-modes—a type of localized quasiparticle—hold great promise for topological quantum computing[1]. Tunnelling spectroscopy in electrical transport is the primary tool for identifying the presence of Majorana zero-modes, for instance as a zero-bias peak in differential conductance[2]. The height of the Majorana zero-bias peak is predicted to be quantized at the universal conductance value of $2e^2/h$ at zero temperature[3] (where $e$ is the charge of an electron and $h$ is the Planck constant), as a direct consequence of the famous Majorana symmetry in which a particle is its own antiparticle. The Majorana symmetry protects the quantization against disorder, interactions and variations in the tunnel coupling[3–5]. Previous experiments[6], however, have mostly shown zero-bias peaks much smaller than $2e^2/h$, with a recent observation[7] of a peak height close to $2e^2/h$. Here we report a quantized conductance plateau at $2e^2/h$ in the zero-bias conductance measured in indium antimonide semiconductor nanowires covered with an aluminium superconducting shell. The height of our zero-bias peak remains constant despite changing parameters such as the magnetic field and tunnel coupling, indicating that it is a quantized conductance plateau. We distinguish this quantized Majorana peak from possible non-Majorana origins by investigating its robustness to electric and magnetic fields as well as its temperature dependence. The observation of a quantized conductance plateau strongly supports the existence of Majorana zero-modes in the system, consequently paving the way for future braiding experiments that could lead to topological quantum computing.**

A semiconductor nanowire coupled to a superconductor can be tuned into a topological superconductor with two Majorana zero-modes localized at the wire ends[1,8,9]. Tunnelling into a Majorana mode will show a zero-energy state in the tunnelling density-of-states, that is, a zero-bias peak (ZBP) in the differential conductance ($dI/dV$)[2,6]. This tunnelling process is an 'Andreev reflection', in which an incoming electron is reflected as a hole. Particle–hole symmetry dictates that the zero-energy tunnelling amplitudes of electrons and holes are equal, resulting in a perfect resonant transmission with a ZBP height quantized at $2e^2/h$ (refs 3, 4, 10), irrespective of the precise tunnelling strength[3–5]. The Majorana nature of this perfect Andreev reflection is a direct result of the well-known Majorana symmetry property 'particle equals antiparticle'[11,12].

This predicted robust conductance quantization has not yet been observed[6,7,13,14]. Instead, most of the ZBPs have a height considerably less than $2e^2/h$. This discrepancy was first explained by thermal averaging[15–18], but that explanation does not hold when the peak width exceeds the thermal broadening (about $3.5k_BT$)[13,14]. In that case, other averaging mechanisms, such as dissipation[19], have been invoked. The main source of dissipation is a finite quasiparticle density-of-states within the superconducting gap, often referred to as a 'soft gap'. Substantial advances have been achieved in 'hardening' the gap by improving the quality of materials, eliminating disorder and interface roughness[20,21], and better control during device processing[22,23], all guided by a more detailed theoretical understanding[24]. We have recently solved all these dissipation and disorder issues[21], and here we report the resulting improvements in electrical transport leading to the elusive quantization of the Majorana ZBP.

Figure 1a shows a micrograph of a fabricated device and schematics of the measurement set-up. An InSb nanowire (grey) is partially covered (two out of six facets) by a thin superconducting aluminium shell (green)[21]. The 'tunnel-gates' (coral red) are used to induce a tunnel barrier in the non-covered segment between the left electrical contact (yellow) and the Al shell. The right contact is used to drain the current to ground. The chemical potential in the segment covered with Al can be tuned by applying voltages to the two long 'super-gates' (purple).

Transport spectroscopy is shown in Fig. 1b, which displays $dI/dV$ as a function of voltage bias $V$ and magnetic field $B$ (aligned with the nanowire axis), while fixed voltages are applied to the tunnel- and super-gates. As $B$ increases, two levels detach from the gap edge (at about 0.2 meV), merge at zero bias and form a robust ZBP. This is consistent with the Majorana theory: a ZBP is formed after the Zeeman energy closes the trivial superconducting gap and re-opens a topological gap[8,9]. The gap re-opening is not visible in a measurement of the local density-of-states because the tunnel coupling to these bulk states is small[25]. Moreover, the finite length (about 1.2 μm) of the proximitized segment (that is, the part that is superconducting because of the proximity effect from the superconducting Al coating) results in discrete energy states, turning the trivial-to-topological phase transition into a smooth crossover[26]. Figure 1c shows two line-cuts from Fig. 1b extracted at 0 T and 0.88 T. Importantly, the height of the ZBP reaches the quantized value of $2e^2/h$. The line-cut at zero bias in the lower panel of Fig. 1b shows that the ZBP height remains close to $2e^2/h$ over a sizable range in $B$ field (0.75–0.92 T). Beyond this range, the height drops, most probably because of a closure of the superconducting gap in the bulk Al shell.

We note that the sub-gap conductance at $B = 0$ (black curve, left panel, Fig. 1c) is not completely suppressed down to zero, reminiscent of a soft gap. In this case, however, this finite sub-gap conductance does not reflect any finite sub-gap density-of-states in the proximitized wire. It arises from Andreev reflection (that is, transport by dissipationless Cooper pairs) due to a high tunnelling transmission, which is evident from the above-gap conductance ($dI/dV$ for $V > 0.2$ mV) being larger than $e^2/h$. As this softness does not result from dissipation, the Majorana peak height should still reach the quantized value[27]. In Extended Data Fig. 1, we show that this device tuned into a low-transmission regime,

- Also a proposal and experiment to realize a single chiral Majorana mode at the edge of a quantum anomalous Hall system through proximity effect with an s-wave superconductor

Wang,Zhou,Lian,Zhang, 2015

Lin He et al, 2017

# Chiral Majorana fermion modes in a quantum anomalous Hall insulator–superconductor structure

Qing Lin He,[1*†] Lei Pan,[1†] Alexander L. Stern,[3] Edward C. Burks,[4] Xiaoyu Che,[1] Gen Yin,[1] Jing Wang,[5,6] Biao Lian,[6] Quan Zhou,[6] Eun Sang Choi,[7] Koichi Murata,[1] Xufeng Kou,[1,8*] Zhijie Chen,[4] Tianxiao Nie,[1] Qiming Shao,[1] Yabin Fan,[1] Shou-Cheng Zhang,[6*] Kai Liu,[4] Jing Xia,[3] Kang L. Wang[1,2*]

2017

# Editorial Expression of Concern

2021

On 21 July 2017, *Science* published the Report "Chiral Majorana fermion modes in a quantum anomalous Hall insulator—superconductor structure" by Q. L. He *et al.* (*1*). Since that time, raw data files were offered by the authors in response to queries from readers who had failed to reproduce the findings. Those data files did not clarify the underlying issues, and now their provenance has come into question. While the authors' institutions investigate further, we are alerting readers to these concerns.

**H. Holden Thorp**
Editor-in-Chief

- Peaks could be due to disorder
- Essential point, zero bias peaks necessary but not sufficient condition to establish Majoranas

Pan and Das Sarma, 2020

Zhang et al, 2101.11456

- No smoking gun evidence yet

- But fair to say  Majoranas not ruled out even in these platforms
- Further experiments may give better results

# Other platforms

- Historically, first suggestion for possible Majorana excitations was in $\nu$ =5/2 FQHE

Moore-Read state, 1991

- e/4 excitations could be shown to have non-abelian braiding statistics

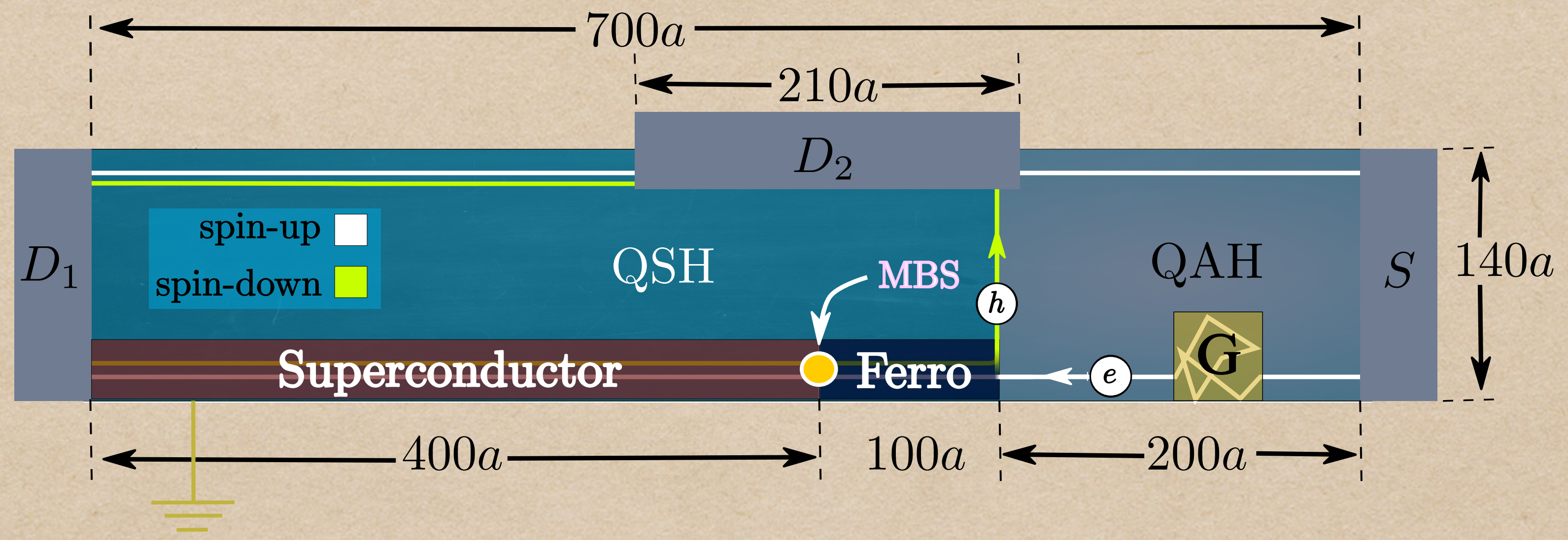
- 2D spinless p-wave superconductor with vortices forming Majorana excitations

