Overview of Fractional Quantum Hall Effect

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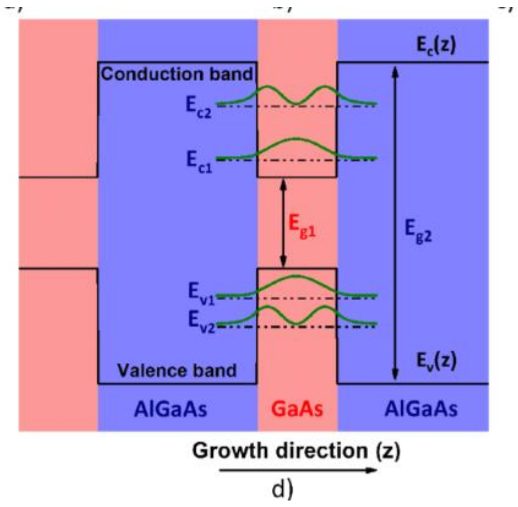
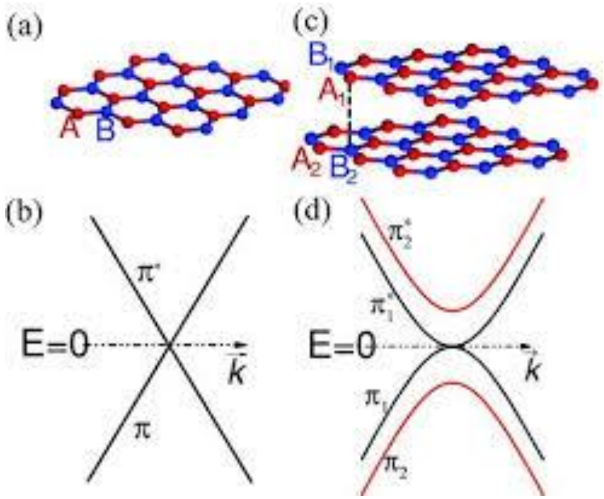
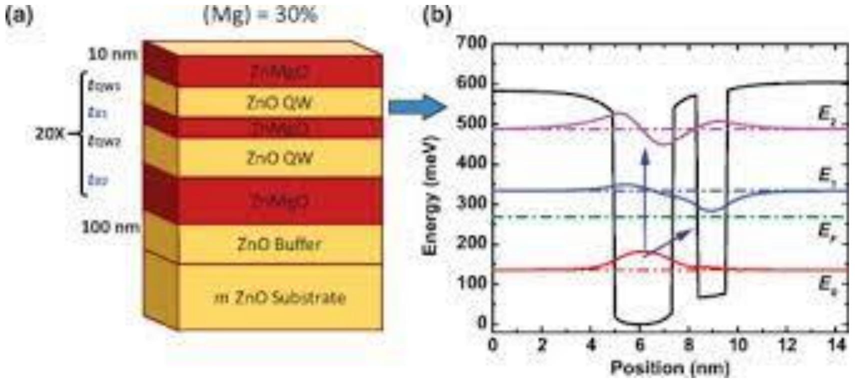
Experimental Systems



GaAs quantum well



Monolayer and Bilayer Graphenes



ZnO-based system

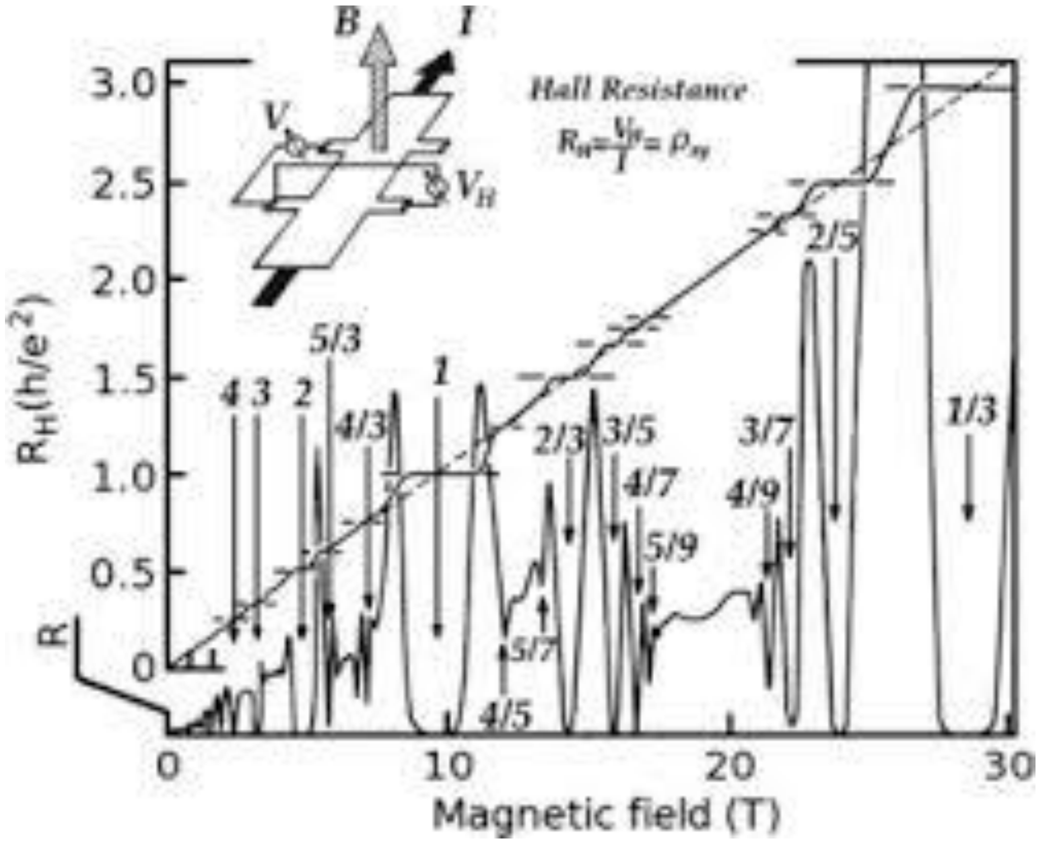
These are the major quasi two-dimensional electron systems where fractional quantum Hall effect is realized.

Fractional Quantum Hall Effect

$$N_\Phi = \frac{BL_xL_y}{\frac{hc}{e}}$$

- Filling factor is noninteger, specifically, $\nu = \frac{N_e}{N_\phi} < 1$
- Quantum Hall Effect was unexpected because all electrons have same kinetic energy and there is no energy gap.

What does produce the bulk-gap then?



It is the Coulomb interaction that breaks the degeneracy.

$$\nu = \frac{n}{2n \pm 1} \qquad R_H = \frac{h}{e^2\nu}$$

The Coulomb interaction is now ***nonperturbative***, as the kinetic energy of the electrons is quenched.

Infact, Coulomb intercation is the ***only*** *energy (if we ignore spin as* the magnetic field is very high).

Willett et al, PRL, 1987

Laughlin's Theory

$$Hamiltonian: H = P_{LLL} \sum_{i<j}^{N} \frac{e^2}{|\vec{r}_i - \vec{r}_j|}$$

(Nonperturbative)

$$H_0 = \frac{\left(\vec{P} + \left(\frac{e}{c}\right) \vec{A}\right)^2}{2m_b}, \qquad \vec{\nabla} \times \vec{A} = B\,\hat{z}$$

Physics is independent of the choice of gauge

$$Symmetric\ gauge: \ \vec{A} = \left(\frac{B}{2}\right)[-y, x, 0]$$

$$z = \frac{(x - iy)}{\ell}, \qquad \ell^{-2} = \frac{eB}{\hbar c}$$

$$H_0 = \frac{\hbar\omega_c}{2}\left[-4\frac{\partial}{\partial z}\frac{\partial}{\partial z^*} + \frac{|z|^2}{4} - z\frac{\partial}{\partial z} + z^*\frac{\partial}{\partial z^*}\right]$$

$$H_0|n, l\rangle = \epsilon_n\,|n, l\rangle, \epsilon_n = \left(n + \frac{1}{2}\right)\hbar\omega_c, \qquad L_z|n, l\rangle = \hbar(l - n)|n. l\rangle$$

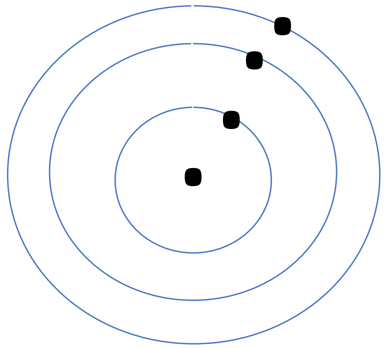
$$\phi_{0,m}(z, z^*) = \langle z, z^*|0, m\rangle = \frac{1}{\sqrt{2\pi 2^m m!}}\, z^m \exp\left[-\frac{|z|^2}{4}\right]$$

Wave functions for the lowest Landau level

$$|\phi_{0,m}(z,z^*)|^2 = P(r) \sim r^{2m} \exp\left[-\frac{r^2}{2}\right]$$

$$\frac{\partial P(r)}{\partial x} = 0 \Rightarrow r_m = \sqrt{2m}\,\ell$$

Area of an annulus: $\pi(r_{m+1}^2 - r_m^2) = \pi[2(m+1) - 2m]\ell^2 = 2\pi\ell^2$



$$Total\ Area = N_a\, 2\pi\ell^2 \Rightarrow Flux, \Phi = B 2\pi N_a \ell^2 = \frac{N_a\, hc}{e} = N_a \Phi_0$$

Flux per annulus is $\Phi_0 \Rightarrow$ each orbital is associated with one flux quantum.

Degeneracy for the lowest LL: N_Φ, ***and per unit area***: $1/2\pi\ell^2$.

Single-particle orbitals correspond to analytic functions, f(z).

