Composite fermions: Wavefunctions, numerical strategies, and some results on an inverse problem

Sreejith G J

IISER Pune

Abhishek Anand (PhD IISER Pune)
Aditya Kolhatkar (BSMS IISER Pune → Cornell)
Rushikesh Patil (BSMS IISER Pune → UC Santa Barbara)
G J Henderson, B Karmakar, K Kudo, S Das, S Pu, M Fremling,
G S Jeon, J K Jain, S H Simon, A C Balram, Y H Wu

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PRL 125, 076802 (2020)


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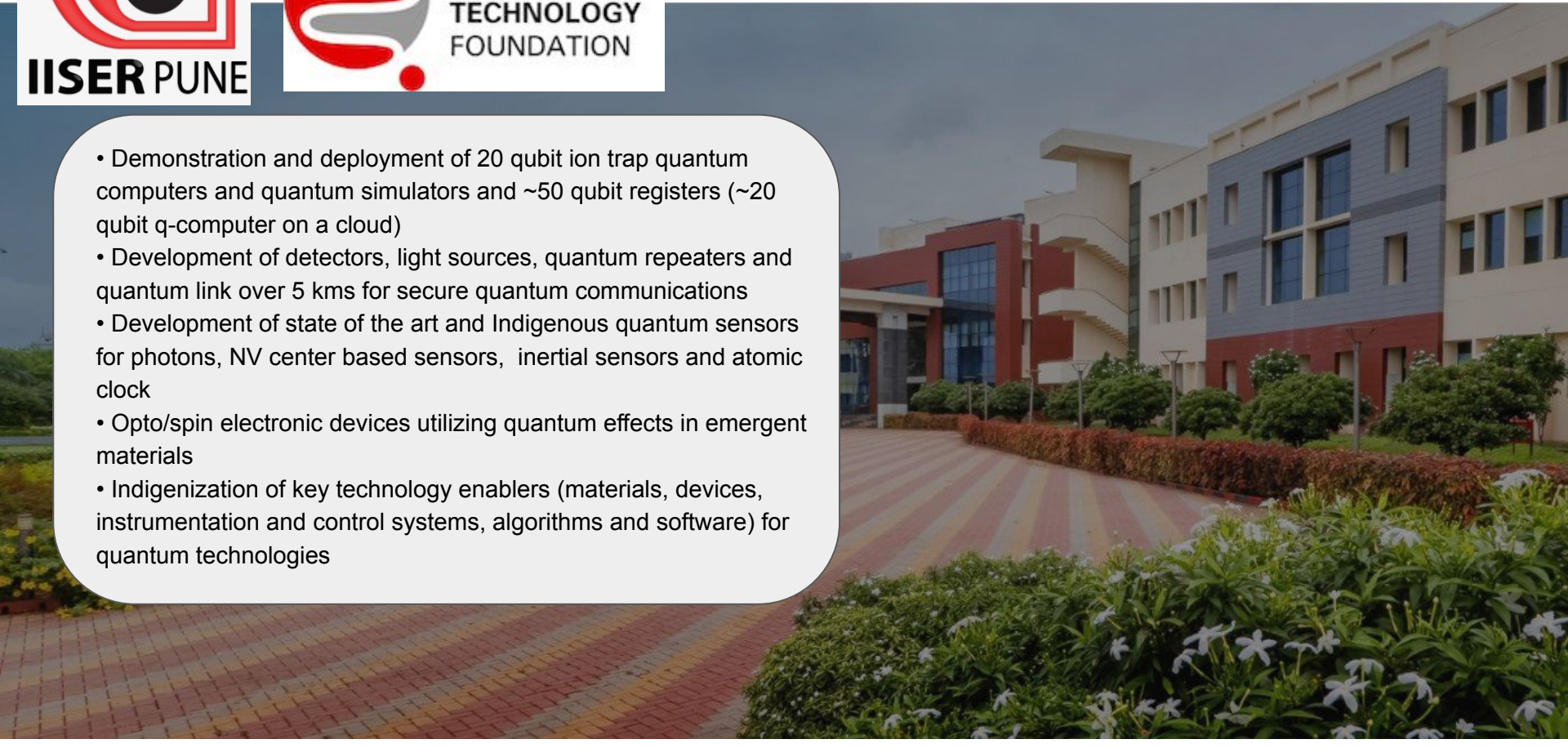
I-HUB QUANTUM TECHNOLOGY FOUNDATION

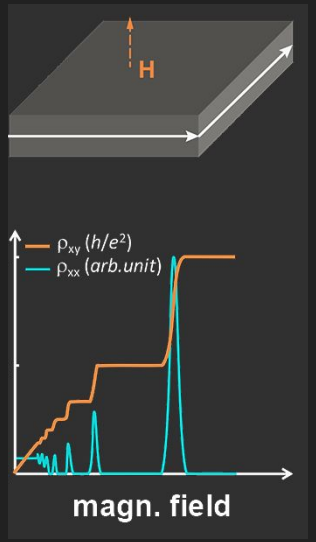


- Demonstration and deployment of 20 qubit ion trap quantum computers and quantum simulators and ~50 qubit registers (~20 qubit q-computer on a cloud)
- Development of detectors, light sources, quantum repeaters and quantum link over 5 kms for secure quantum communications
- Development of state of the art and Indigenous quantum sensors for photons, NV center based sensors, inertial sensors and atomic clock
- Opto/spin electronic devices utilizing quantum effects in emergent materials
- Indigenization of key technology enablers (materials, devices, instrumentation and control systems, algorithms and software) for quantum technologies

Plan

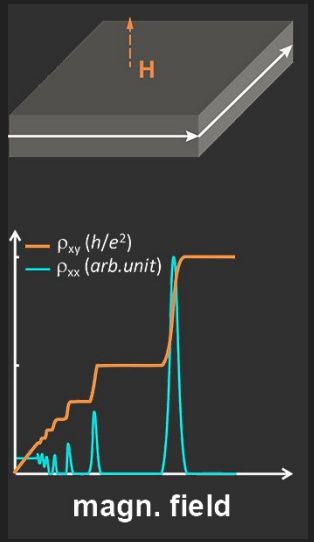
- QHE, FQHE, Variational wavefunctions, Laughlin State, Composite fermions, Excitations, **Entanglement spectra**
- Correlations and parent Hamiltonians
- An inverse problem:
  - Root partitions and constraints on the Hamiltonians
- **A modified problem and a formally exactly solvable model on multiple Landau levels**
- ***Extensions beyond - to Pfaffian*** *(arxiv: 2206.07789)*