Read -Green, 2000

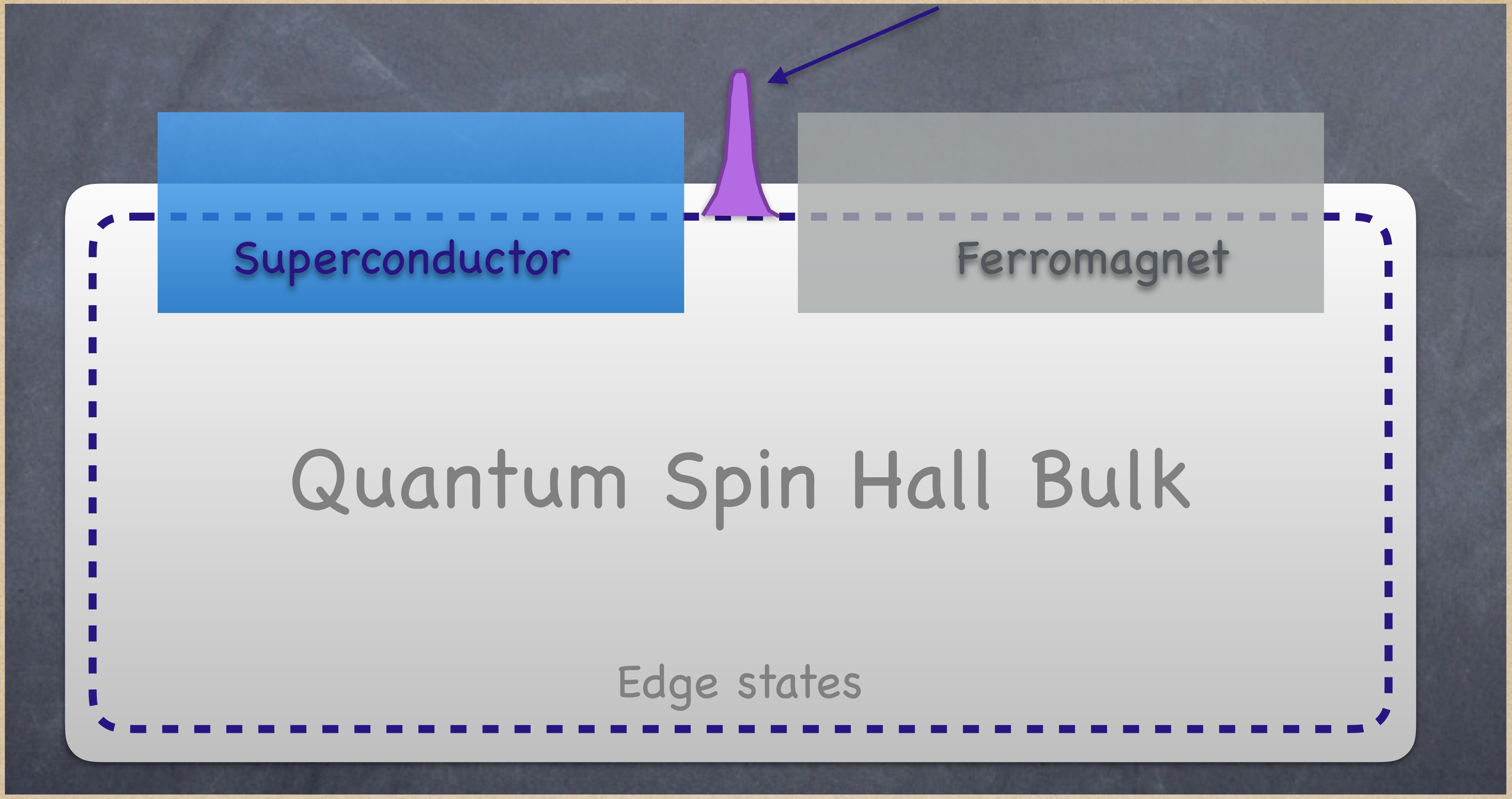
Room for more proposals

Perhaps 2D platforms more desirable

# Our recent work




- Chiral injection using a QAH edge to detect Majorana bound state at the edge of a quantum spin Hall insulator - separates incoming electron channel from outgoing hole channel




- Well-known that MBS can be trapped at the edge of a QSH bulk by proxmitising to superconductor and ferromagnet

- Helical edge state of quantum spin Hall insulator gapped out by proximity to s-wave superconductor on one side and a Zeeman field which can be applied parallel or perpendicular to spin quantization axis ( taken to be out of plane)

$$H = (v_F p_x \sigma_z - \mu)\eta_z + \Delta\eta_x + g_\perp \sigma_x + g_\parallel \sigma_z$$

- $g_\perp$ (mass term) hybridises the modes and opens a mass gap whereas $g_\parallel$ changes energy of right and left mover without mixing them

$$\mathcal{H} = (v_F \hat{p}_x \sigma_z - \mu)\tau_z + \overbrace{g_1(x)\sigma_z + g_2(x)\sigma_x}^{\text{Boost} \qquad \text{Mass gap}} + \underbrace{\Delta(x)\tau_x}_{\text{BdG gap}} + \overbrace{M(x)\sigma_x}^{\text{Barrier}}$$

$g_1 = 0$

$$\Delta_p(k) = \frac{\hbar v_F\, k\, \Delta_0}{\sqrt{(\hbar v_F k)^2 + g_2^2}}$$

$$\Delta_s(k) = \frac{g_2\, \Delta_0}{\sqrt{(\hbar v_F k)^2 + g_2^2}}$$

Topological : $g_2 < \sqrt{\Delta_0^2 + \mu^2}$

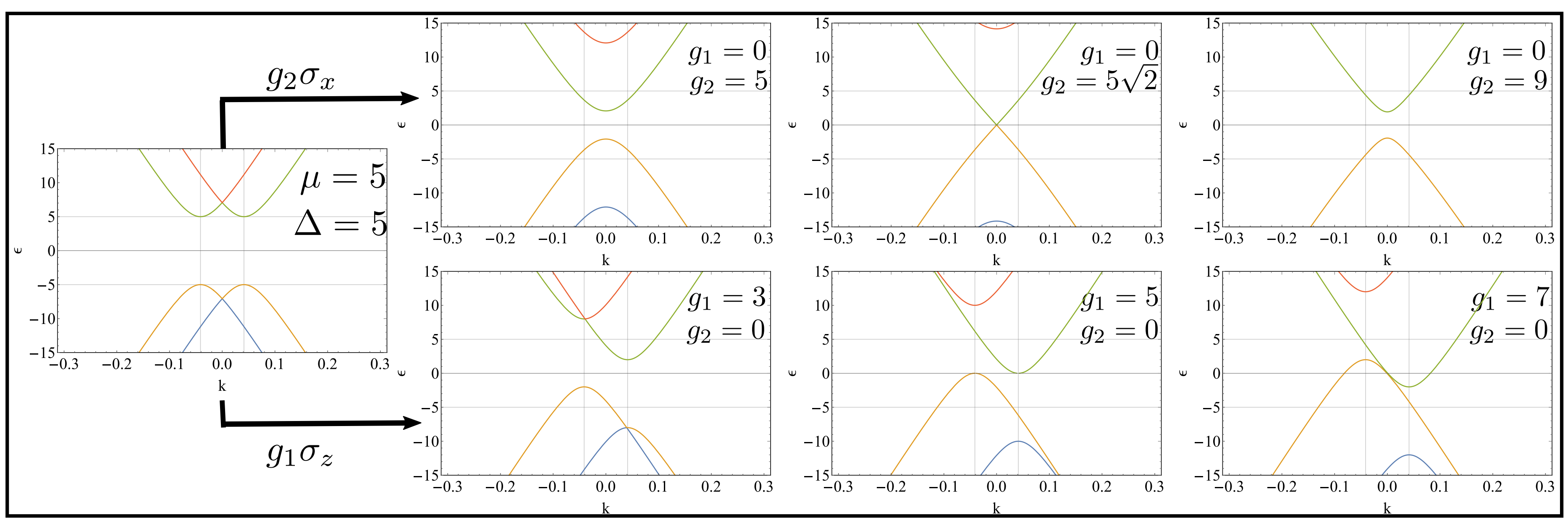
Non-Topological : $g_2 > \sqrt{\Delta_0^2 + \mu^2}$