$\boldsymbol{Wave\ Function\ for\ fillig\ factor\ \nu = 1}$:

$Filling\ Factor\ \nu = \frac{N}{N_\Phi} = 2\pi n_e \ell^2$

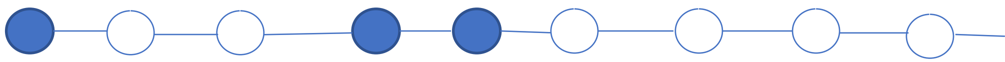
$Noninteracting\ problem, or, the\ interaction\ is\ unimportant.$

$$\Psi_{\nu=1} = \begin{bmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ z_1^N & \cdots & z_N^N \end{bmatrix} \exp\left[-\frac{\sum_{i=1}^{N}|z_i|^2}{4}\right] = \prod_{i<j=1}^{N}(z_i - z_j) \exp\left[-\frac{\sum_{i=1}^{N}|z_i|^2}{4}\right]$$

Fractional Filling factor $\boldsymbol{\nu = \frac{1}{3}}$:

$Say, N_\Phi = 9, N = 3$

$$\{\lambda\} \in \frac{9!}{3!\,6!}$$



(Lowest Landau Level)

Ground state:

$$\Psi_{\nu=\frac{1}{3}} = \sum_{\{\lambda\}} C_\lambda \, |3 \times 3\,|_\lambda,$$

$Degeneracy\ is\ broken\ due\ to\ the\ Coulomb\ interaction.$

$C_\lambda\ may\ be\ obtained\ by\ the\ method\ of\ exact\ diagonalization\ for\ a\ few\ electrons, N < 20.$

However, this is not sufficient for complete understanding!

$Also, sometimes\ exact\ result\ cannot\ be\ understood\ physically.$

Laughlin's Theory for FQHE

1) $Manybody\ wave\ function\ consists\ of\ occupying\ single-particle\ orbitals.$

$$\Psi = f(\{z_i\})\exp\left[-\frac{\sum_{i=1}^{N}|z_i|^2}{4}\right] \qquad \Psi = f(\{z_i\})$$

2) $f(\{z_i\})\ should\ have\ Jastrow\ form:\ \ f(z_i - z_j)$

3) $Wavefunction\ should\ be\ antisymmetric\ for\ the\ exchange\ of\ particles.$

$$f\left(z_i - z_j\right) = \sum_{\alpha=1,3,\ldots} \prod_{i<j}^{N} \left(z_i - z_j\right)^{\alpha}$$

4) $Wavefunction\ should\ be\ eigenstate\ of\ total\ angular\ momentum$
$as\ the\ Hamiltonian\ is\ rotationally\ symmetric.$

$$M = \sum_{i=1}^{N}\left(z_i\frac{\partial}{\partial z_i} - z_i^*\frac{\partial}{\partial z_i^*}\right) \Rightarrow\ \alpha\ \ should\ be\ any\ one\ odd\ integer.$$

$$\Psi_L = \prod_{i<j}^{N}\left(z_i - z_j\right)^{m},\ m\ odd-integer$$

Laughlin's Theory for FQHE

A **Groundstate** wavefunction for FQHE state: $\Psi_L = \prod_{i<j}^{N} (z_i - z_j)^m$,

$Which\ filling\ factor\ \nu\ is\ represented\ by\ \Psi_L?$

$Maximum\ exponent\ of\ any\ z_i:\ \ m(N-1) \Rightarrow N_\Phi = m(N-1)$

$The\ filling\ factor\ \nu = \lim_{N\to\infty} \frac{N}{N_\Phi} = \lim_{N\to\infty} \frac{N}{m(N-1)} = \frac{1}{m}$

Quasiholes and Quasiparticles (Collective Excitations):

1) A ***quasihole*** $may\ be\ created\ by$ ***increasing*** $one\ unit\ of\ flux, i.e., \Phi_0$ =hc/e .

2) A ***quasiparticle*** $may\ be\ created\ by$ ***decreasing*** $one\ unit\ of\ flux.$

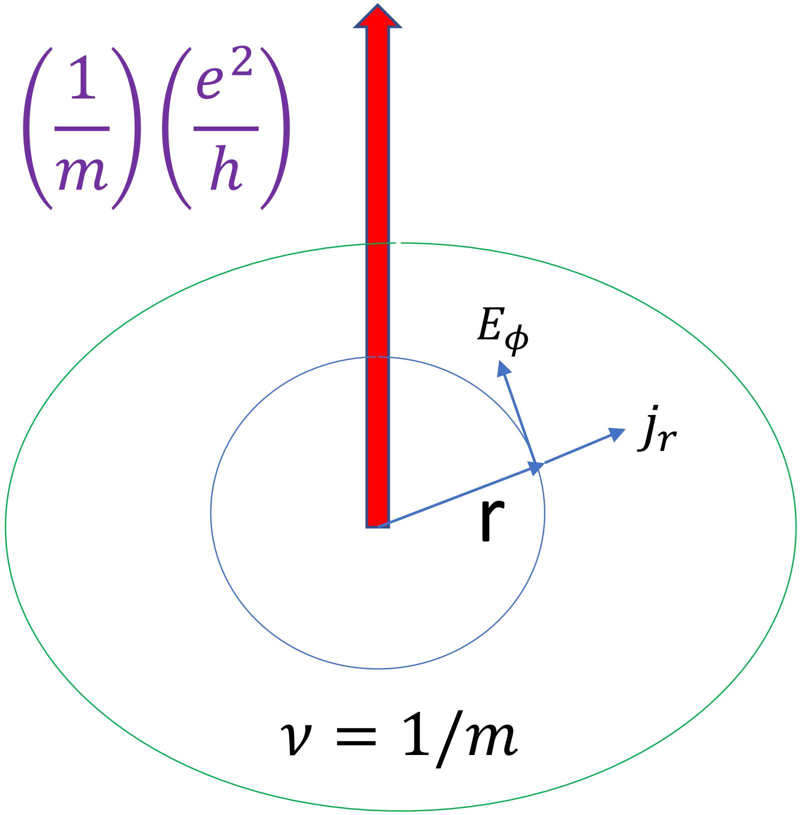
Adiabatic Insertion of one unit of Flux

$$\oint \vec{E} \cdot \vec{dl} = -\frac{\partial \Phi}{\partial t} \Rightarrow E_\phi (2\pi r) = -\frac{\partial \Phi}{\partial t}$$

$$j_r = \sigma_H \, E_\phi, \quad \sigma_H = \left(\frac{1}{m}\right)\left(\frac{e^2}{h}\right)$$

Charge displaced from the center:

$Q = 2\pi r \int j_r \, dt = -\sigma_H \Phi_0$ = -e/m



Charge-Vortex duality: One vortex is associated with e/m charge.

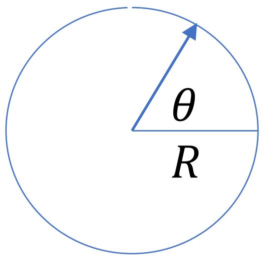
Laughlin's Quasihole Wavefunction:

$$\Psi_L^{qh} = \prod_{i=1}^{N} (z_i - \eta)\, \Psi_L^{\frac{1}{m}} = \prod_{i=1}^{N} (z_i - \eta) \prod_{i<j}^{N} (z_i - z_j)^m$$

All electrons see a zero at the complex point $\eta = Re^{-i\theta}$.

How does this wave function describe quasihole charge of e/m?

Consider a loop of radius R on which η changes from θ = 0 to θ = 2π adiabatically.



Acquired Berry Phase: $\gamma = i \oint d\theta \left\langle \Psi_{qh}^{L} \middle| \frac{\partial}{\partial\theta} \middle| \Psi_{qh}^{L} \right\rangle = 2\pi N_{encl}$

$Ahronov - Bohm\ phae\ for\ a\ particle\ of\ charge\ e^*\ in\ magnetic\ field\ B$:

$$\phi = BA\frac{e^*}{\hbar c} = 2\pi\left(\frac{e^*}{e}\right)N_{\Phi} = 2\pi\left(\frac{e^*}{e}\right)N_{encl}\, m$$

$Equating\ these\ two,$ $\quad e^* = \frac{e}{m}$

$$\Psi_L^{qh} = \prod_{i=1}^{N}(z_i - \eta)\, \Psi_L^{\frac{1}{m}} = \prod_{i=1}^{N}(z_i - \eta)\prod_{i<j}^{N}(z_i - z_j)^m$$

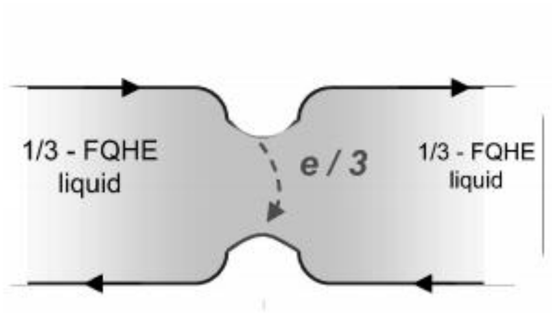
$Quasihole\ charge = e/m$

$$\Psi_L^{qp} = \prod_{i=1}^{N}\left(2\frac{\partial}{\partial z_i} - \eta^*\right)\prod_{i<j}^{N}(z_i - z_j)^m$$