QHE, IQHE



Klitzing et al, PRL 1980
Laughlin PRB 1981
Halperin 1982
Thouless PRB 1983

QHE, IQHE



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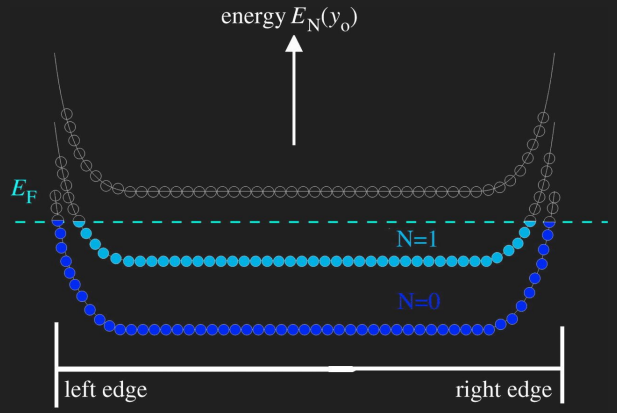


$$z^m \exp(-|z|^2)$$

Angular momentum

$$\text{Degeneracy at 1T} = \frac{B}{h/e} \sim 2.4 \times 10^{10}/\text{cm}^2$$

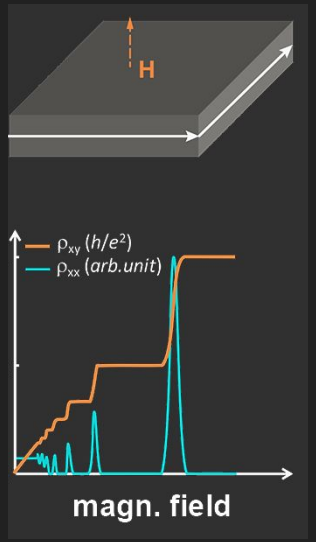
$$\text{Length scales} \sim \sqrt{\frac{\hbar}{eB}}\ 30nm$$

$$\text{cycl. gap} = \frac{\hbar eB}{m} \sim \mathcal{O}(1)K$$

$$R_{xy} = \frac{V_x}{I_y} \sim \frac{h}{ne^2}$$

$$25812/n \text{ Ohms}$$

QHE, IQHE



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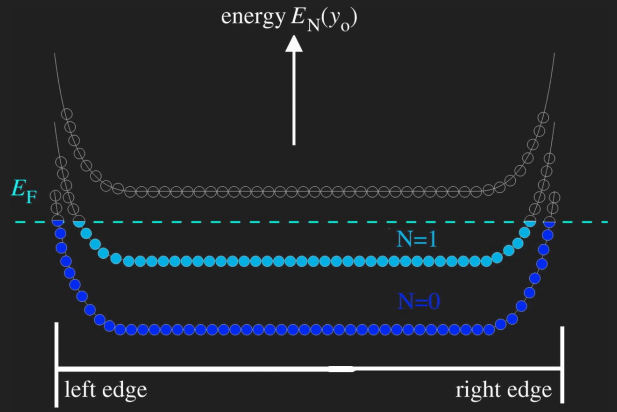
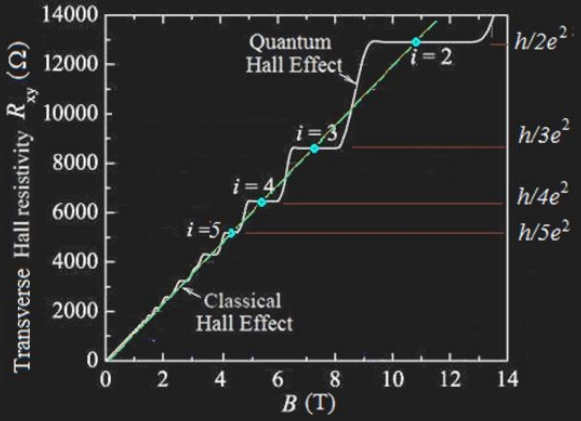
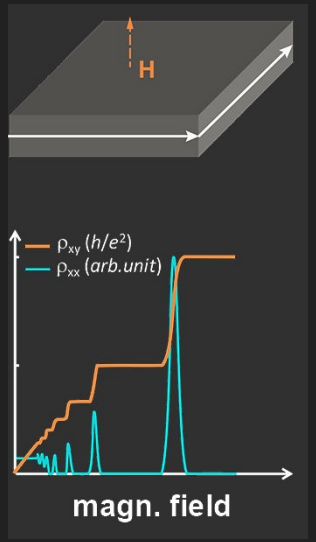
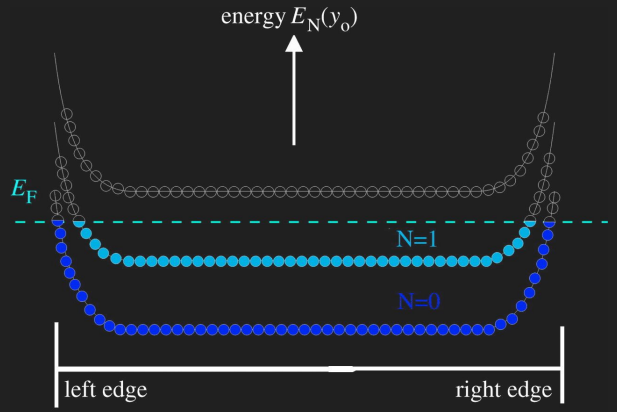