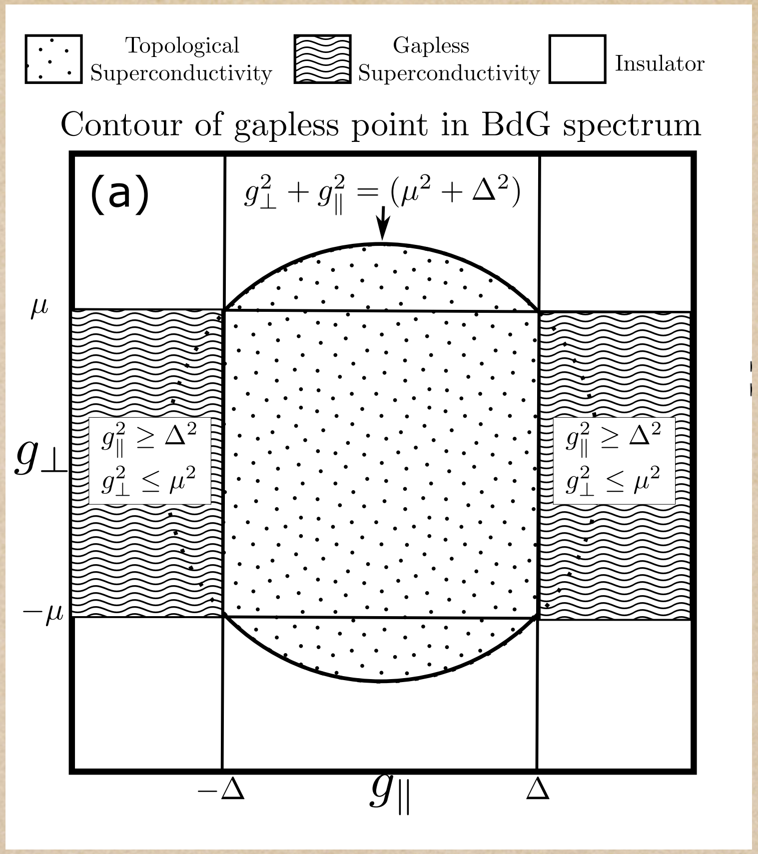
$g_2 = 0$

Topological : $g_1 < \Delta_0$

Non-Topological : $g_1 > \Delta_0$

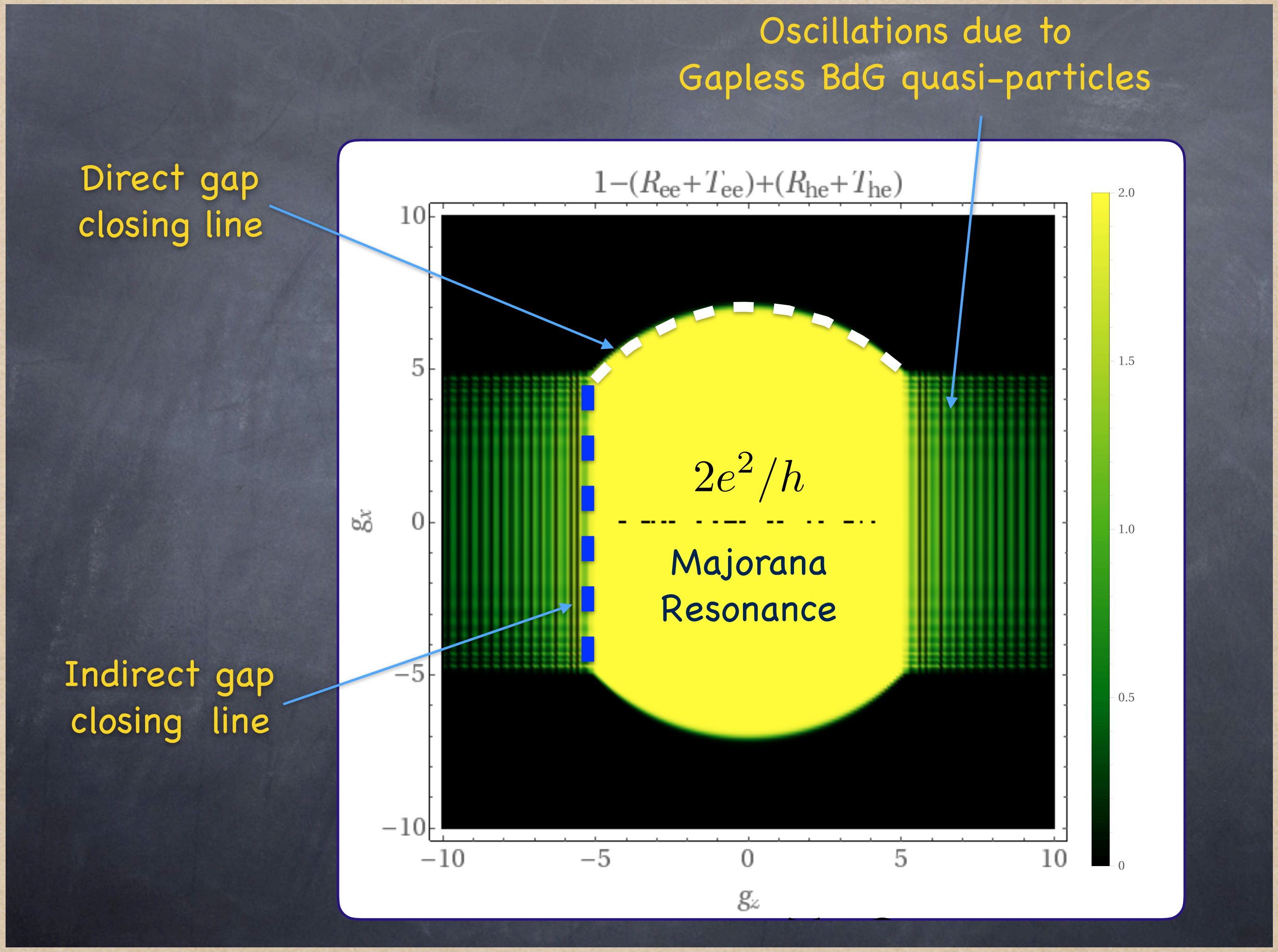
BdG






- Obtain phase diagram as function of $g_\perp$ and $g_\parallel$ using the BdG spectrum
- Gapped topological superconductor for the dotted region
- gapless superconductor for curved lines region