$Quasiparticle\ charge = -e/m$

These quasiparticles and quasiholes obey Abelian fractional statistics: π/m

Experimental Realization of Fractional Charge



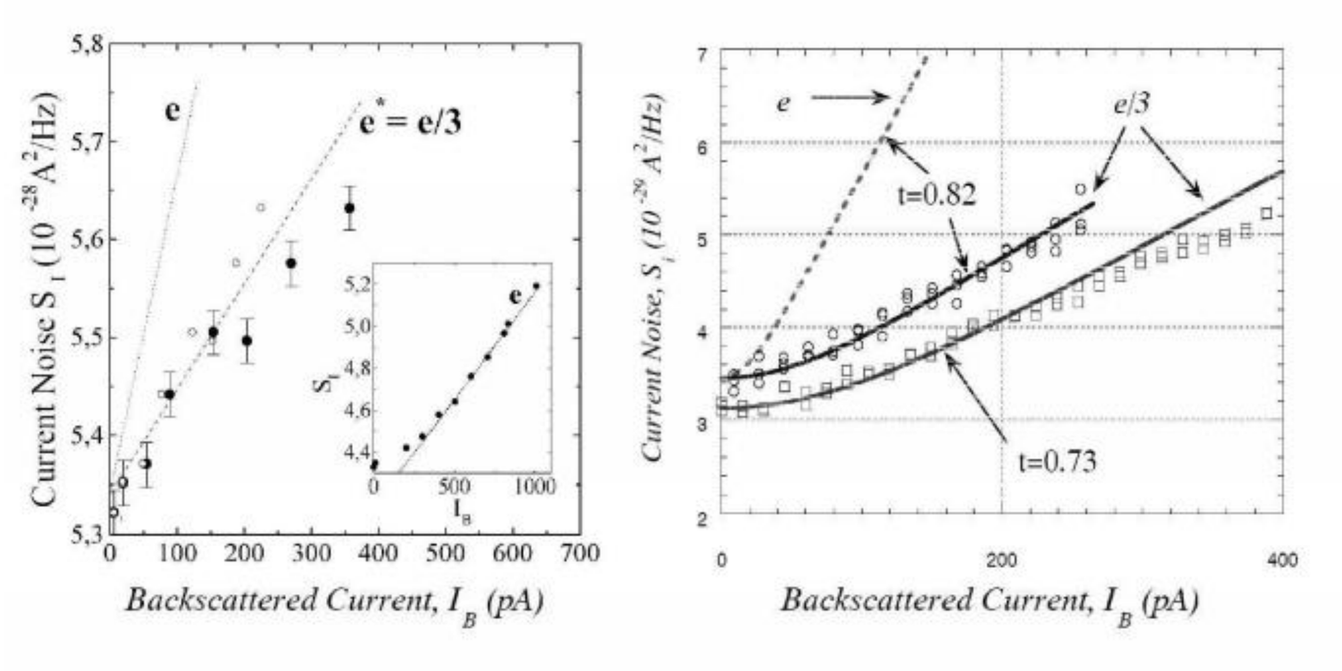
Transfer trough
1/3 FQHE fluid

(weak bakscattering):

$q = e/3$

$$\langle (\Delta I)^2 \rangle = 2\,(e/3)\,I_B\,\Delta f$$

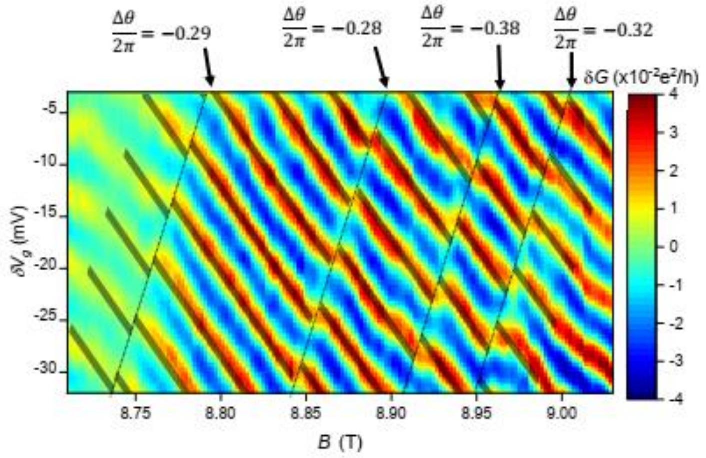
Shot-noise Experiment



Saminadayar et al, PRL, 1997

Direct observation of anyonic braiding statistics

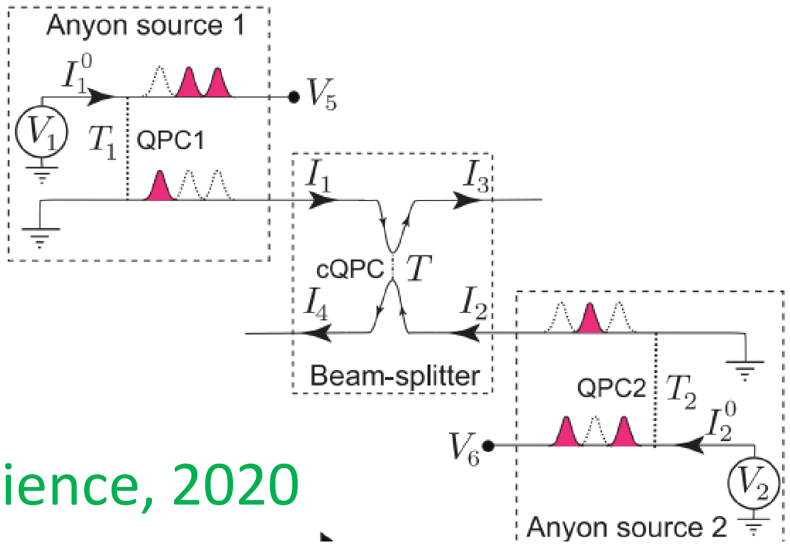
Namura, Liang, Gardner and Manfra, Nat. Phys. 931 (2020)



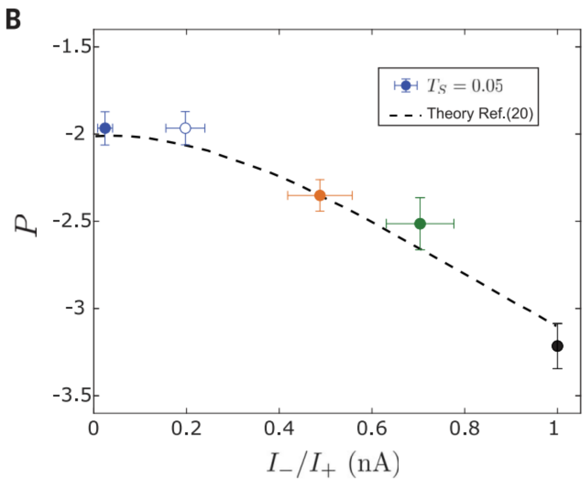
Fabry-Perot Interferometer

Fractional statistics in anyon collisions

H. Bartolomei[1]*, M. Kumar[1]*†, R. Bisognin[1], A. Marguerite[1]‡, J.-M. Berroir[1], E. Bocquillon[1], B. Plaçais[1], A. Cavanna[2], Q. Dong[2], U. Gennser[2], Y. Jin[2], G. Fève[1]§



Science, 2020



Measurement of current correlations resulting the scattering of anyons in beam-splitter.

Rosenow, Levkivskyi, Halperin, 2020

Realization of Abelian Fractional statistics for 1/3 state.

Laughlin's Theory for FQHE is very much successful, yet it has limitations:

It cannot describe other fractional states such as 2/5, 3/7,... by any simple generalization.

Composite-Fermion Theory by Jain:

Jain, PRL, 1989

Hamiltonian: $H = \sum_{j=1}^{N} \frac{\left(\vec{P_j} + \left(\frac{e}{c}\right)\vec{A_j}\right)^2}{2m_b} + \sum_{i<j}^{N} \frac{e^2}{\epsilon|\vec{r_i} - \vec{r_j}|}$

$$H\,\Psi(\vec{r}_1, \cdots, \vec{r}_N) = E\Psi(\vec{r}_1, \cdots, \vec{r}_N)$$

Gauge Transformation: $\Psi = \prod_{j<k}^{N} \left(\frac{z_j - z_k}{|z_j - z_k|}\right)^2 \Psi_{CS} = \exp\left[2i\sum_{k\neq j} \ln\left(\frac{z_j - z_k}{|z_j - z_k|}\right)\right]\Psi_{CS}$

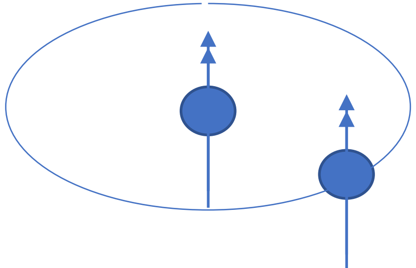
Halperin, Lee, Read, PRB, 1993

$$H' = \sum_{j=1}^{N} \frac{\left(\vec{P_j} + \left(\frac{e}{c}\right)\vec{A_j} - \left(\frac{e}{c}\right)\vec{a_j}\right)^2}{2m_b} + \sum_{i<j}^{N} \frac{e^2}{\epsilon|\vec{r_i} - \vec{r_k}|}$$

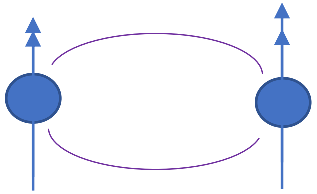
$$\vec{a_j} = 2i\,\frac{\Phi_0}{2\pi}\sum_{k\neq j} \vec{\nabla}_j \ln\left(\frac{z_j - z_k}{|z_j - z_k|}\right)$$

$$H'\;\Psi_{CS} = E\;\Psi_{CS}$$

Singular Flux tube: $b_j = \vec{\nabla}_j \times \vec{a_j} = 2\,\Phi_0 \sum_{k\neq j} \delta^{(2)}(\vec{r_j} - \vec{r_k})$



Extra Phase: 2π



***Particles are still Fermion*,**
called composite Fermion.

Mean field (if the electron density is large): $\langle b \rangle = 2\Phi_0 n_e$

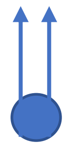
$Effective\ magnetic\ field:\ B^* = B - 2\Phi_0 n_e$

$Filling\ factor, \nu = \frac{n_e \Phi_0}{B}$ (fraction of a LL filled)

$$\frac{n_e\Phi_0}{\nu^*} = \frac{n_e\Phi_0}{\nu} - 2\Phi_0 n_e \Rightarrow \nu = \frac{\nu^*}{2\nu^*+1},\ (\nu^*\ can\ be\ both + ve\ and\ - ve)$$

$$If\ \nu^* = \pm n, \quad \nu = \frac{n}{2n \pm 1}$$ (IQHE of Composite Fermions $\Rightarrow$ $FQHE\ of\ electrons$)

$$Wavefunction:\ \Psi_\nu = \prod_{j<k} \left(\frac{z_j - z_k}{|z_j - z_k|} \right)^2 \chi_{\pm n}(\{z_i, z_i^*\})$$



This wavefunction is, however, not correct for two different reasons:

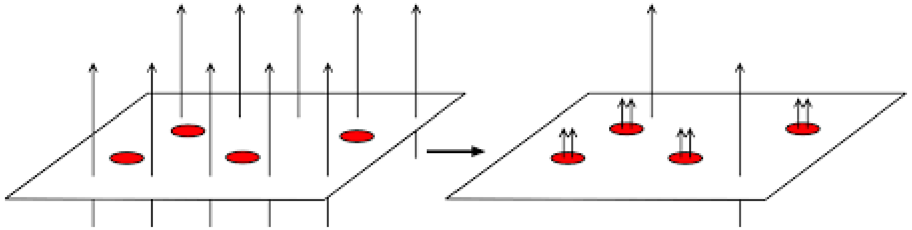
1) Phase term will not be important for electron density distribution.
2) Wavefunction in the lowest Landau level must be analytic.