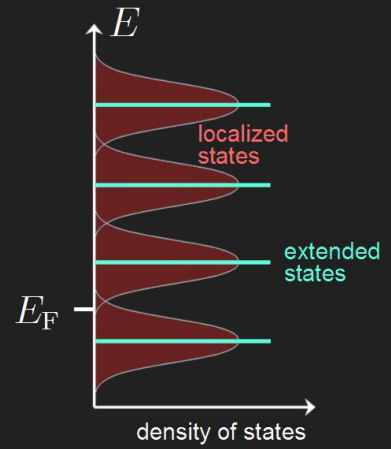


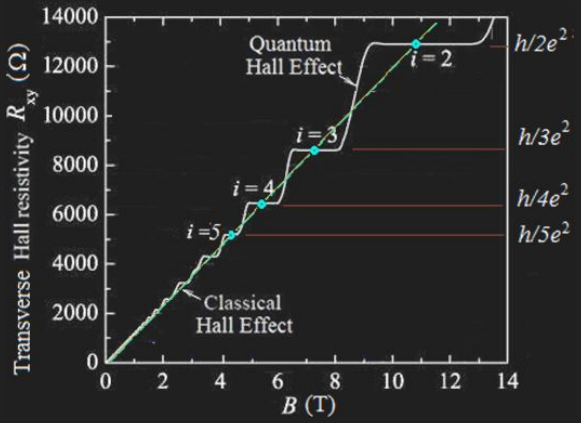
QHE, IQHE

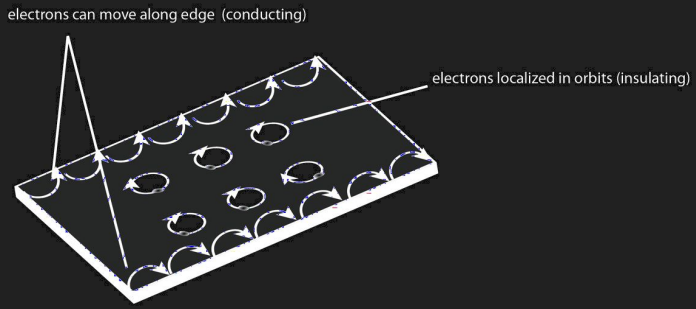


Klitzing et al, PRL 1980
Laughlin PRB 1981
Halperin 1982
Thouless PRB 1983
Chalker Coddington, Prange
Joynt, ….
Huckstein (RMP 1995)









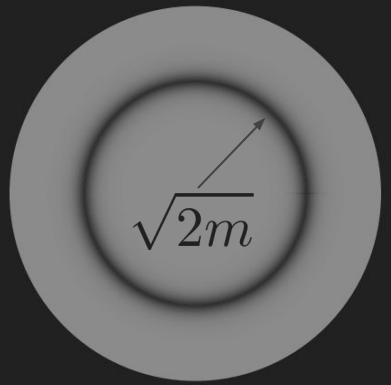
Beautiful physics bringing together ideas of percolation, localization, topology, critical phenomena, random matrices…

Background - Landau quantization & many body wavefunctions

Angular momentum *m = 0,1,2….*

$$\psi_m \sim z^m \mu(|z|)$$

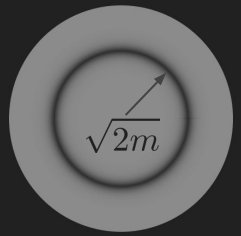
Measure that depends on the geometry



$$\exp\left(-\frac{|z|^2}{4}\right)$$

Background - Landau quantization & many body wavefunctions

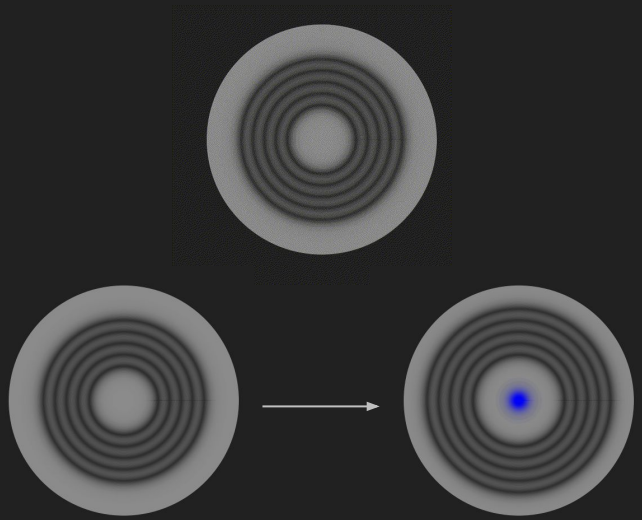
$$\psi_m \sim z^m \mu\left(|z|\right)$$



$$c_1 \left|m_1, m_2, m_3\right\rangle + c_2 \left|n_1, n_2, n_3\right\rangle + \ldots$$

$$C_1 S[z_1^{m_1} z_2^{m_2} z_3^{m3}] + C_1 S[z_1^{n_1} z_2^{n_2} z_3^{n_3}] + \ldots$$

Background - Inserted flux

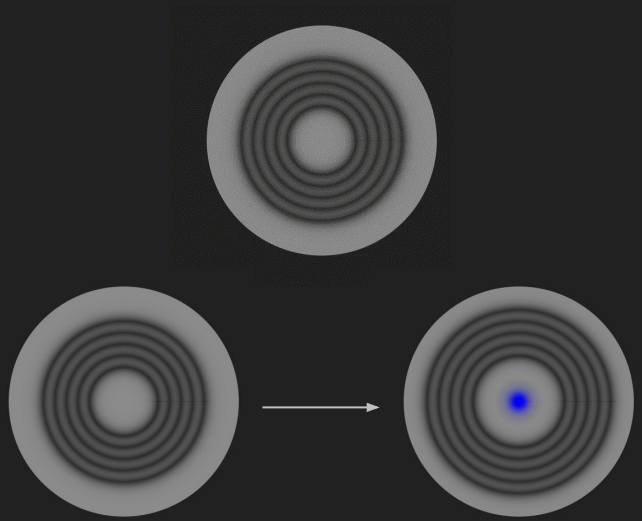


Insertion of flux moves each single particle states out by 1 orbital

- If Landau level is fully filled → Many body state is “incompressible” → Gapped
- Gap ensures applicability of adiabatic insertion. Electrons stay in the radially drifting orbitals.

Laughlin PRB 1981; Halperin PRB 1982; Byers Yang PRL 1961;

Background - Inserted flux



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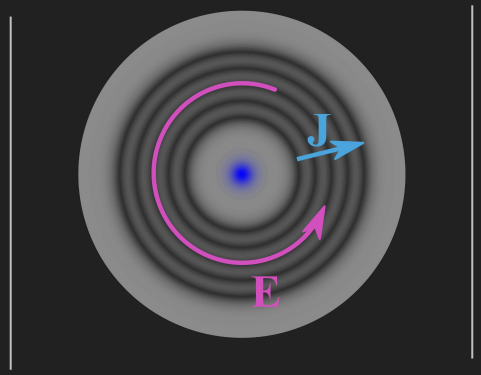

$$\sigma_H = \frac{J}{E} = \frac{e}{\Phi_0} = \frac{e^2}{h}$$

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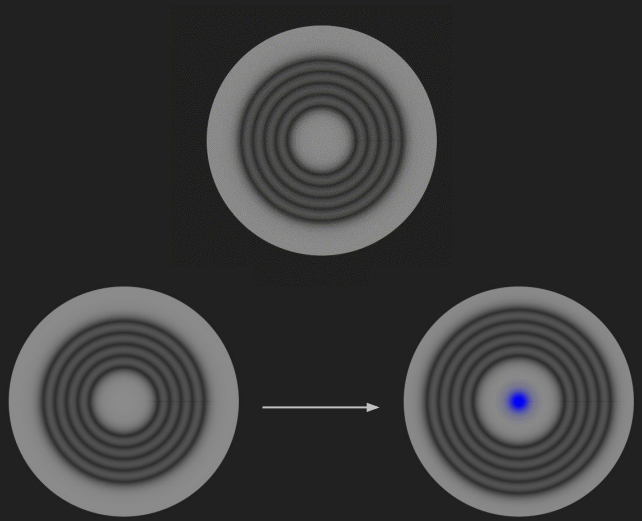