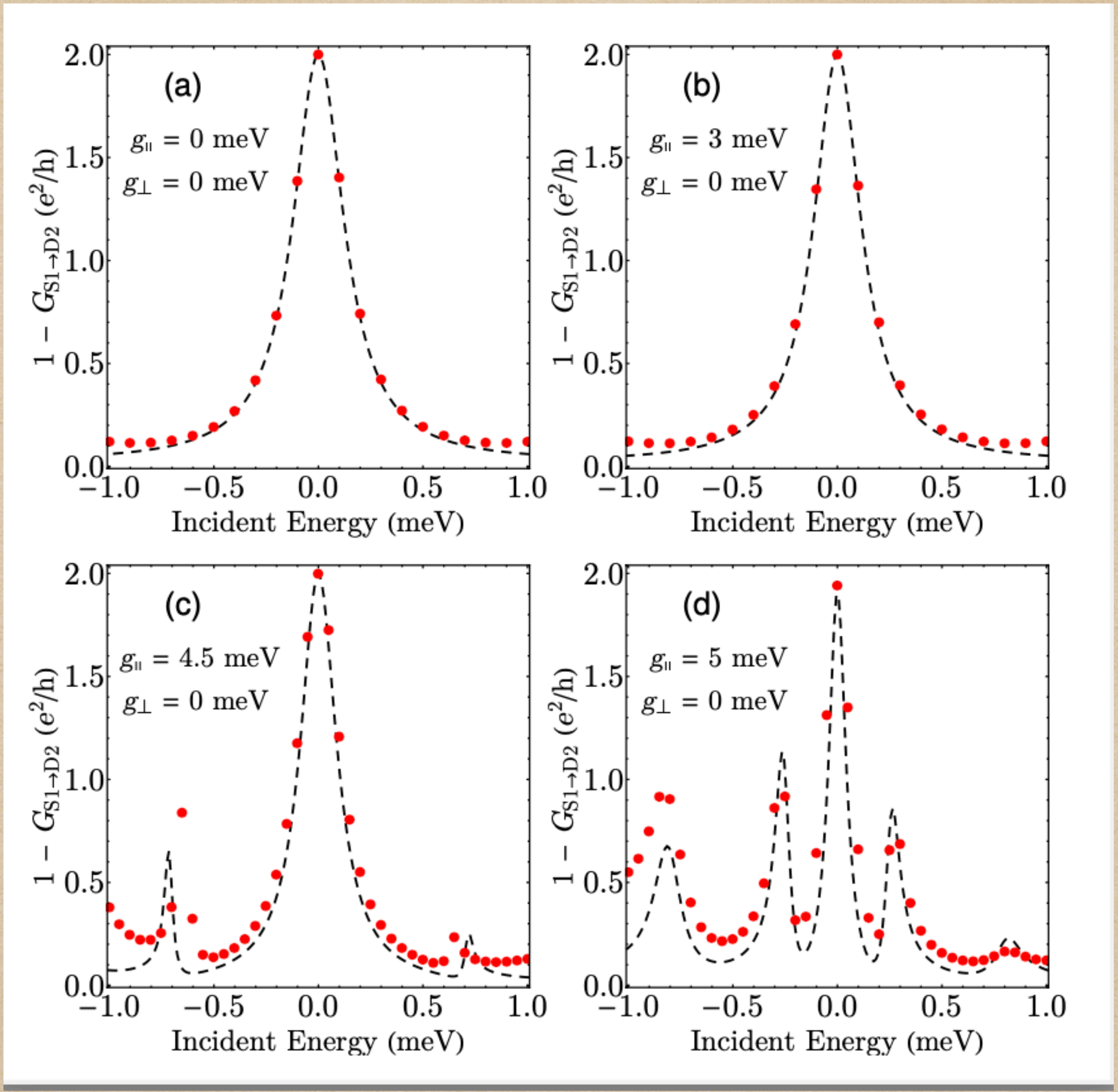
# Conductance in units of $2e^2/h$



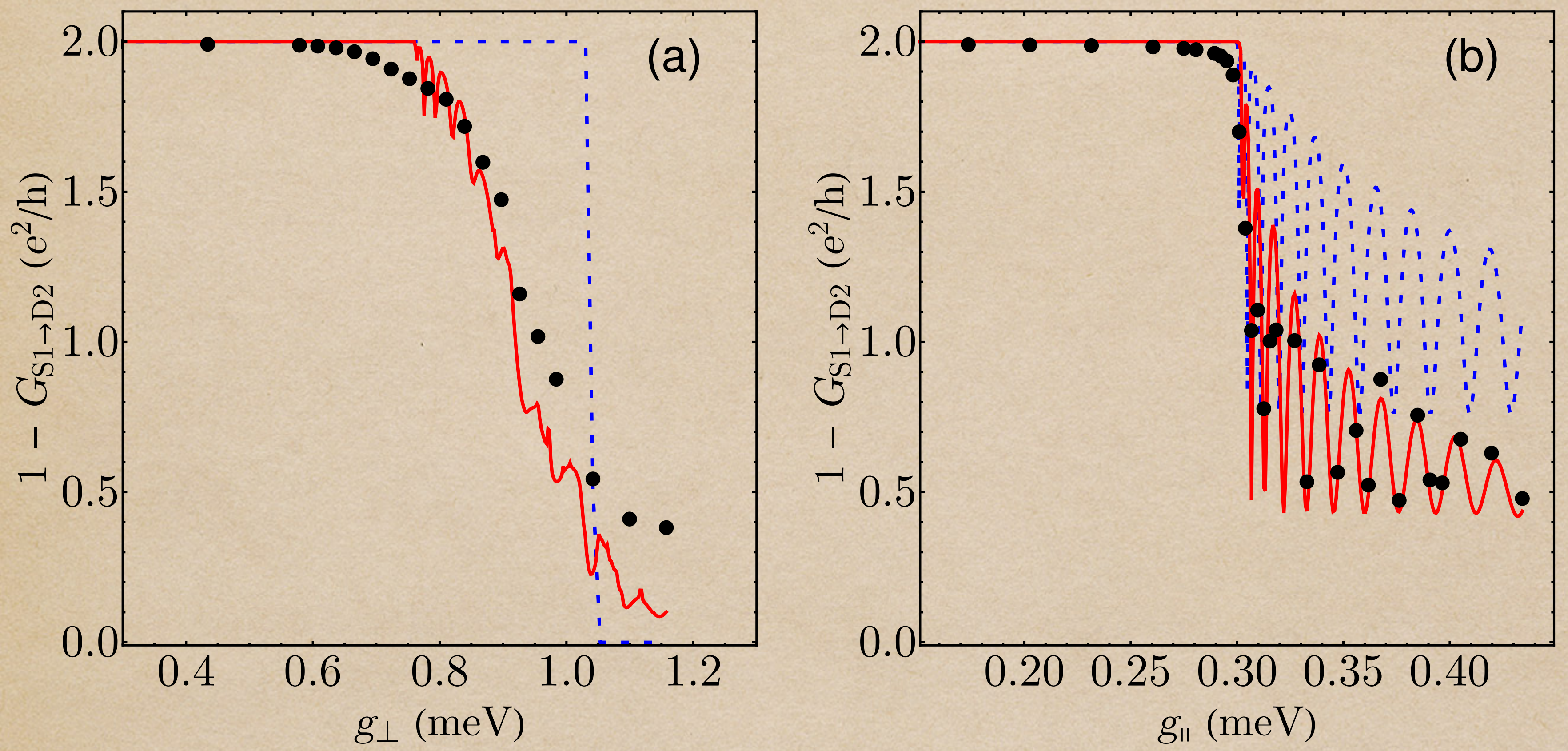

$g_{\perp}$

$g_{\parallel}$

- Phase diagram from the zero energy bias conductance scan
- Transport phase diagram matches BdG spectrum



(a)
$g_{\parallel}$ = 0 meV
$g_{\perp}$ = 0 meV
(b)
$g_{\parallel}$ = 3 meV
$g_{\perp}$ = 0 meV
(c)
$g_{\parallel}$ = 4.5 meV
$g_{\perp}$ = 0 meV
(d)
$g_{\parallel}$ = 5 meV
$g_{\perp}$ = 0 meV
$1 - G_{S1\to D2}$ ($e^2$/h)
Incident Energy (meV)



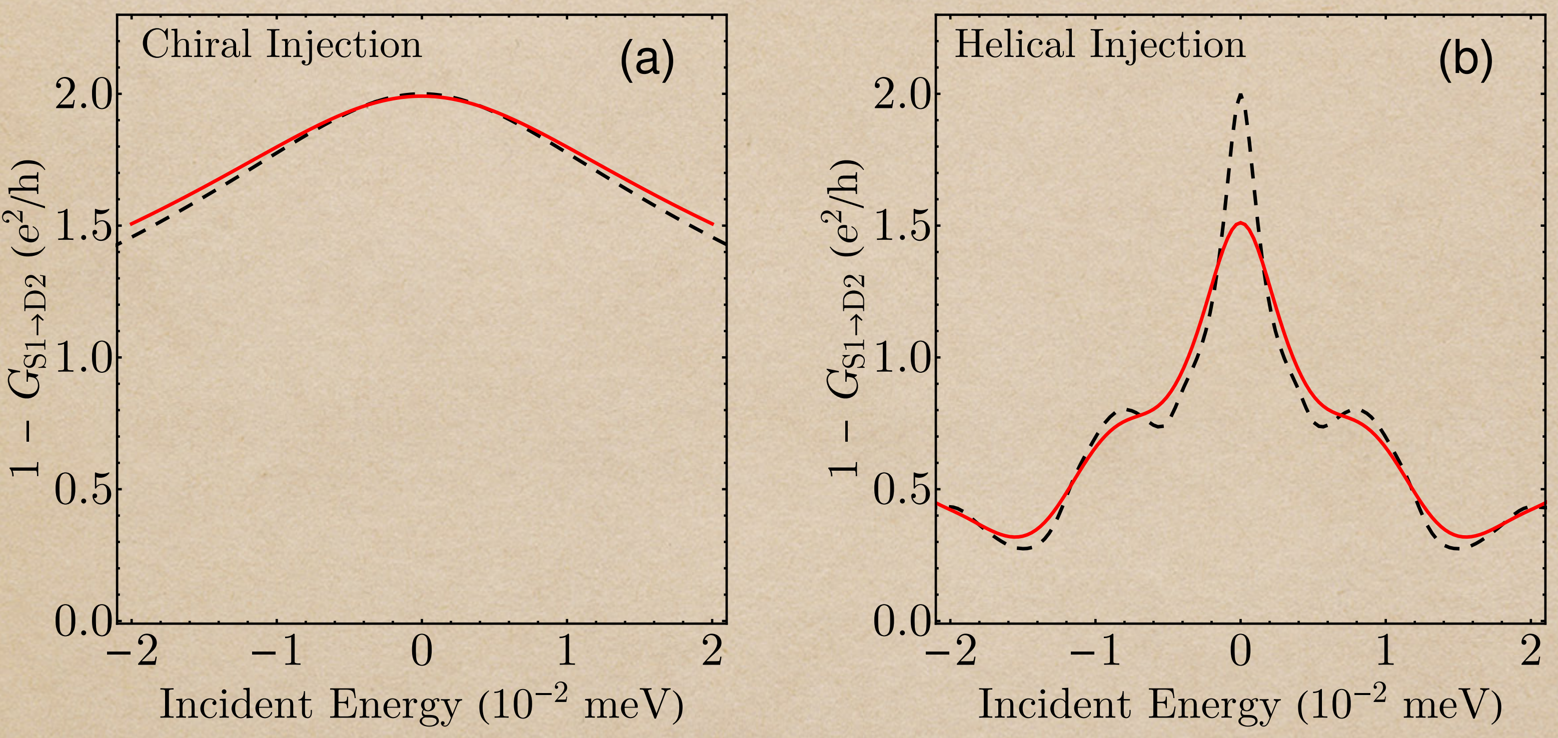

*Blue* $T = 0$

*Red, disorder averaged*

*Black,* $T = 12mK$ *disorder averaged*

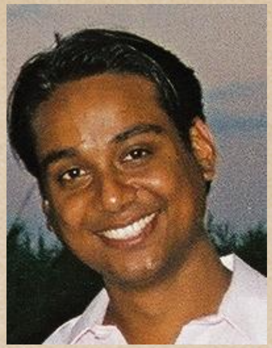
- Can see that transition as a function of $g_\parallel$ remains sharp at finite temperatures as compared to transition as a function of $g_\perp$

- Important point by using QAH insulator edge as a source, we can separate incoming and outgoing current, so impervious to disorder