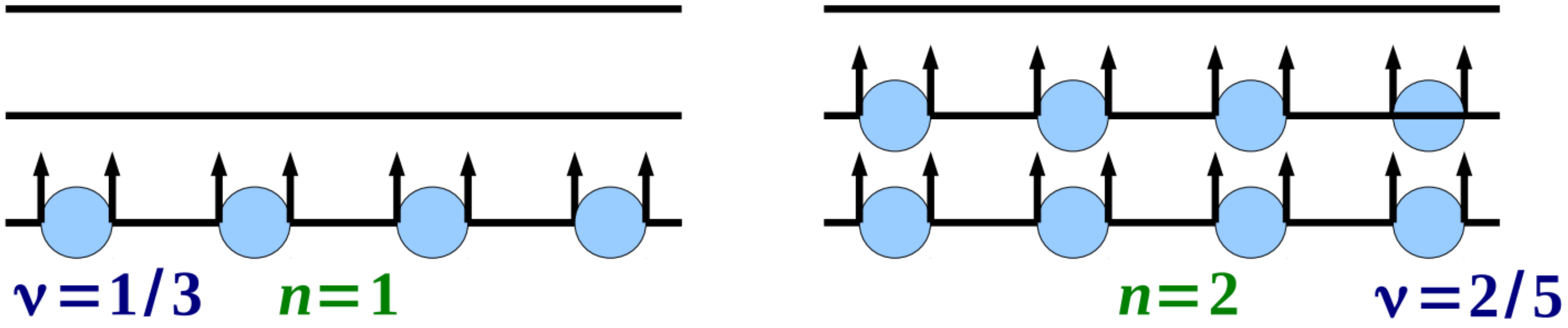
$$Modification:\ \Psi_\nu = P_{LLL} \prod_{j<k} (z_j - z_k)^2 \chi_{\pm n}(\{z_i, z_i^*\})$$ (Vortices are not singular flux tubes, they have got some finite size)

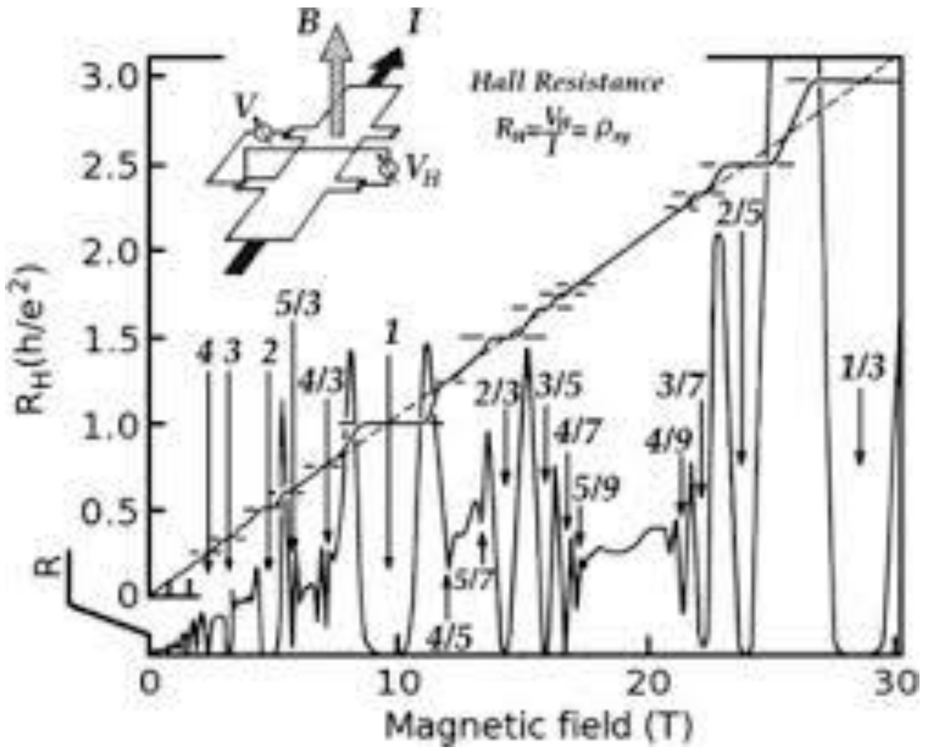
Jain, PRL, 1989

This is the composite fermion wavefunction proposed by Jain for FQHE states in the LLL

(After projection, this wavefunction becomes formally complicated)







$$\nu = \frac{n}{2n \pm 1}$$

1/3,2/5,3/7,.... 2/3,3/5,4/7,.....

$$Bulk\ gap:\ \omega_c^* = \frac{eB^*}{m_b c} \propto 1/|n|$$

Edge Modes:

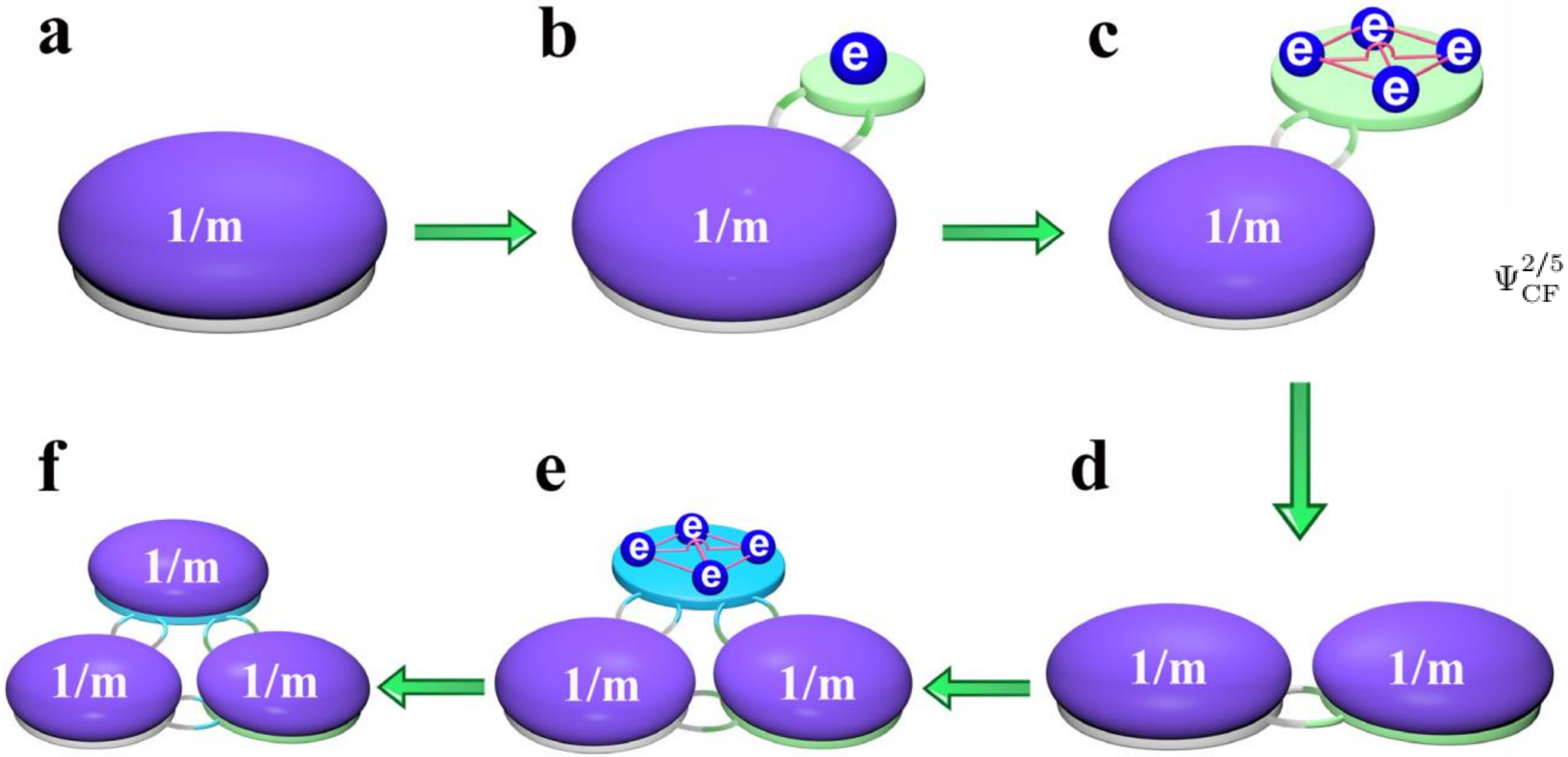
1) *If $n > 0$, there are n edge modes each carying quasiparticles of charge $\frac{e}{2n+1}$,* all along **downstream**.

2) *If $n < 0$, one **downstream** mode carying charge e and $(n-1)$ **upstream** modes each* carrying charge e/(2n-1).

Connection between Laughlin and Composite Fermion Theories

$$O_L = \prod_j \frac{\partial}{\partial z_j} \quad \rightarrow \quad O_{qp} = \sum_j \left(\frac{\partial}{\partial z_j} \prod_{l \neq j} \frac{\partial}{\partial (z_l - z_j)}\right)$$

Mandal, JPCM, 2018



$$\Psi_{\rm CF}^{2/5} = \prod_{i<j\leq N} U_{i,j}^2 \mathcal{A}\left[\left(\prod_{k<l\leq N/2} U_{k,l}\, U_{k+N/2,l+N/2}\right) \times \prod_{m\leq N/2}\left(\sum_{i\neq m} \frac{1}{U_{m,i}}\right)\right]$$

$$U_{i,j} = z_i - z_j$$

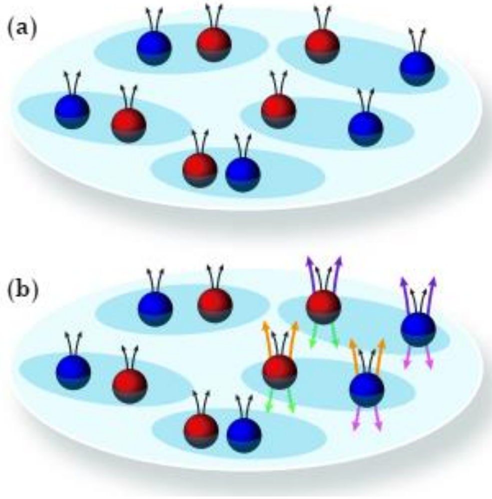
Possibility of non-Abelian FQHE in the lowest Landau level

$$\Psi^{2/5}_{\rm Gf} = \prod_{i<j\le N} U^2_{i,j}\, \mathcal{A}\left[\left(\prod_{k<l\le N/2} U_{k,l}\; U_{k+N/2,l+N/2}\right) \times \prod_{m\le N/2} \frac{1}{U_{m,m+N/2}}\right]$$

Exact ground-state of certain kind of **three-body** potential.