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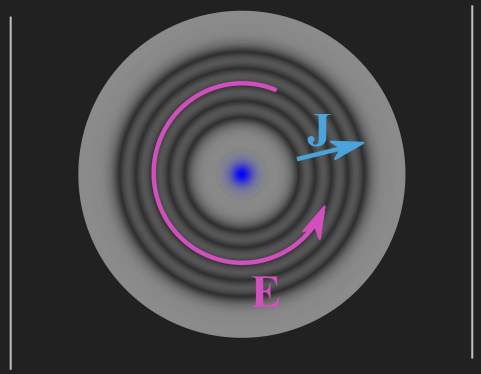

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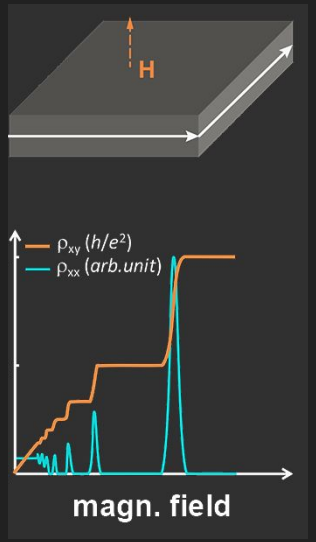
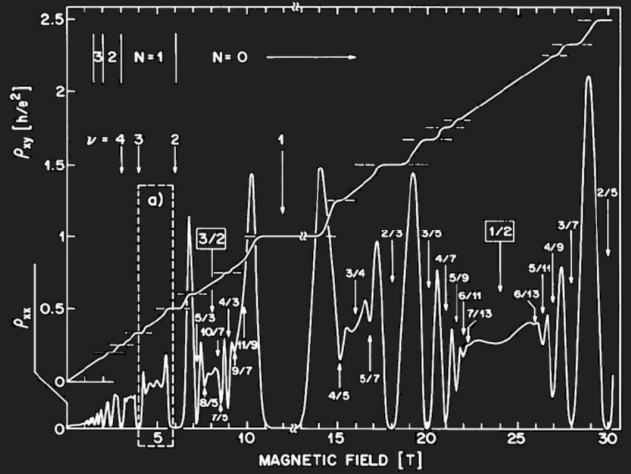
QHE, FQHE

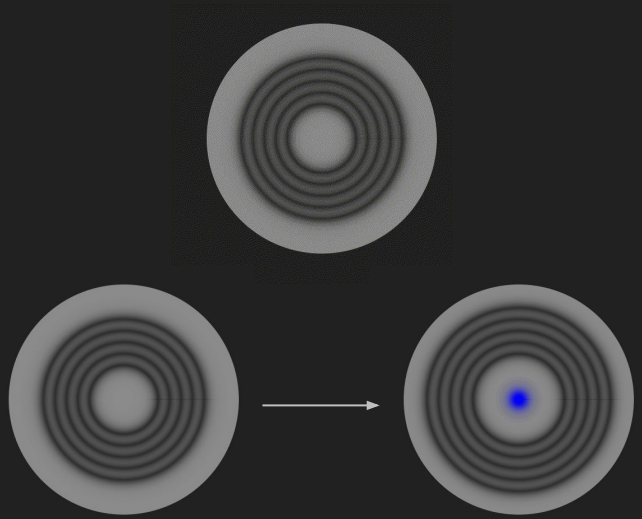


Tsui Stormer Gossard, PRL 1982;
Laughlin 1983
Haldane 1983

- Rich set of distinct phases
- Phases characterized not by broken symmetry
- No local order parameter
- FQHE: Rich physics arising entirely from strong interactions - peculiar in its tractability in terms of quantum many body wavefunctions



Background - Inserted flux



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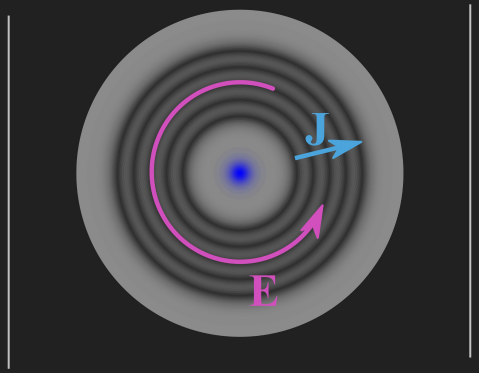

$$\sigma_H = \frac{J}{E} = \frac{e}{\Phi_0} = \frac{e^2}{h}$$

$$\sigma_H = \nu \frac{e^2}{h}$$

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Laughlin PRB 1981; Halperin PRB 1982; Byers Yang PRL 1961; Pruisken, Chalker, Coddington...
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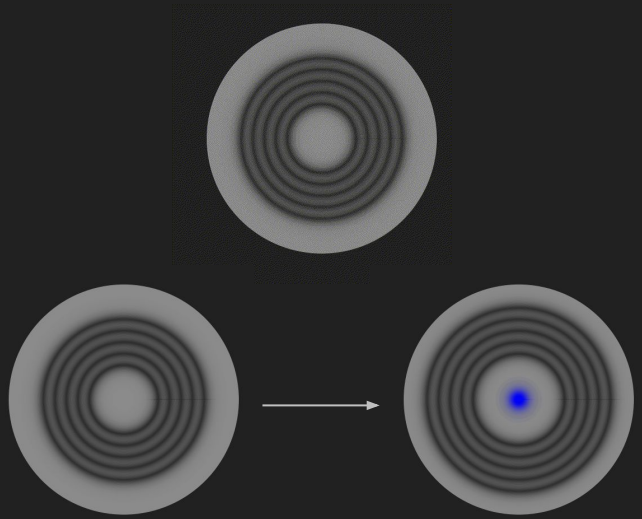

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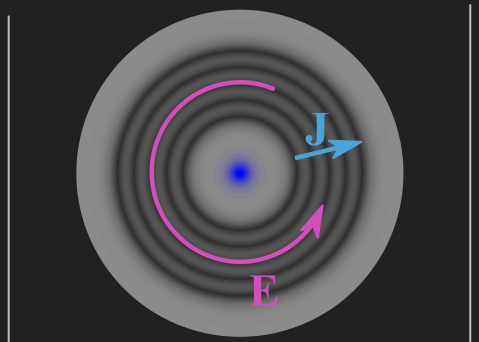

$$\sigma_H = \frac{J}{E} = \frac{e}{\Phi_0} = \frac{e^2}{h}$$

$$\sigma_H = \nu \frac{e^2}{h}$$

But the fractionally occupied systems cannot be gapped *in the absence of interactions* !