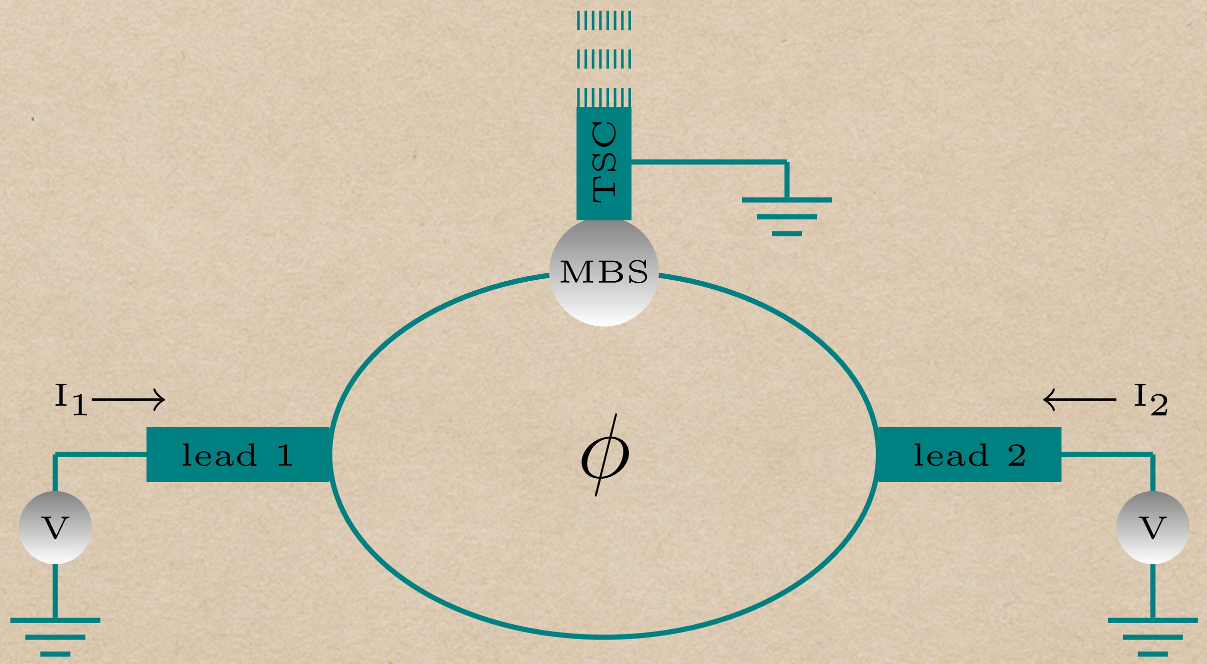

$$T = 0(black)$$
$$T = 12mK(red)$$

- Disorder averaged comparison at zero and finite temperatures - chiral injection retains height of peak at finite temperatures much better

- Main point is there is a much bigger phase space in the 2 D platform where there is a robust possibility for seeing the Majorana
- Ref : Chiral detection of Majorana bound states at the edge of a quantum spin Hall insulator - Vivekananda Adak, Aabir Mukhopadhyay, Suman Jyoti De, Udit Khanna, S. R and Sourin Das cond-mat/2106.04596, PRB 106, 045422 (2022).

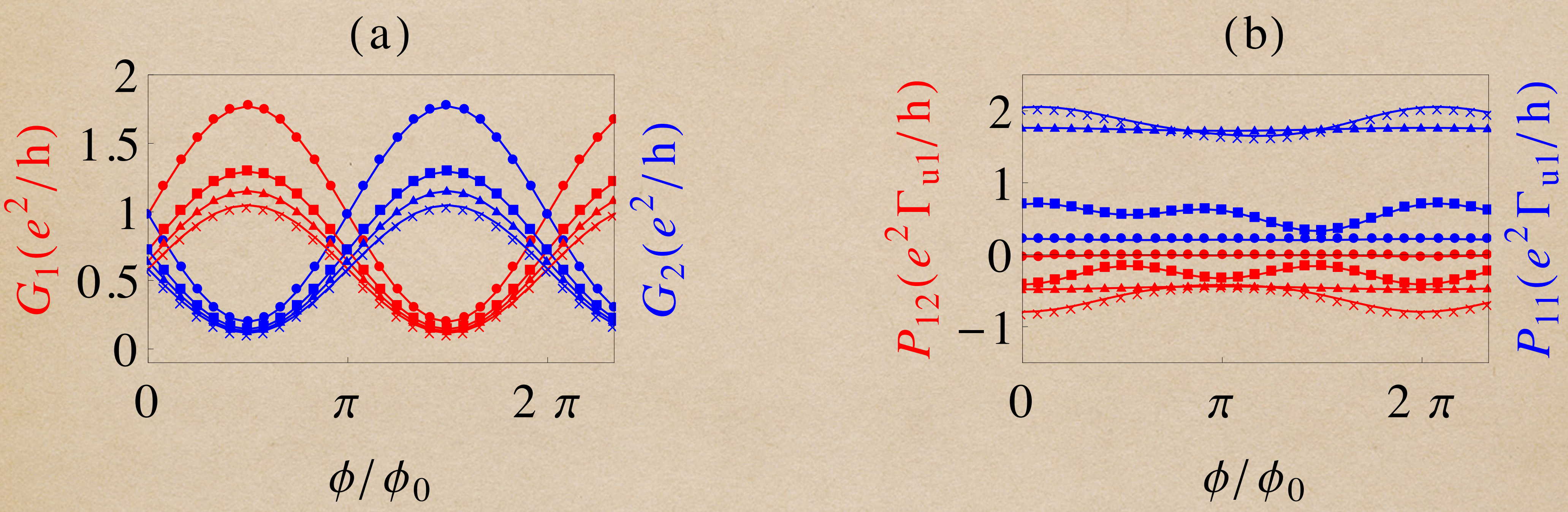
# Fingerprints of Majorana bound states in Aharanov-Bohm geometry




- Ref: K. M. Tripathi, Sourin Das and S.R, Phys. Rev. Lett. 116, 166401 (2016).

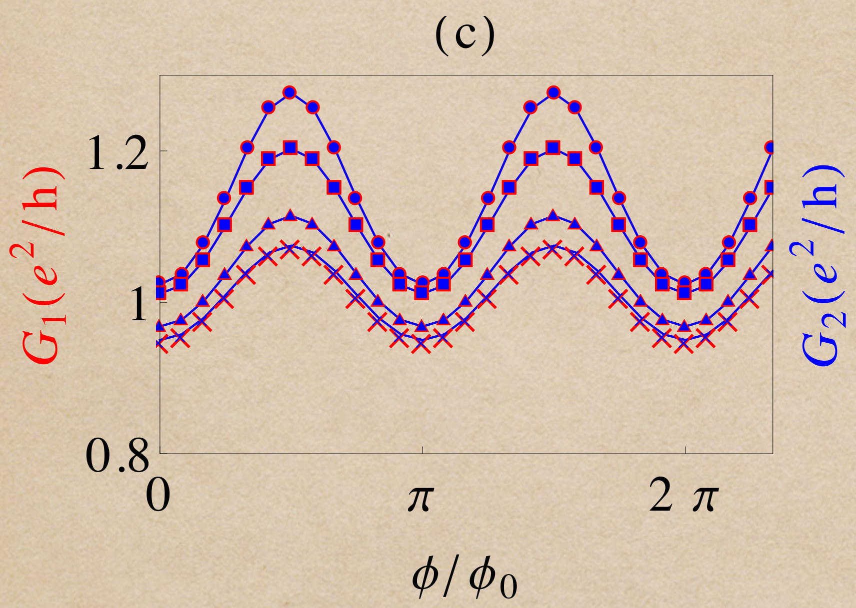
- Important point is that this geometry also, the conductance is measured not only as a function of the voltages, but also as a function of the flux through the ring
- Gives clear contrast between MBS and other spurious ABS

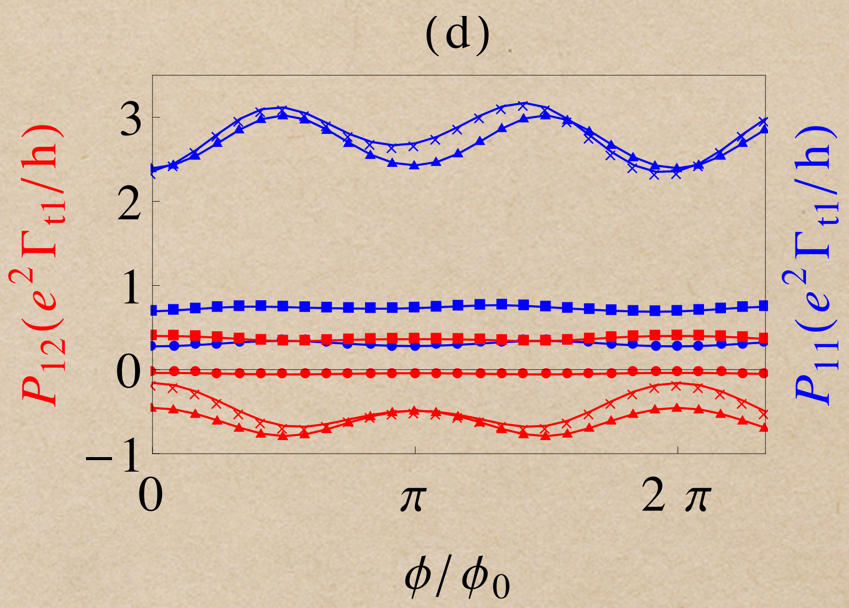
# Majorana bound state




- Total conductance is $e^2/h$, but anti-correlated on both wires
- Noise cross correlations negative

# Andreev bound state




- Equal currents on both wires with maximum of $2e^2/h$ for conductance
- Noise cross correlations can be positive or negative

- Important point, in both cases, MBS is not a single measurement, but exists in a range
- Signal is robust and harder to mimic by non MBS sources
- But still at one end of the Topological SC
- Perhaps true test is signals of MBS at both ends

# Parafermions

# More exotic anyons

- Motivation - Braiding of Majoranas cannot lead to universal quantum computation - it does not allow for all possible unitary operations
- Can one engineer universal topological quantum computer?