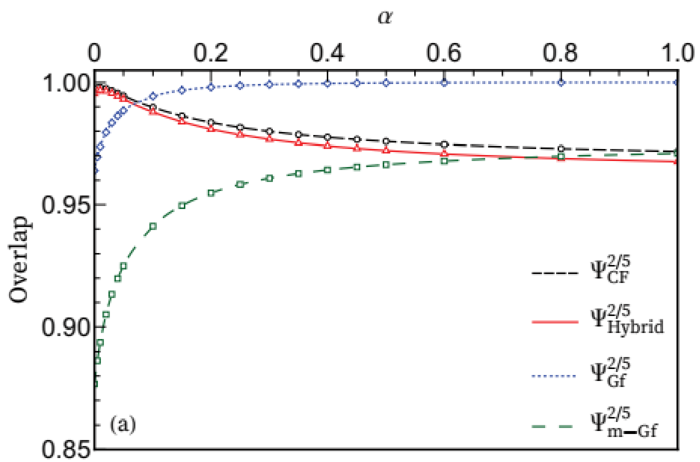
Simon et al, PRB, 2007

$$U_{i,j} = z_i - z_j$$



$$\Psi^{2/5}_{\rm CF} = \prod_{i<j\le N} U^2_{i,j}\, \mathcal{A}\left[\left(\prod_{k<l\le N/2} U_{k,l}\, U_{k+N/2,l+N/2}\right) \times \prod_{m\le N/2} \left(\sum_{i\neq m} \frac{1}{U_{m,i}}\right)\right]$$

Jain, PRL, 1989;
SSM, JPCM, 2018



$$\Psi^{2/5}_{\rm m-Gf} = \prod_{i<j\le N} U^2_{i,j}\, \mathcal{A}\left[\left(\prod_{k<l\le N/2} U_{k,l}\; U_{k+N/2,l+N/2}\right) \times \left(\prod_{m\le N/2} \frac{1}{U_{m,m+N/2}}\right) \sum_{l_1<l_2\le N/2} \left(\frac{U^2_{l_1,l_1+N/2} U^2_{l_2,l_2+N/2}}{U^2_{l_1,l_2} U^2_{l_1+N/2,l_2+N/2}}\right)\right]$$

Das, Das, SSM, PRB(Letter), 2022

$$H_\alpha = \alpha V_{3-{\rm b}} + (1-\alpha) V_C$$

$$\Psi^{2/5}_{\rm Hybrid} = C_1 \Psi^{2/5}_{\rm Gf} + C_2 \Psi^{2/5}_{\rm m-Gf}$$

- The Hybrid wavefunction behaves quite same as way as the composite fermion wavefunction.
- However, while the nature of quasiparticles in the composite fermion theory is Abelian, the quasiparticle nature corresponding to the hybrid wavefunction is non-Abelian.
- Two-body Coulomb interaction along with a suitable 3-body interaction may provide the FQHE state as **non-Abelian** for 2/5 state in the **lowest Landau level**.

Filling Factor 1/2

$\nu = \frac{n}{2n \pm 1} \Rightarrow \nu = \frac{1}{2}\ as\ n \rightarrow \infty$ $\quad Bulk\ gap \propto \frac{1}{n} = 0$ $\quad B^* = 0$

$At\ \frac{1}{2}\ filling, the\ composite\ fermions\ feel\ zero\ net\ magnetic\ field.$

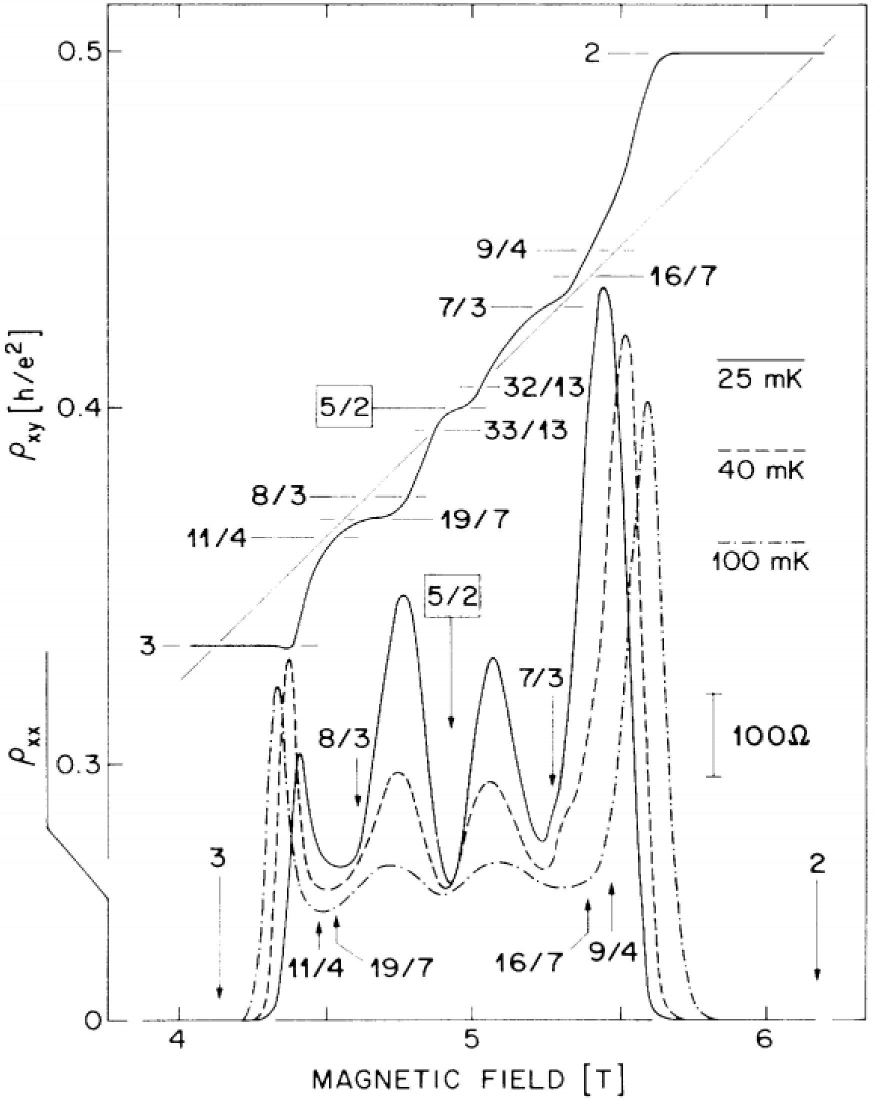
Therefore, weakly interacting CFs at zero net magnetic field should also form **Fermi surface**.

$\text{Near}\ \frac{1}{2}\ \text{filling, cyclotron orbit of particles should be determined by}\ B^*.$

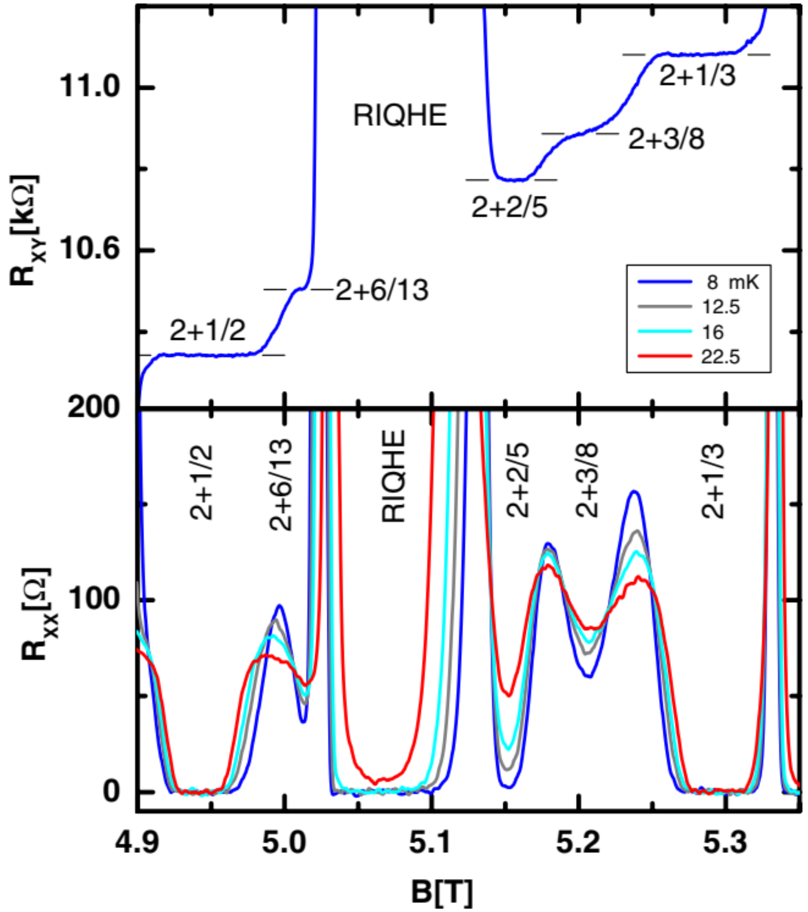
This fact is experimentally observed by elegantly designed experimental setups!

Filling factor ½ is **not** a quantized fractional quantum Hall state.

Fractional Quantum Hall States in the Second Landau Level



Willett et al, PRL, 1987



(Monolayer GaAs)

Kumar et al, PRL, 2010

Filling factor ½ is one of the most prominent states!!