- If Landau level is fully filled → Many body state is "incompressible" → Gapped
- Gap ensures applicability of adiabatic insertion. Electrons stay in the radially drifting orbitals.

Laughlin PRB 1981; Halperin PRB 1982; Byers Yang PRL 1961; Pruisken, Chalker, Coddington...
Laughlin PRL 1983; Halperin PRL 1983; Arovas Schrieffer Wilczek 1984; Kivelson Rocek 1985

Gapped states: Composite fermion wavefunctions

$$KE + \frac{1}{r}$$

Jain PRL 1989, Halperin Lee Read 1993

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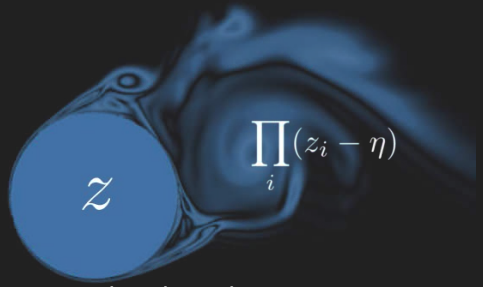

In the ground state, every electron is in the proximity of *2p* number of vortices of other electrons. → "Holomorphicity" of single particle states allows to relate correlation holes as vortices.

Statistically, |wavefunction| is higher when the electron coordinate is in the proximity of "zeros" of other electrons. Within the space of such correlated states, interactions are unimportant.
⇒ *These composite fermions are weakly interacting*

Statistically the electrons feel a reduced effective field

$$B^* = B - 2p\rho$$

Jain PRL 1989, Halperin Lee Read 1993

Gapped states: Composite fermion wavefunctions

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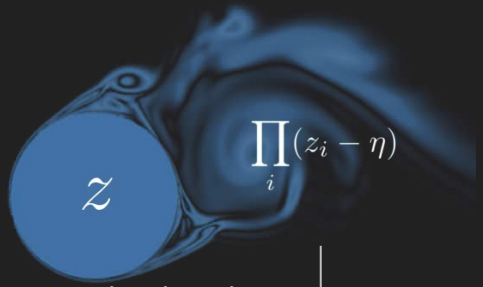

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2p vortices of every electron on every other

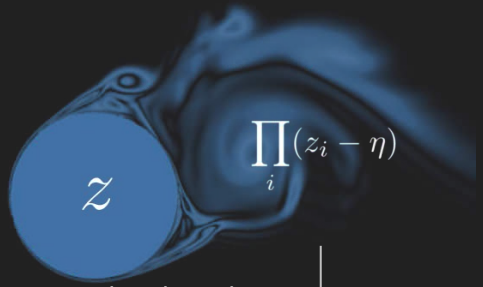
$$\prod_{i<j=1}^{N} (z_i - z_j)^{2p} \times \Psi_n$$

WF of *n* filled Landau levels of fermions

Jain PRL 1989, Halperin Lee Read 1993

Gapped states: Composite fermion wavefunctions

$$KE + \frac{1}{r}$$



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$$\prod_{i<j=1}^{N} (z_i - z_j)^{2p} \times \Psi_n$$

The wf has a small component in higher LLs.
Projection of this to the LLL is energetically better.

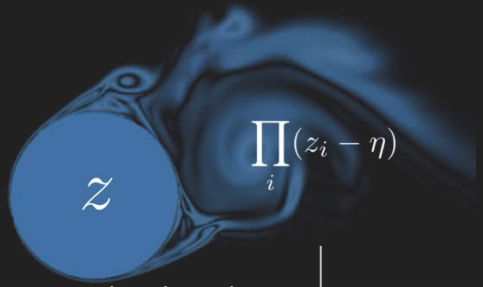
$$\mathcal{P}_{\mathrm{LLL}} \prod_{i<j=1}^{N} (z_i - z_j)^{2p} \times \Psi_n$$

$$\nu = \frac{n}{2pn+1}$$

Jain PRL 1989
Jain Kivelson Trivedi PRL 1990

Gapped states: Composite fermion wavefunctions

$$KE + \frac{1}{r}$$



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$$\nu = \frac{n}{2pn+1}$$

Example: Laughlin ⅓ state corresponds to 2p=2 fluxes attached to electrons. These objects fill one LL ie *n=1.*

$$\prod_{i<j=1}^{N} (z_i - z_j)^2 \times \prod_{i<j=1}^{N} (z_i - z_j)$$

Jain PRL 1989
Jain Kivelson Trivedi PRL 1990

Aside: Beyond the simplest cases

$$\mathcal{P}_{\mathrm{LLL}} \prod_{i<j=1}^{N} (z_i - z_j)^{2p} \times \Psi_n$$

$$\nu = \frac{n}{2pn+1}$$

Describes almost all single component states below *v*=1/2p

1/3, 1/5, 1/7…

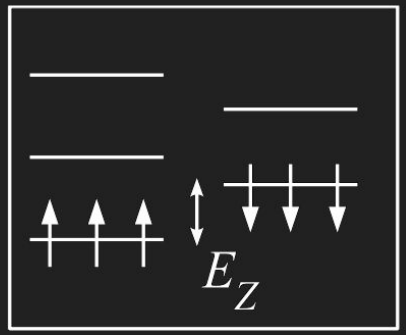
2/5, 3/7,4/9…

$$\mathcal{P}_{\mathrm{LLL}} \prod_{i<j=1}^{N} (z_i - z_j)^{2p} \times \overline{\Psi_n}$$

$$\nu = \frac{n}{2pn-1}$$

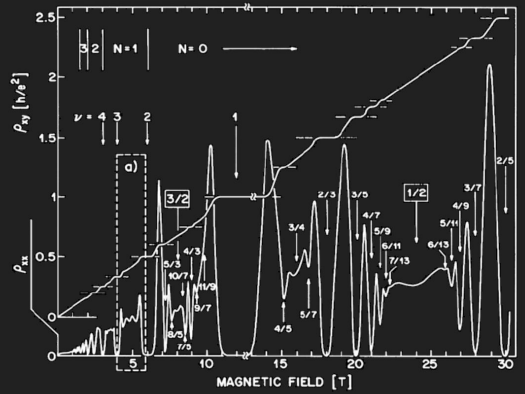
Single component states above *v*=1/2p

2/3, 3/7…

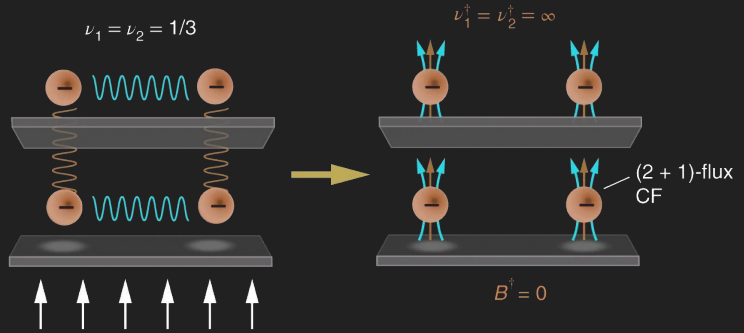


Spin unpolarized states

R R Du et al PRL 1995, Park Jain PRL 1998, Ashoori Nat Phy 2020 …, S Davenport, Simon PRB 2012