# Parafermions

*P. Fendley*, 2012

- Generalisations of the Majorana modes, which are $Z_2$ anyons or Ising anyons to $Z_N$ anyons
- 1D quantum clock model with flip $\tau$ and shift $\sigma$ operators
- $$H = -J\sum_{j=1}^{L-1}(\sigma_j^\dagger\sigma_{j+1} + h.c) - h\sum_{j=1}^{L}(\tau_j^\dagger + \tau_j)$$
- $$\sigma_j^N = 1,\ \tau_j^N = 1,\ \sigma_j\tau_j = \tau_j\sigma_j e^{2\pi i/N}$$

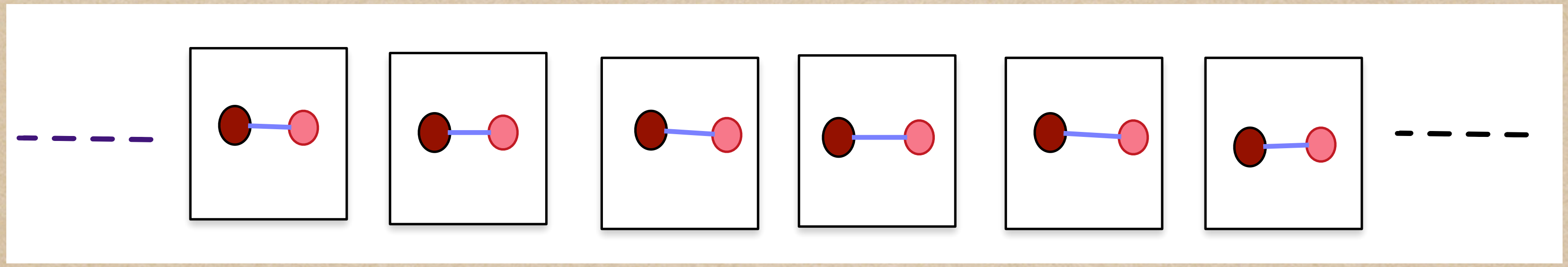
- Jordan-Wigner transformation to rewrite in terms of `parafermion' operators

$$\alpha_{2j-1} = \sigma_j \Pi_{i<j} \tau_i, \quad \alpha_{2j} = -e^{i\pi/N} \tau_j \sigma_j \Pi_{i<j} \tau_i$$

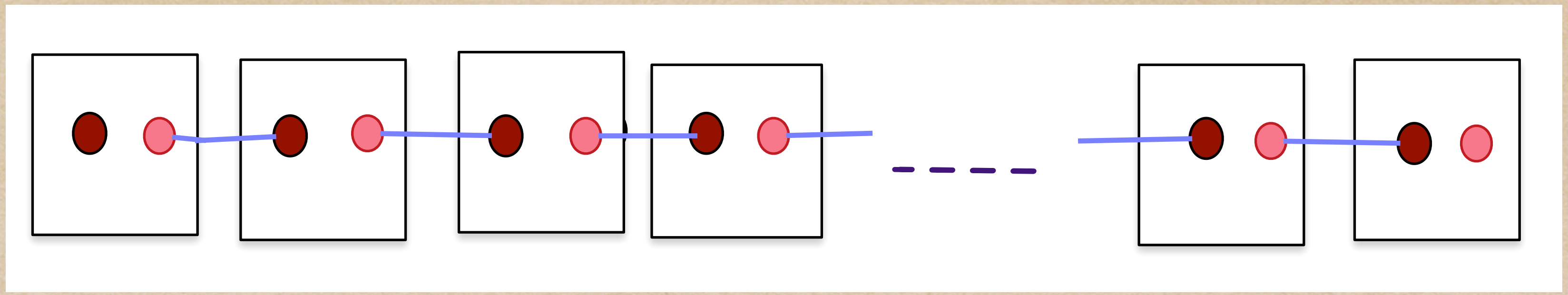
$$\alpha_i^N = 1, \;\; \alpha_i^\dagger = \alpha_i^{N-1}, \;\; \alpha_j \alpha_k = \alpha_k \alpha_j e^{2\pi i/N(sgn(k-j))}$$

$$H = J \sum_j^{L-1} (e^{-i\pi/N} \alpha_{2j}^\dagger \alpha_{2j+1} + h.c.) + h \sum_{j=1}^{L} (e^{i\pi/N} \alpha_{2j-1}^\dagger \alpha_{2j} + h.c.)$$

- Point is that although not solvable in general because of the complicated commutation relations, easy to solve in 2 limits



- When J=0, h>0, parafermion operators couple spin at same clock site and we get a non-degenerate ground state - trivial phase (maximally disordered limit of clock model)



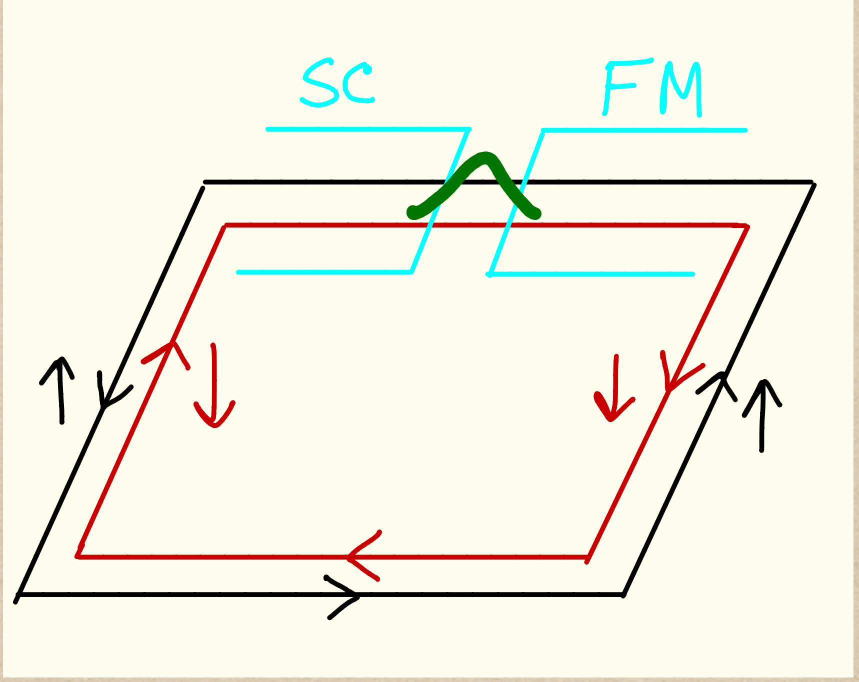
- But when h=0, J>0, the J term couples parafermions on two different spins (maximally ordered limit)
- Implies dangling parafermion modes at the two ends
- N-fold degenerate due to the possible eigenstates of the spin formed by the two end modes - topological phase
- Two phases for suitable ranges of J and h

- So main point is that parafermions are the simplest generalizations of Majorana modes

$$\gamma_i^2 = 1, \quad \gamma_i \gamma_j = (-1)\gamma_j \gamma_i$$

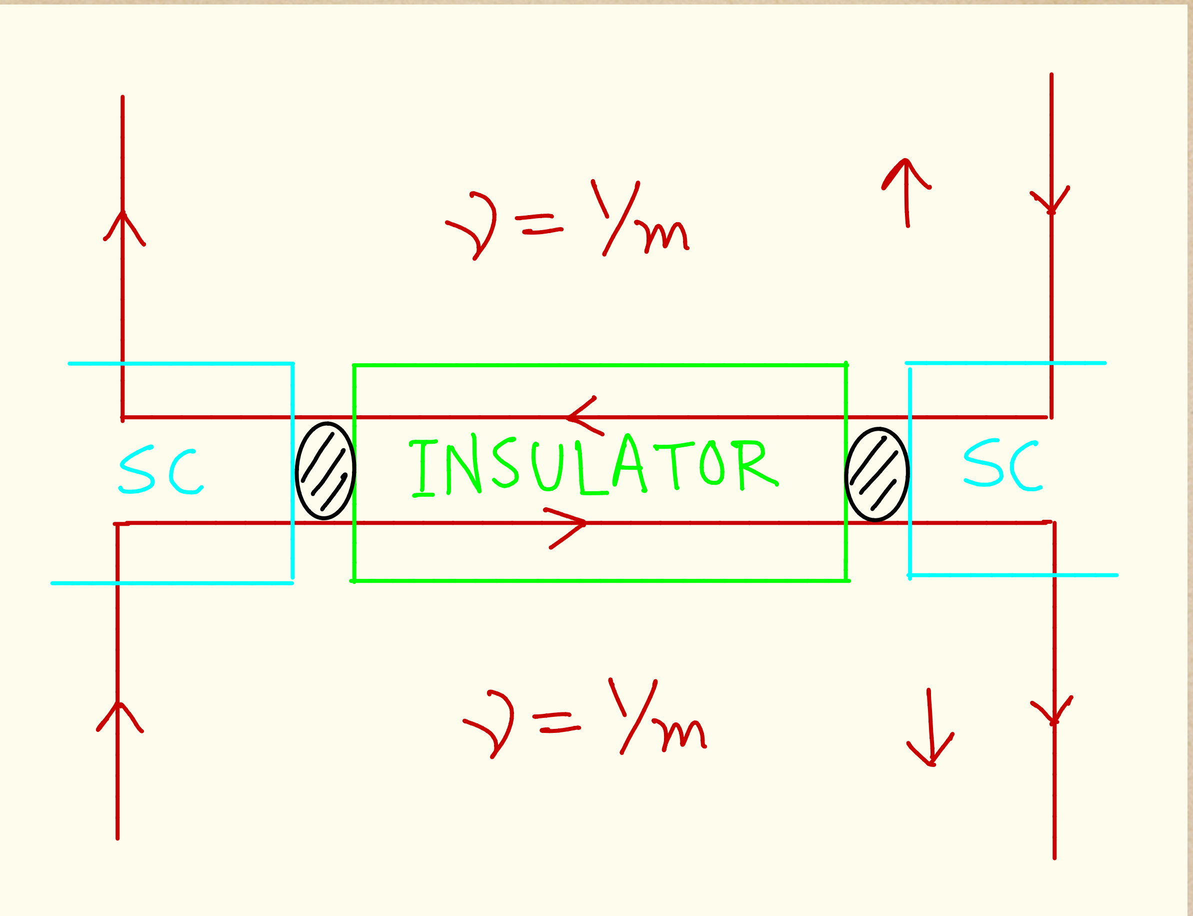
$$\alpha_i^N = 1, \quad \alpha_j \alpha_k = e^{(2\pi i/N) sgn(j-k)} \alpha_k \alpha_j$$