States are not in the sequence of the lowest Landau level!

Early Theoretical Understanding of ½ Filled Second LL

$z = \frac{x - iy}{\ell}$

- ***Electron Operator***: $\psi_e(z) = e^{i\sqrt{2}\,\phi(z)}\,\psi(z)$

Charged Bosonic operator

Neutral Majorana-Fermionic operator

Moore and Read, NPhB, 1991

- ***Wave Function***: $\Psi_{\mathrm{MR}}(z_1, \cdots, z_N) = \langle \psi(z_1) \cdots \psi(z_N) \rangle \langle \prod_i e^{i\sqrt{2}\phi(z_i)}\, e^{-i\sqrt{2}\rho_0 \int dr' \phi(z')} \rangle$

Pfaffian Order: $= Pf\left(\frac{1}{z_i - z_j}\right) \prod_{i<j}^{N} (z_i - z_j)^2 \; \exp(-\sum_i |z_i|^2/4)$

chiral $p_x + i\,p_y$ wave pairing

Composite fermion

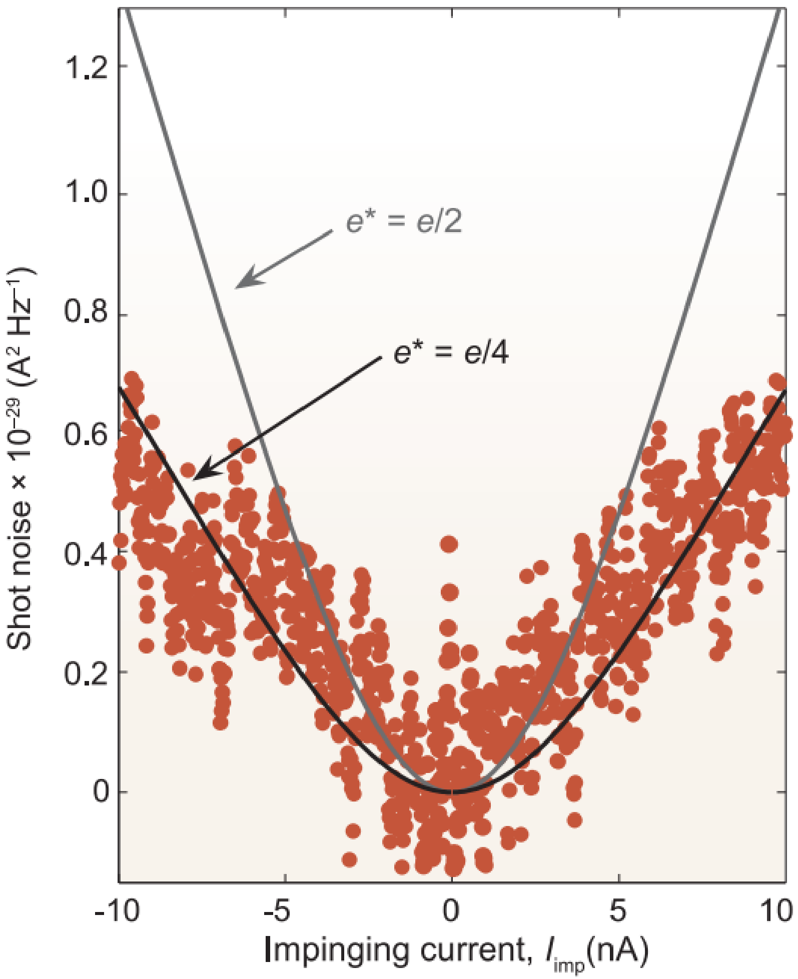
- **Pairing** causes Bulk-gap and **bulk-edge** correspondence provides a neutral **Majorana mode** along with bosonic charge mode.

Quasiparticle Charge: e/4, instead of e/2

NonAbelian braiding statistics of quasiparticles.

Fusion: $\boldsymbol{\Phi}_i \times \boldsymbol{\Phi}_j = \sum_k N_{ij}^k \boldsymbol{\Phi}_k$

Experimental Observations of Fractional Charge of Quasiparticles



Dolev et al, 2008

However, there are several other topological orders, including Abelian, also predict e* = e/4 charge.

So, e/4 charge is not conclusive for Pfaffian order.

Theoretical Debate for 5/2 state

- Fractional charge e/4 has been predicted for several other models including Abelian theory for 5/2 state.
- Particle-Hole conjugate of **MR Pfaffian** state, called **Anti-Pfaffian**, is degenerate with it for Coulomb potential.
  - Landau-Level-Mixing induced **3-body** interaction can lift this degeneracy.
    APf is topologically distinct from Pf: The former provides **upstream** Majorana mode.

Lee et al, PRL,2007
Levin et al, PRL,2007

Very hard to predict the clear picture as their energies are very close.

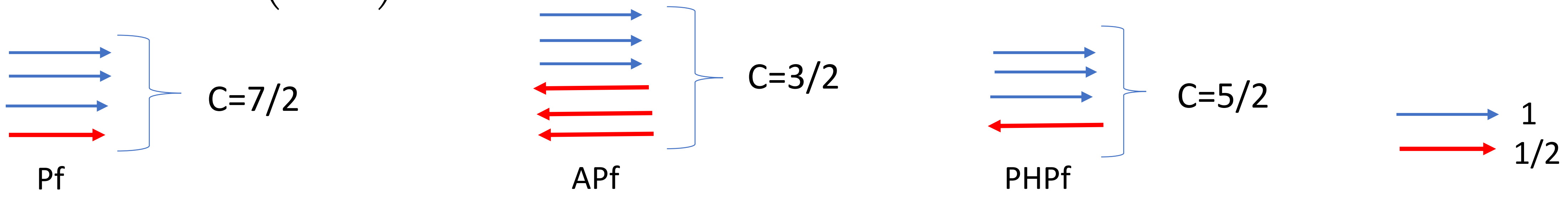
- Another competing state: **Particle-Hole symmetric Pfaffian** (PHPf) predicting Majorana mode.

Son, PRX,2015
Zucker and Feldman,PRL, 2016

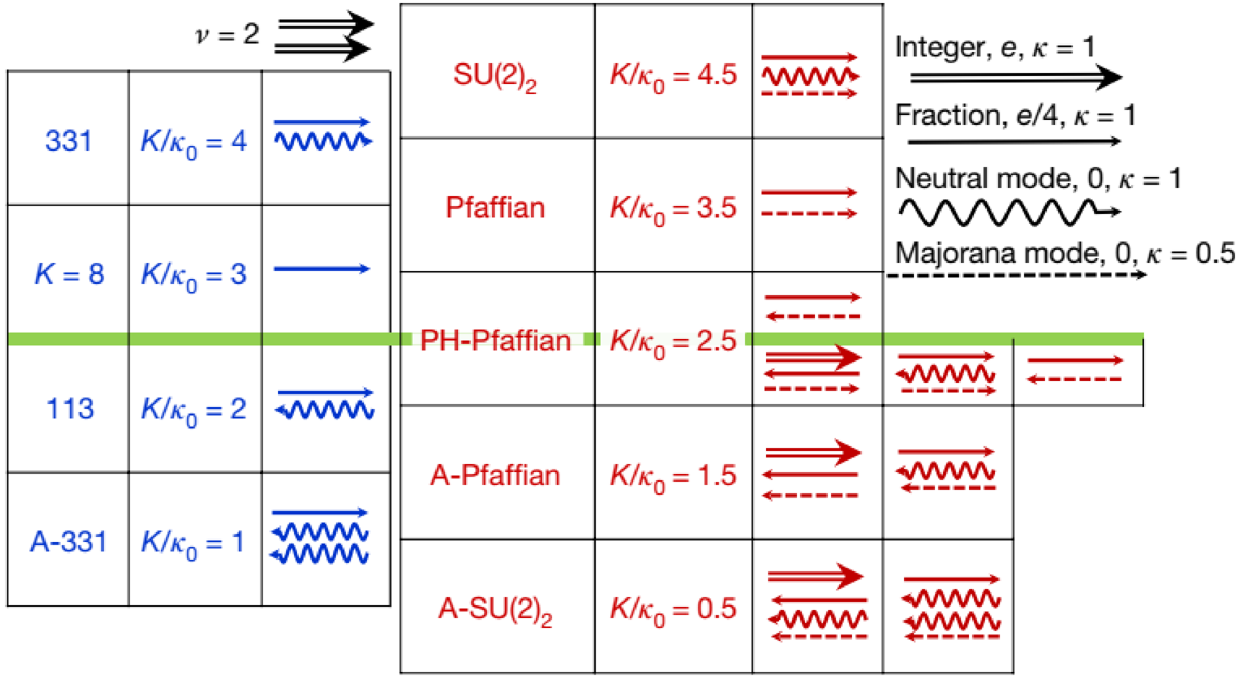
- These competing states can be distinguished in the measurement of **thermal Hall conductance** through edge.

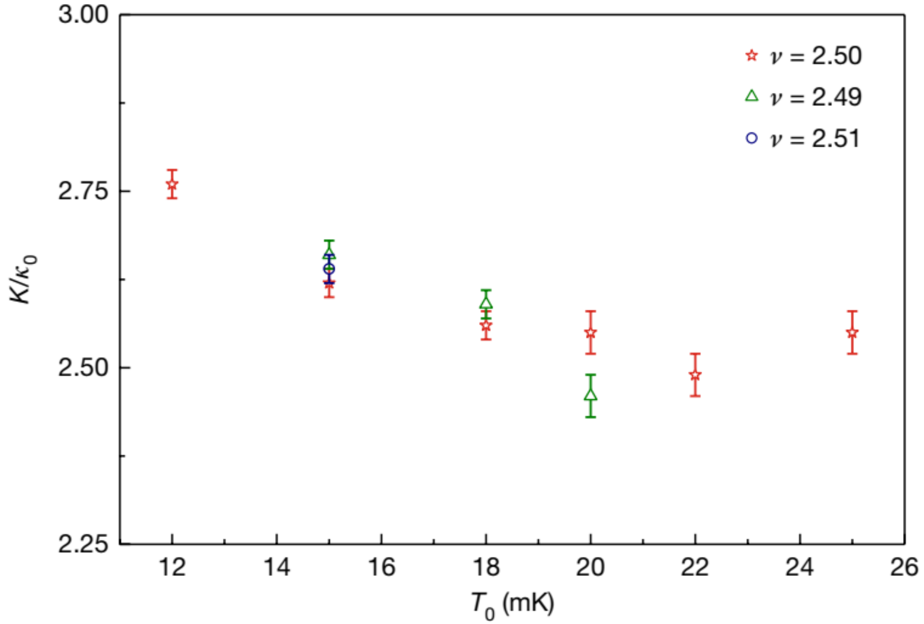
$$\kappa_H = C\left(\frac{\pi^2 k_B^2 T}{3h}\right) \qquad C = C_d - C_u$$

C is the net central charge of the ground state.