Multicomponent cases: Valley, Spin, Multilayer



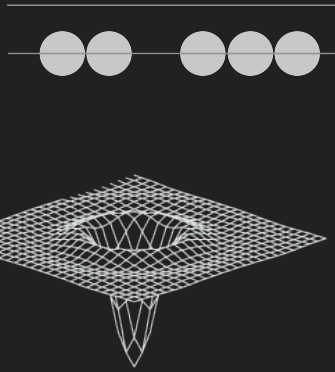
Bolotin et al Nature 2009, Liu et al Nature Physics 2019, Li et al Nature Physics 2019

Describing low energy excitations

Mapping from the FQHE of electrons to IQHE of composite fermions works far beyond the gapped states. Though tedious, these can be numerically compared with finite ED calculations. All symmetry quantum numbers and

Quasiholes of charge *e/(2pn+1)*

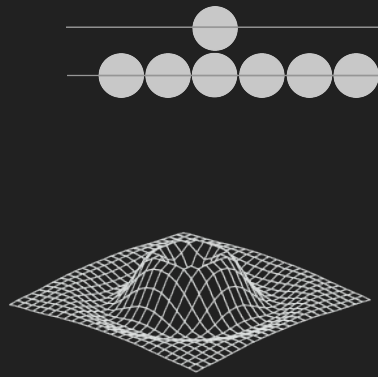
$$\mathcal{P}_{\rm LLL} \prod_{i<j=1}^{N} (z_i - z_j)^{2p} \times$$



K Park, 2000

Quasiholes of charge *e/(2pn+1)*

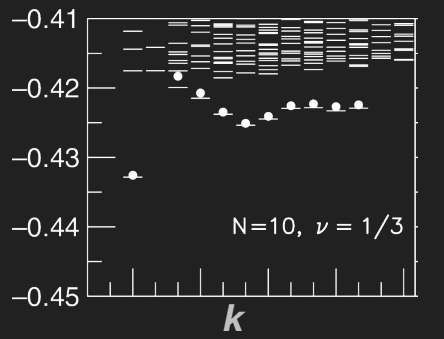
$$\mathcal{P}_{\rm LLL} \prod_{i<j=1}^{N} (z_i - z_j)^{2p} \times$$



K Park, 2000

Particle-hole exciton mode

$$\mathcal{P}_{\rm LLL} \prod_{i<j=1}^{N} (z_i - z_j)^{2p} \times$$



Kamila, Jain 1998

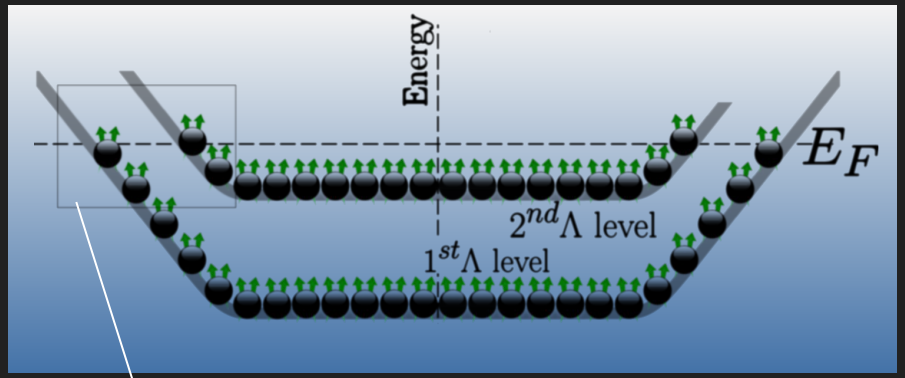
Charged, spin skyrmions

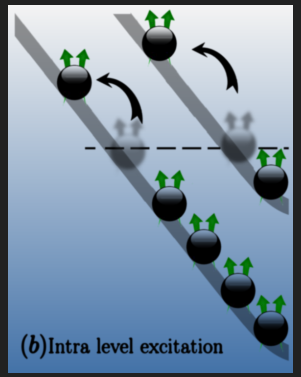
$$\mathcal{P}_{\rm LLL} \prod_{i<j=1}^{N} (z_i - z_j)^{2p} \times$$

Sondhi et al PRB 1993

Balram et al Nat Comm 2015

Wavefunctions of gapless edge excitations





Edge of each CF Landau level corresponds to a chiral bosonic 1D mode with velocity determined by the confinement potential.

Filling fraction

$$\nu = \frac{n}{2pn+1}$$

is associated with *n* filled CF Landau levels.
⇒ Its edge is described by *n* chiral bosons.

Wen 1992, Wen 1995, Chang, Rev Mod Phys 2003, Moore Haldane PRB 1997,Wu Sreejith Jain PRB 2012, Sreejith, Jolad, Sen, Jain PRB 2011

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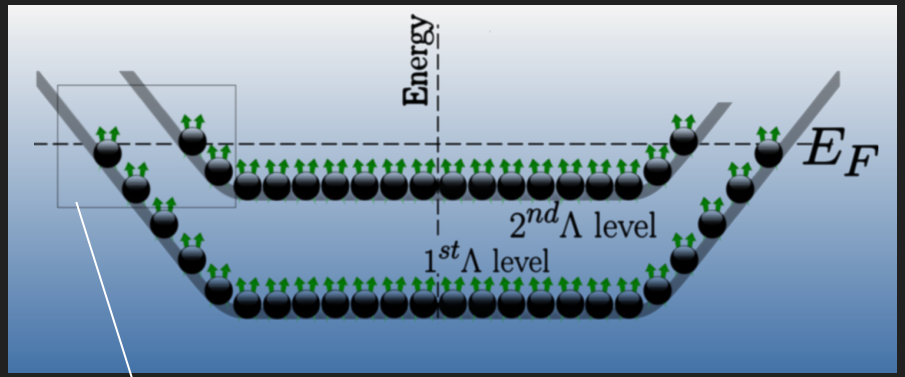