# Engineering parafermion modes




- Expect parafermions at edges of fractional topological insulators with superconductors and ferromagnets

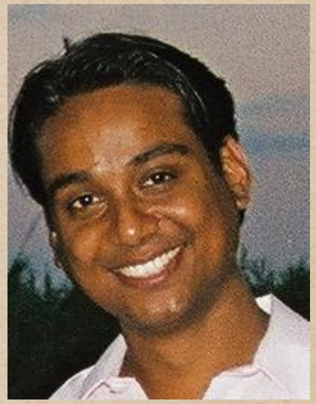
M . Cheng,2012



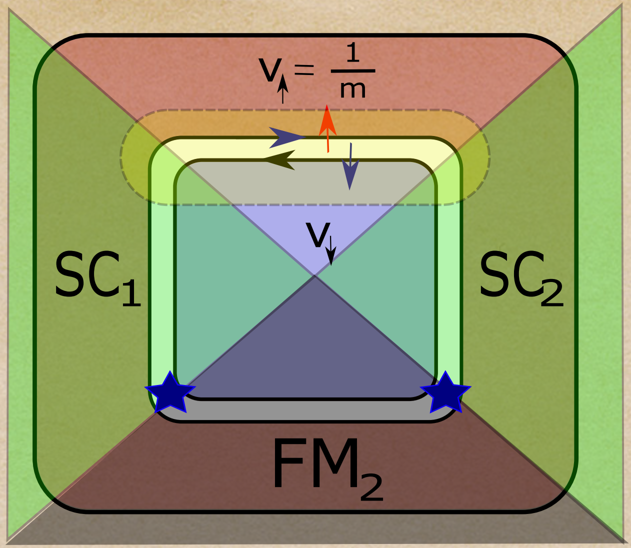

- More promising - realisation in fractional quantum Hall edges

Clarke, Alicea and Shtengel,2012

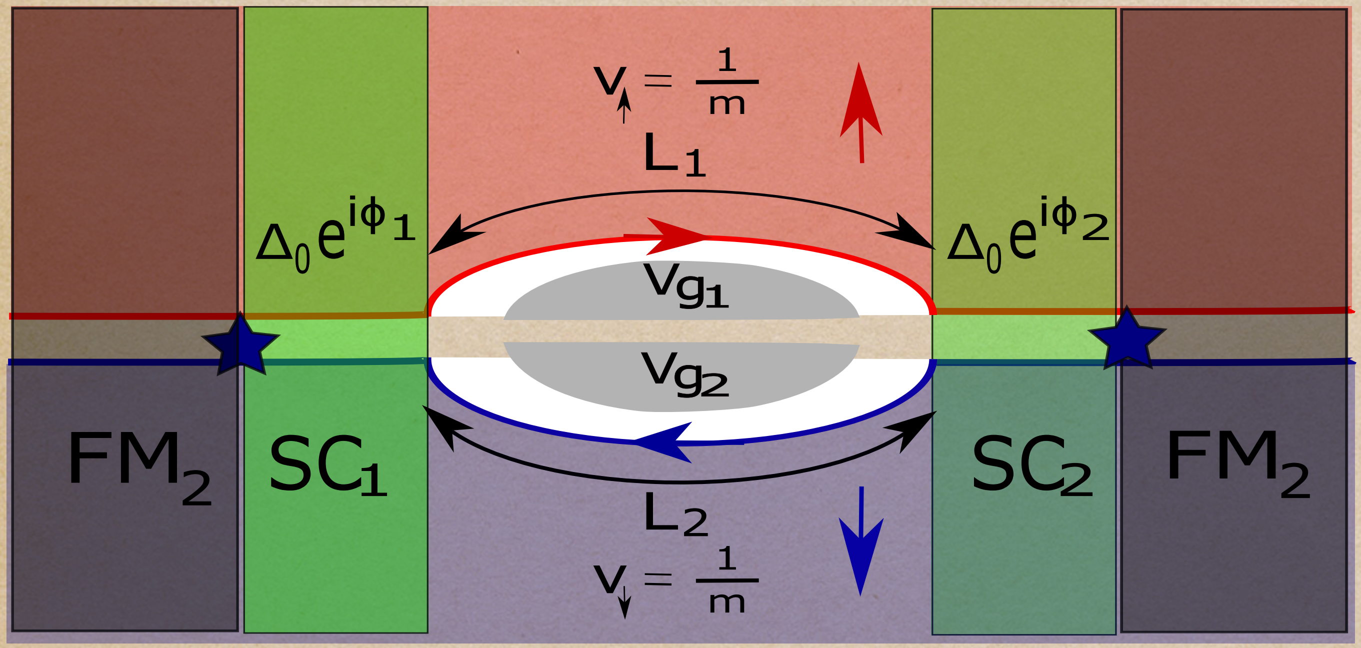
# Our recent work

- Spontaneous fractional Josephson Current from parafermions,
  Kishore Iyer, Amulya Ratnakar, Aabir Mukhopdhyay, Sourin Das, Sumathi Rao, cond-mat/2208.05504

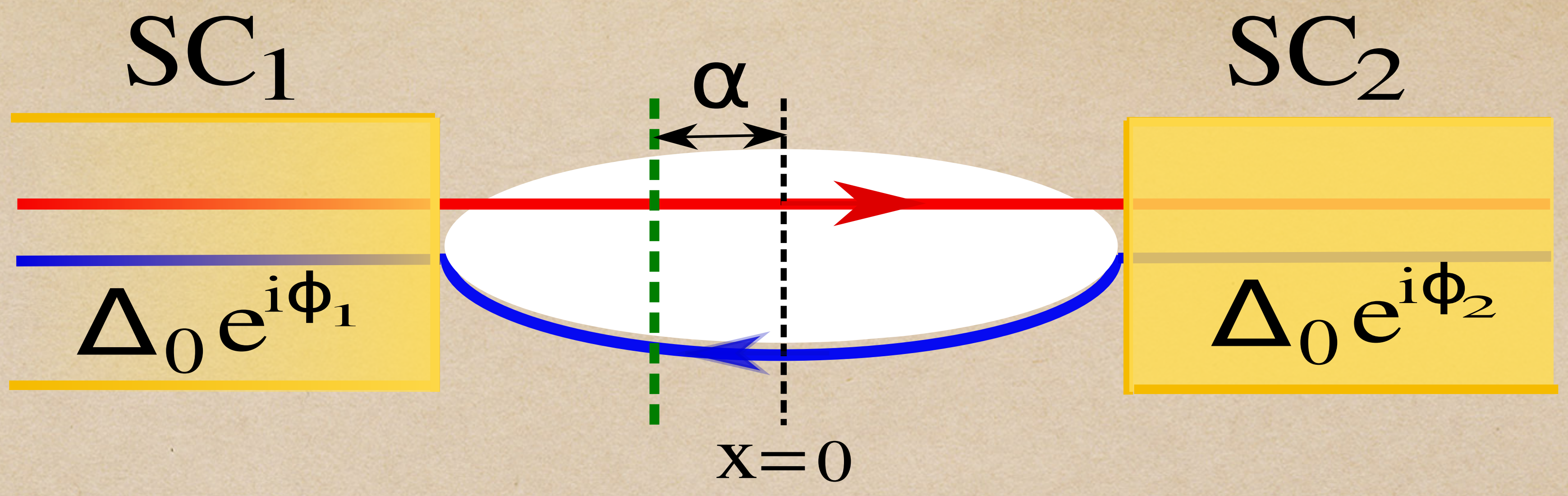



Similar to Lindner, Berg, Refael and Stern, 2012




Similar to Clarke, Alicea and Shtengel, 2012

- Two concentric rings of $\nu_{\uparrow/\downarrow} = 1/m$ FQH states
- Edges proximitised by SC and FM




$$\psi_{R,\uparrow}(x=0) = e^{-i\Phi} e^{i\phi_1} \psi^\dagger_{L,\downarrow}(x=0)$$
$$\psi_{R,\uparrow}(x=L_1) = e^{-i\Phi} e^{i\phi_2} \psi^\dagger_{L,\downarrow}(x=L_2)$$

- For $\nu = 1$ case, expect Majoranas at the edges will hybridize to give the $4\pi$ Josephson effect as a function of $\phi_1 - \phi_2 = \phi$

- ABS spectrum can be calculated for different lengths using scattering matrices

$$E = \pm\Delta_0 \cos\left[\frac{E}{\Delta_0}\frac{\langle L\rangle}{L_{SC}} \pm \left(\frac{\mu\delta L}{\hbar v_F} - \frac{\phi}{2}\right)\right]$$

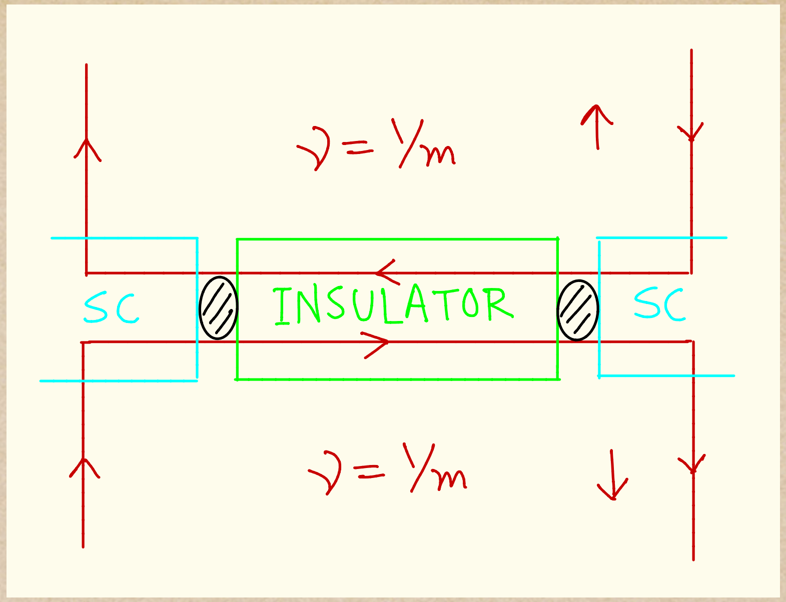
- Main point here is that $\delta L = L_1 - L_2$ is additive with the phase difference $\phi = \phi_1 - \phi_2$
- Hence leads to spontaneous Josephson effect even in absence of phase difference