Measurement of Thermal Hall Conductivity



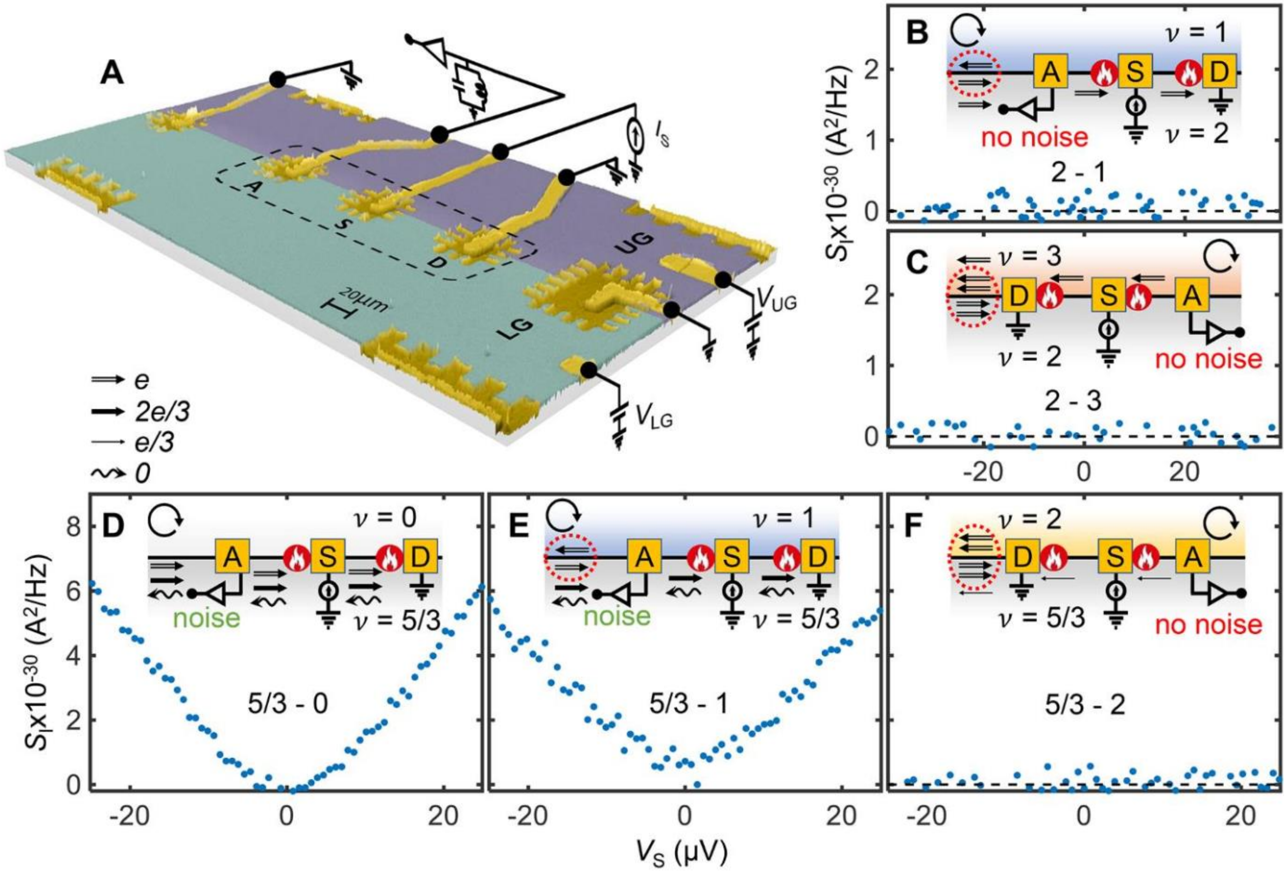


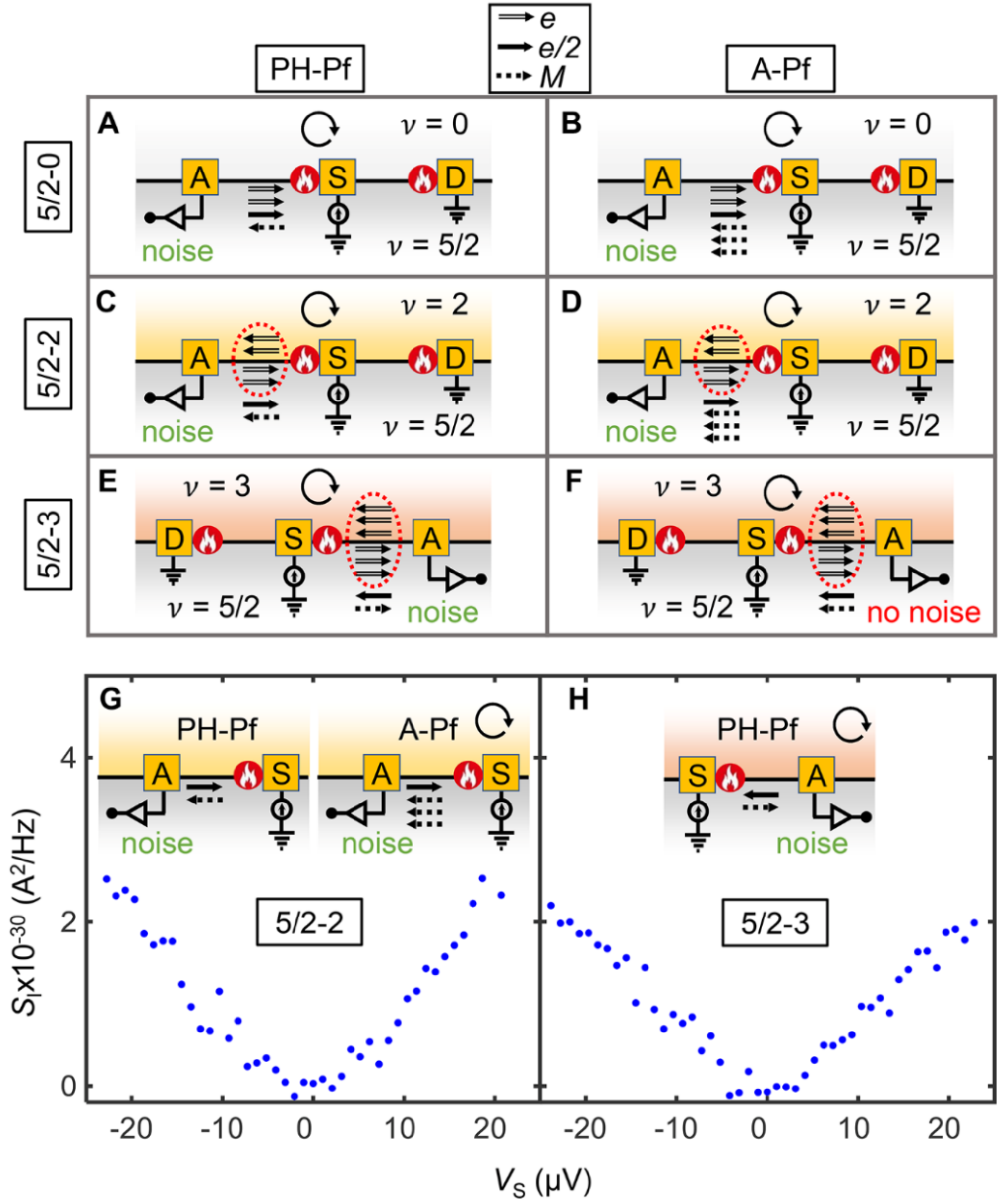
Banerjee et al, Nature, 2018

$$\kappa_0 = \frac{\pi^2 k_B^2 T}{3h}$$

In contrast to the numerical verdict, measurement suggests PHPf topological order!!

Shot-Noise Experiment





PHPf order is recommended!

Dutta et al, Science, 2022

- No clear understanding of experimental results.

$$\kappa = \frac{e^2/\epsilon\ell_0}{\hbar\omega_c}$$

- Numerical studies are generally for relatively low values of Landau-level-mixing (LLM) parameter: $\kappa < 1$, while experimental systems typically have $\kappa = 0.8 - 1.8$.

$$\hat{H}_{\text{eff}}(\kappa) = \sum_m \left[V_m^{(2)} + \kappa\, \delta V_m^{(2)}\right] \sum_{i<j} \hat{P}_{ij}(m) + \sum_m \kappa\, V_m^{(3)} \sum_{i<j<k} \hat{P}_{ijk}(m),$$

$V_m^{(2)} \Rightarrow$ *Coulomb pseudopotentials in the second LL.*

$\delta V_m^{(2)} \Rightarrow$ *two body pseudopotentials arising due to LLM.*

$\delta V_m^{(3)} \Rightarrow$ *three body pseudopotentials arising due to LLM.*

Peterson and Nayak, PRB, 2013

The pseudopotentials are estimated for $m \leq 8$ diagrammatically, using κ as perturbing parameter, i.e., terms with κ^2 are ignored.