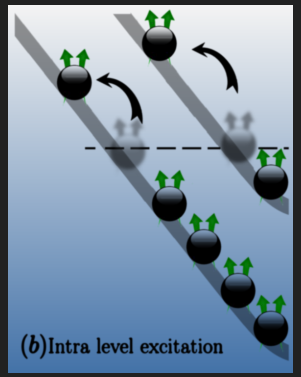

Edge structures can be significantly more complex due to remnant interactions and confinement effects - lead to edge reconstructions, fractionalization at the edge even in IQHE…

See
PRL 125, 076802 (2020)

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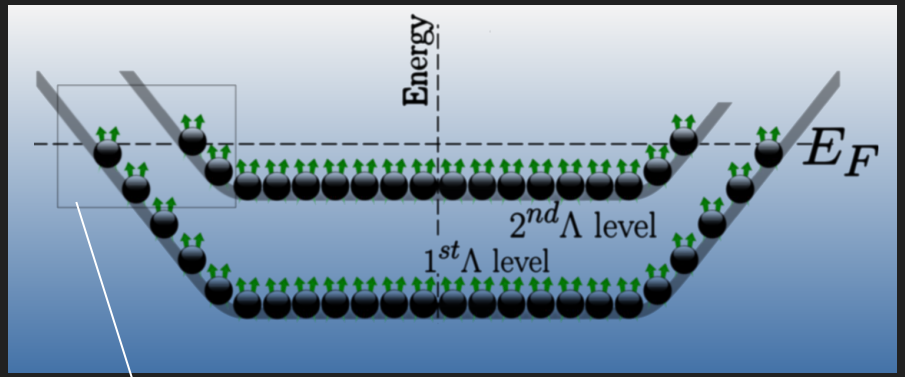
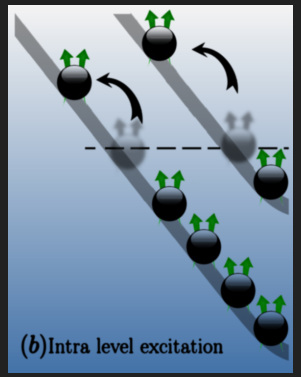
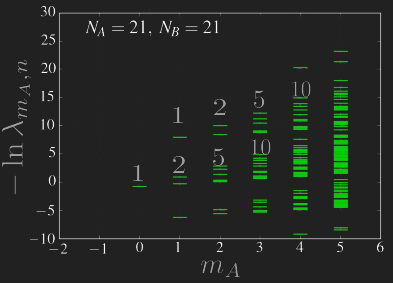




Also shows in the entanglement spectrum



Henderson, Sreejith, Simon 2021, Li, Haldane 2007; Dubail, Read Rezayi 2012; Anand, Rushikesh, Balram, Sreejith 2022;

Sreejith, S Jolad, D Sen, Jain PRB 2012

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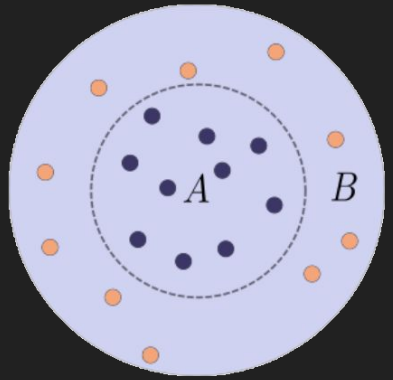
Real space entanglement spectra as a proxy for edge excitations



B essentially acts as a confinement to A in a sense.

A if measured will be found most likely in the GS of A or in one of the low energy excitations of A → which are the edge excitations of A.

The eigenvalues, eigenstates of the reduced density matrix gives the probability of finding A in these low energy edge excitation states. Close energy eigenstates should have similar probability of occurrence.

Reduced density matrix is block diagonal in angular momenta (for rot-symmetric cut)

⇒ **Counting** of low “entanglement energy ” (*-log eigenvalue*) states convey the information about the counting of edge excitations at each angular momenta.

Li, Haldane 2007,
Dubail Read Rezayi 2012, Qi Katsura Ludwig 2011, Henderson, Sreejith, Simon PRB 2021, Rodriguez, Simon, Slingerland PRL 2012, A Chandran et al PRB 2011, Sterdyniak et al PRB 2012

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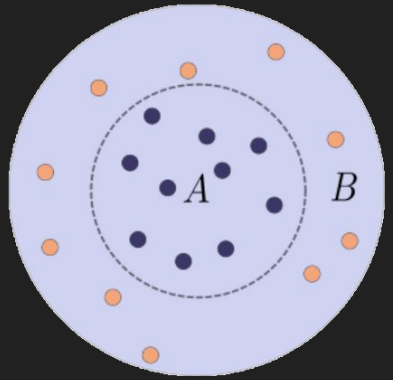


Counting of low “entanglement energy ” (*-log eigenvalue*) states contain the information about the counting of edge excitations at each angular momenta.

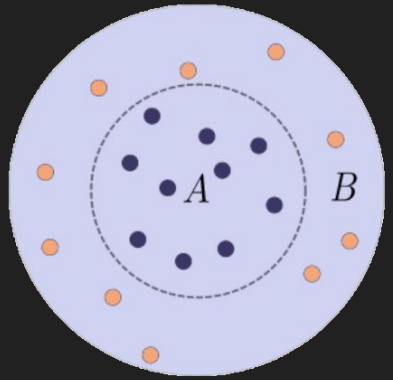
In simple states, edge counting can be inferred from the ansatz. In cases where there is an ansatz for the GS alone, ES can be an unbiased way to obtain identify the edge counting.

→ Computing the entanglement spectra is computationally expensive in general.

Reduced density matrix is huge ~ dim(A) x dim(A) for even very small systems.

Li, Haldane 2007,
Dubail Read Rezayi 2012, Qi Katsura Ludwig 2011, Henderson, Sreejith, Simon PRB 2021, Rodriguez, Simon, Slingerland PRL 2012, A Chandran et al PRB 2011, Sterdyniak et al PRB 2012

Efficient computation of Real space entanglement spectrum



Eigenvalues of the reduced density matrix can be efficiently computed for wavefunctions that can be written as product of Slater determinants

→ includes CF states
→ Parton states (next talk)

IDEA

$$\sum_{i=1}^{\dim_A} \sum_{j=1}^{\dim_B} c_{ij} \, |i\rangle \, |j\rangle$$

Difficult

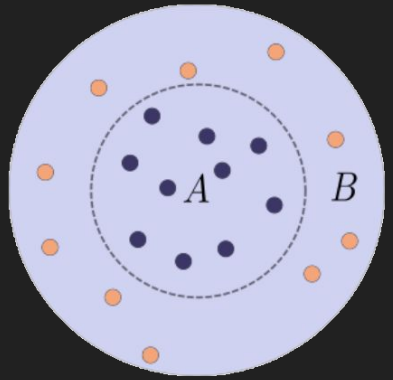
$$\sum_{i=1}^{\Lambda_A} \sum_{j=1}^{\Lambda_B} c_{ij} \, |i\rangle \, |j\rangle$$

Tractable if we can explicitly write down states i, j and c_ij
And if Λ_B, Λ_A are small.