# $Z_6$ Parafermions

$$H_0 = \frac{mv_F}{4\pi} \int dx \, \left[(\partial_x \phi_R)^2 + (\partial_x \phi_L)^2\right]$$

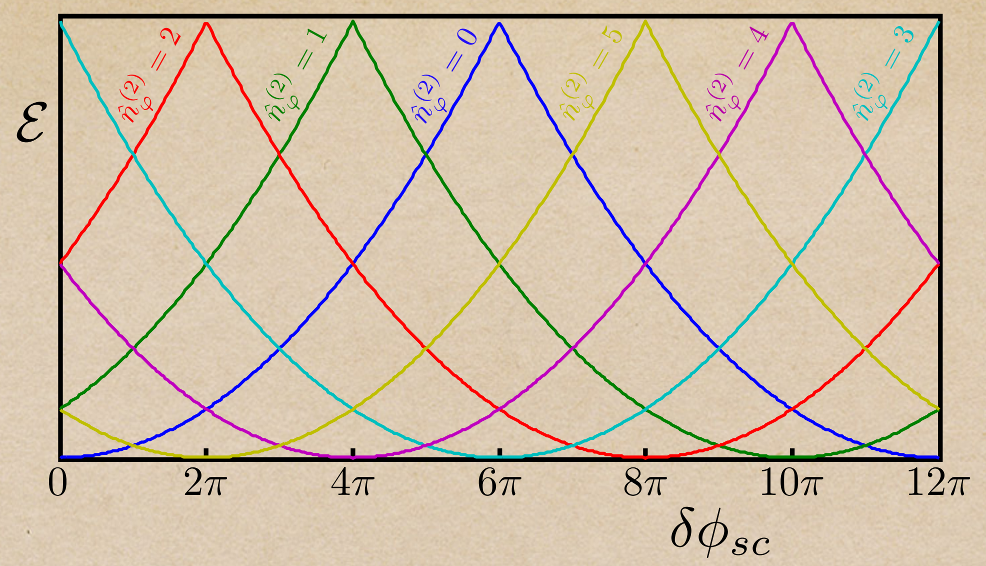
$$H_I = \sum_{i=1,2} \Bigg( \Delta_i \int_{SC_i} dx \, \cos\left[m\left(\phi_R(x) + \phi_L(x)\right)\right]$$
$$+ \mathcal{M}_i \int_{FM_i} dx \, \cos\left[m\left(\phi_R(x) - \phi_L(x)\right)\right] \Bigg)$$

- No free fermion description, need Luttinger liquid physics to describe FQH edge modes




- Essential idea, can 'fractionalise' the local $e/m$ quasi-particles and create zero energy - parafermion modes at the 2 ends (analog of MBS obtained by fractionalizing fermions)

- Need technical details of finite size bosonisation to obtain the correct boundary conditions and construct the parafermion modes
- $$\phi_R(0) + \phi_L(0) = 0$$
  $$\phi_R(L_1) + \phi_L(L_2) = 2\left(\mathrm{mod}\left[\frac{\pi}{m}\left(\hat{n}_2^{SC} - \frac{\sigma}{2\pi}\right), 2\pi\right] - \pi\right)$$
  $$\equiv 2\hat{\eta}$$
- $\hat{n}_2^{SC}$ is the pinned minimum of the fields at the right superconductor and can take $2m$ values and
  $$\sigma/2 = -\cos^{-1}\left(\frac{E}{\Delta_0}\right) + \frac{E\langle L\rangle}{\hbar v_F} \pm \left(\frac{\mu \delta L}{\hbar v_F} - \frac{\phi}{2}\right)$$
- Diagonalised Hamiltonian is now given by
  $$H_{\mathrm{eff}} = \frac{m v_F}{\pi(L_1 + L_2)}\hat{\eta}^2 + \sum_{k>0} \frac{2\pi k v_F}{L_1 + L_2}\left(a_k^\dagger a_k + \frac{1}{2}\right).$$




Clarke, Alicea and Shtengel, 2012

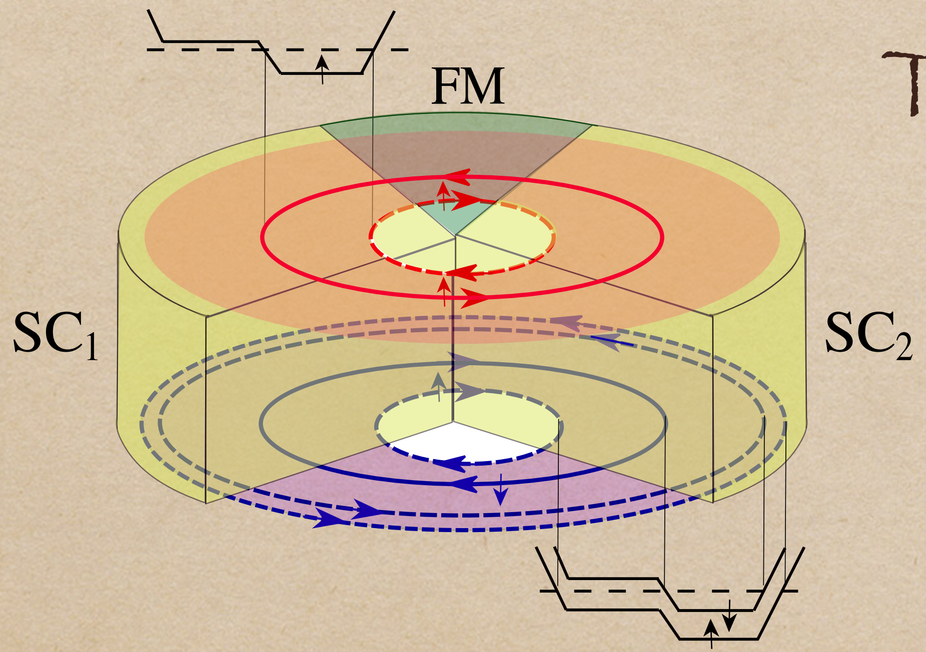
- In the absence of the insulator, the parafermion modes hybridize and one gets the $4\pi m$ Josephson effect

- Same effect now in terms of $\delta L = L_1 - L_2$ instead of the phase difference $\delta\phi_{SC} = \phi_1 - \phi_2$

- Hence, spontaneous fractional Josephson current

- Main point is that we now have another knob to tune Josephson current along with the difference in phase. For $v_F \sim 10^4 m/s$, $\mu \sim 10\text{mev}$, $\delta L \sim$ few $\mu m$

- Perhaps experimentally accessible ?




Top layer at $\nu = 1/m$

Bottom layer at $\nu = 1 + 1/m$

# Glimpse of my collaborators in the last 5 years

# My recent collaborators

- Senior people

- Postdocs and students

# Suman Jyoti De, Works on Majorana modes in topological systems and quantum Hall effect edge reconstruction

- Magnetic flux periodicity in second order topological superconductors, S J De, U. Khanna, S Rao, PRB 101,125429(2020).
- Emergence of spin-active channels at a quantum Hall interface, A Saha, S J De, S Rao, Y Gefen, G Murthy, PRB 103,L081401(2021).
- Chiral detection of Majorana bound states at the edge of a quantum spin Hall insulator, V Adak, A Mukhopadhyay, S J De, U Khanna, S Rao, S Das, PRB 106,045422(2022).
- Complete phase diagram of charge neutral graphene in the quantum Hall regime, S J De, A Das, R Kaul, S Rao, G Murthy.
- Low energy excitations and magnon transmission results between two Graphene quantum Hall ferromagnet (S. J. De, S. Rao and G. Murthy, In preparation).
- Boost driven transition at the edge of a Quantum Spin Hall Superconductor (, S. J. De, A Mukhopadhyay, V Adak, U. Khanna, S. Rao and S. Das, in preparation)

# Faruk Abdulla, Works on topological phase transitions, Weyl semimetals, graphene bilayer and double bilayer quantum Hall phases

- Curvature Function Renormalisation, Topological Phase Transitions and Multi-criticality,F. Abdulla, P. Mohan, and S. Rao, PRB 102, 235129 (2020)
- Fermi Arc Reconstruction at the Interface of Twisted Weyl Semimetal, F. Abdulla, S. Rao, and G. Murthy, PRB B 103, 235308 (2021)
- Time reversal broken Weyl semimetal in the Hofstadter regime, F. Abdulla, A. Das, S. Rao, and G. Murthy, Scipost Phys. Core 5, 014 (2022)
- Topological nodal line semimetals without crystalline symmetries, F. Abdulla, A. Das and G. Murthy ( in preparation)
- Phase diagram of bilayer and double bilayer graphene at charge neutrality in the presence of trigonal warping, F. Abdulla and A. Das, ( in preparation)

# Summary and take-away points

- Non-abelian anyons as quasi-particles in condensed matter systems and why they are relevant in quantum computation
- Majorana modes, have they been found?
- Motivated parafermions and possible ways of looking for them

# Non-abelian anyons are a hot topic!

Thank you all for coming and listening to me!