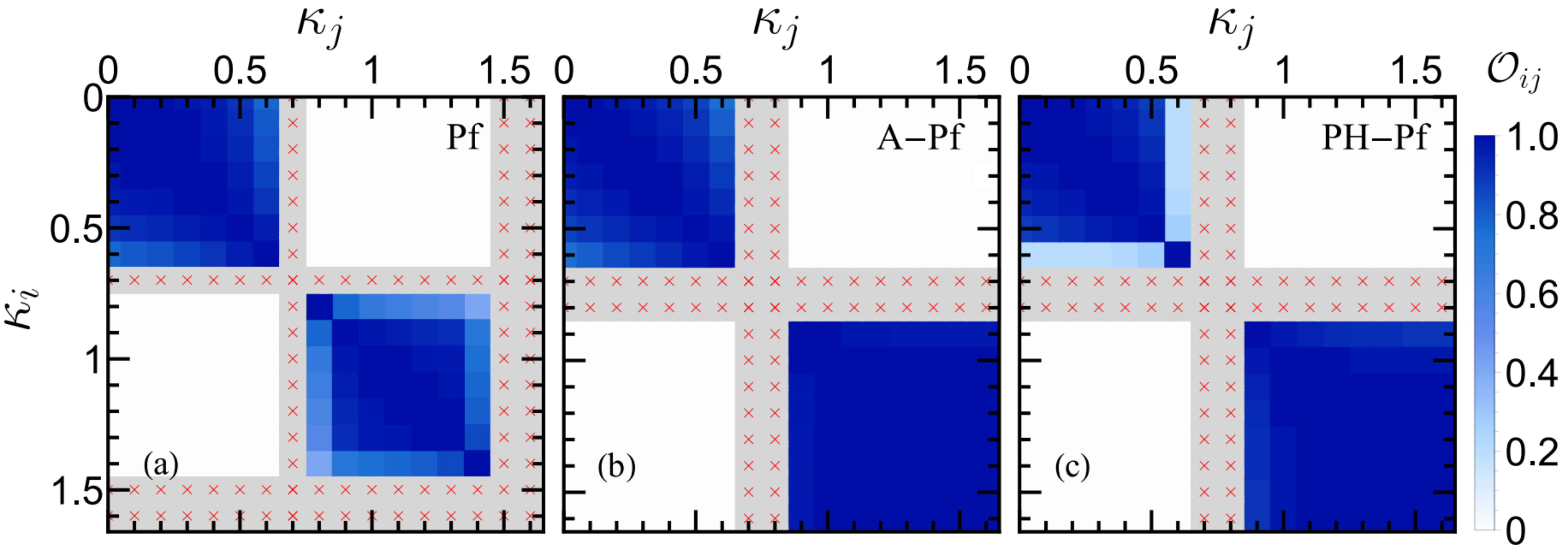
Unless κ is too high or comparing energies with competing states, these pseudopotentials still may be used for moderate κ for qualitative physics.

Phase Transition into a Re-entrant Quantized Phase

- Exact diagonalization at different values of κ : $\Psi_{EX}\ (\kappa_i)$

Three body pseudopotentials for $m \leq 8$.

- If found quantized (Groundstate at L=0) : $O_{ij} = \langle\ \Psi_{EX}(\kappa_i)\ |\Psi_{EX}\ (\kappa_j)\rangle$



Pf: $N = 14, 2Q = 25$ APf: $N = 12, 2Q = 25$ PHPf: $N = 14, 2Q = 27$

Das, Das, **SSM**, arXiv:2206.0441

- Clear separation of two quantized phases, irrespective of three flux shifts.

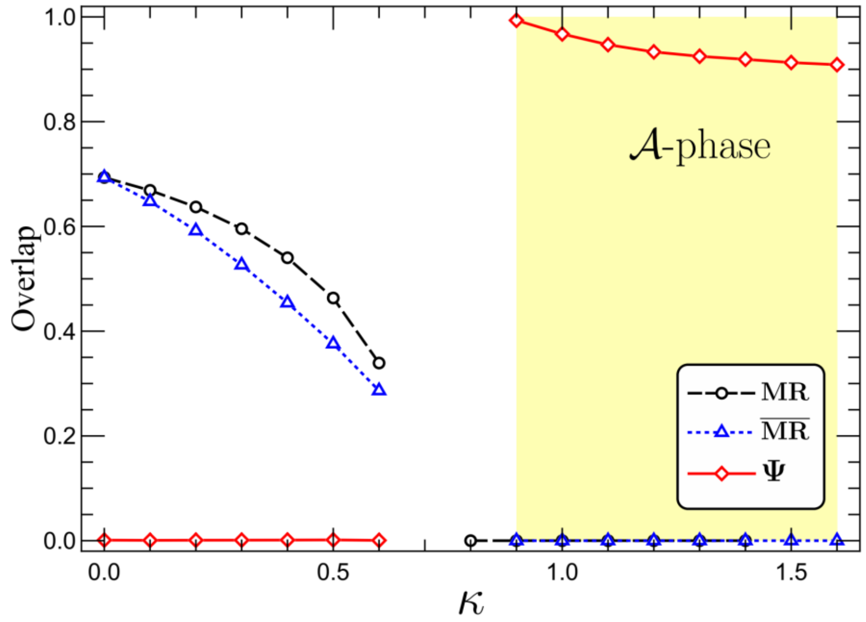
Das, Das and **SSM**, arXiv:2206.0441

Proposed Ground State wave function for the Novel Phase

$$\Psi = \prod_{i<j}^{N} U_{i,j} \; S[\prod_{k=1}^{\frac{N}{2}} \prod_{l=\frac{N}{2}+1}^{N} U_{k,l}^{2} \;]$$

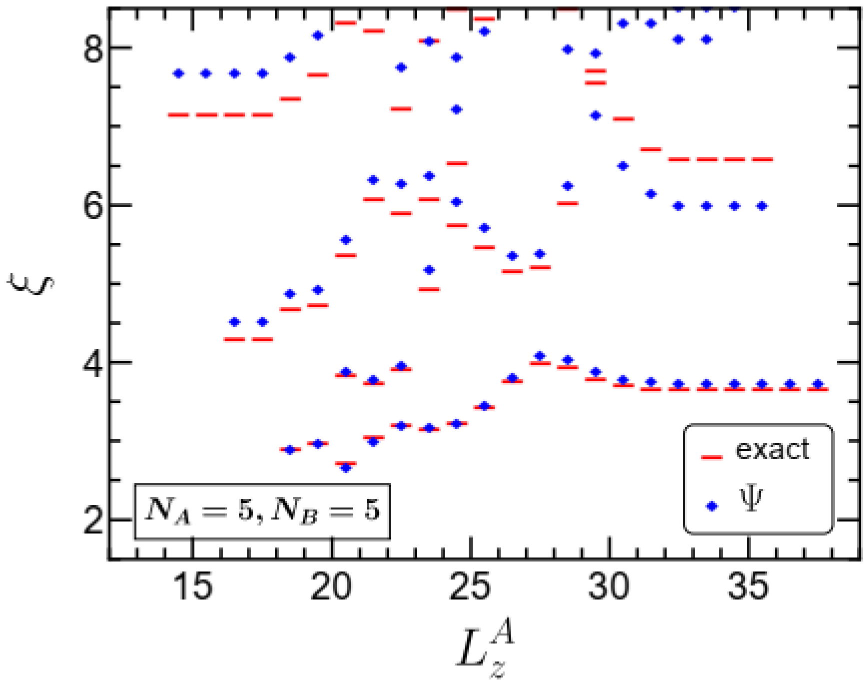
Ψ is consistent with PHPf $-$ shift, $2Q = 2N - 1$

However, Ψ doesn't have particle $-$ hole symmetry.



- **Composite bosons segregate (but no physical segregation) themselves into two groups.**
- **Intra-group non-interacting composite bosons form BEC.**
- **Inter-group composite bosons strongly repel each other.**
- **Like other non-Abelian FQHE ground state wave functions, ignoring minimal ubiquitous Pauli-exclusion term, the rest is nonzero when two or more particles coincide.**
- **Here, macroscopic N/2 particles may coincide.**
- **The proposed wave function Is possibly non-Abelian in nature.**

Entanglement Spectra



The proposed wave function captures the topological order of the state.

Topological Properties of the Proposed Wave Function

$$\Psi(\{u_i, v_i\}) = \prod_{i<j}^{N} (u_i v_j - u_j v_i)$$
$$\times \mathcal{S}\left[\prod_{1\leqslant k,l\leqslant N/2} \left(u_k v_{N/2+l} - u_{N/2+l} v_k\right)^2\right]$$

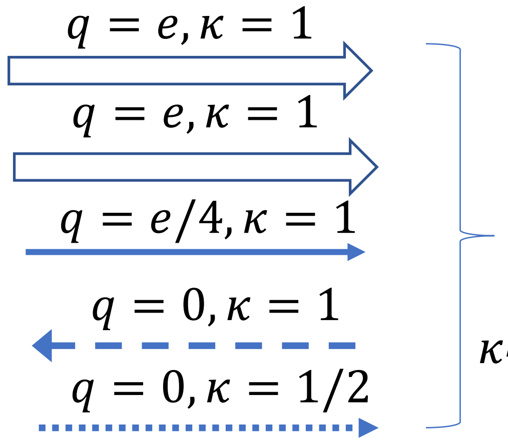
Chern – Simons Lagrangian:

$$L = -\frac{1}{4\pi} a\, K\, \partial a + \frac{1}{2\pi} t\, A\, \partial a$$

$$K = \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix},\ t = \begin{pmatrix} 1 \\ 1 \end{pmatrix},\ l = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad s = \begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix}$$

- Filling factor: $\nu = t^T K^{-1} t = 1/2$
- Quasihole charge: $q/e = l^T K^{-1} t = 1/4$
- Topological Shift: $S = \left(\frac{2}{\nu}\right) t^T K^{-1} s = 1$
- Groundstate Degeneracy: $D = |Det(K)|^g = 8^g$
- Eigenvalues of K: One + ve and one − ve $\Rightarrow$ Central Charge $C = 0$

- Macroscopic $\frac{N}{2}$ bosons may coincide.
- Hidden Z_2 symmetry.
- Neutral mode with central charge $C = 1/2$



C = 3-1+1/2

$$\kappa_T = 2.5\left(\frac{\pi^2 k_B^2 T}{3h}\right)$$

OUTLOOK

- The topological orders for the quantum Hall states in the second Landau level at **moderate to high** Landau-level-mixing strength needs to be explored for knowing true nature of **non-Abelian** FQHE states.

- The nature of transitions between two consecutive IQHE/FQHE states is yet unexplored.

- Nonequilibrium phenomenon between two consecutive IQHE/FQHE states should also be an interesting direction.