A Anand, Rushikesh Patil, AC Balram, GJ Sreejith PRB 2022
G J Henderson, GJ Sreejith, S H Simon PRB 2021, I Rodriguez, S H Simon, J Slingerland PRL 2012

Efficient computation of Real space entanglement spectrum

$$\psi \sim \prod (z_i - z_j)^{2p} \psi_n$$



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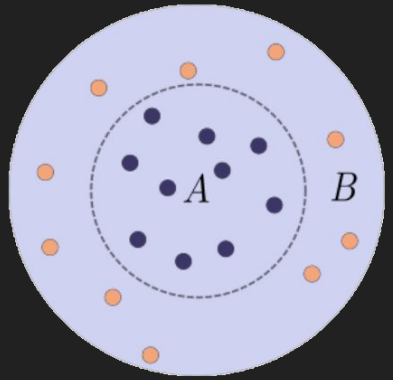

$$S_{k_1 \ldots k_N}(Z) = \sum_{\sigma \in P} \epsilon(\sigma)\, S_{k_{\sigma(1)} \ldots k_{\sigma(N_A)}}(Z_A)\, S_{k_{\sigma(N_A+1)} \ldots k_{\sigma(N)}}(Z_B)$$

A Anand, Rushikesh Patil, AC Balram, GJ Sreejith PRB 2022
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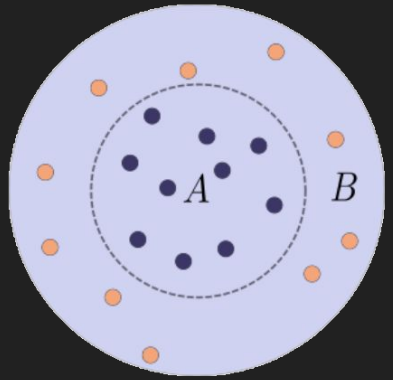

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$$\psi = \sum_{\lambda} \epsilon(\lambda) \xi^A_\lambda(Z_A) \xi^B_\lambda(Z_B)$$

$$\xi^A_\lambda = \prod_{i=1}^{q} S_{M^i_A}(Z_A), \quad \xi^B_\lambda = \prod_{i=1}^{q} S_{M^i_B}(Z_B)$$

A Anand, Rushikesh Patil, AC Balram, GJ Sreejith PRB 2022
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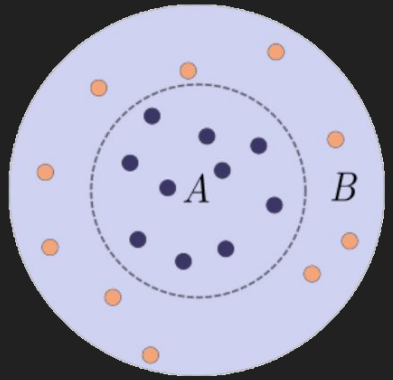

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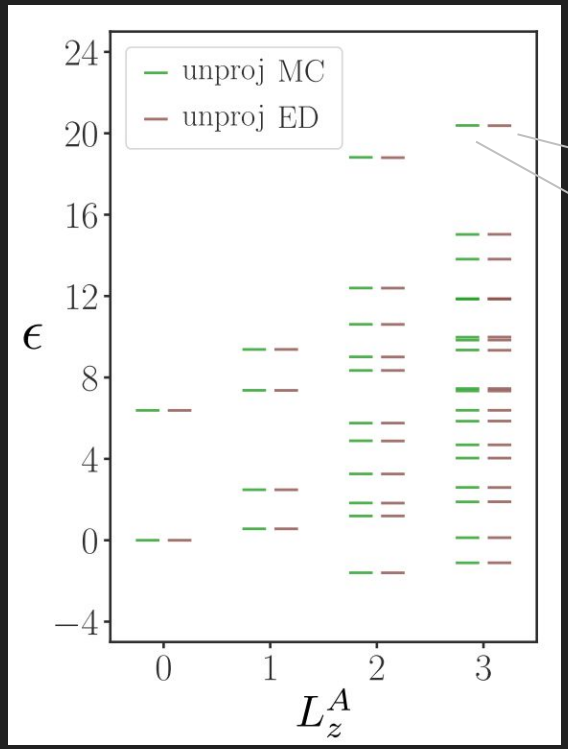
$$M_{jk} = \sum_{i}{}' s_j s_i \langle \xi_i^A | \xi_j^A \rangle \langle \xi_i^B | \xi_k^B \rangle$$

Combinatorial factors

Can be efficiently computed using Monte Carlo methods

I Rodriguez, S H Simon, J Slingerland PRL 2012
A Anand, Rushikesh Patil, AC Balram, GJ Sreejith PRB 2022
GJ Henderson, GJ Sreejith, S H Simon PRB 2021

Efficient computation of Real space entanglement spectrum



Computed on a laptop in ~ 10-20 minutes using the MC method

Computed by brute force integrations and summations:
~200 processors, 2-3 days

(generalization of algorithm in
A Chandran et al 2011)

A Anand, Sreejith in preparation

ES and edge counting in parton states

Good ansatz solutions to the edge states of CF states exists - ES calculation not really needed here. However the algorithm can be used to obtain scaling properties of the entanglement spectra in these systems. (*G J Henderson, G J Sreejith, S H Simon PRB 2021*).

In more general states where an exact parent Hamiltonian is not known, for instance “parton” states of the form below, ES is a natural way to obtain the edge spectrum (details in the next talk by Ajit)

$$\Psi_1 \Psi_2 \Psi_2$$

Such states are expected to carry edge states described by su(n)_k current algebras - related to the non-Abelian statistics.
(Jain PRL 1989, X G Wen PRL 1992) → based on topological scaling limit arguments.

$$[J_l^a, J_m^b] = \imath \sum_c f^{abc} J_{l+m}^c + kl\delta_{l+m,0}\delta_{a,b}.$$

(Di Francesco et al Intro to CFTs)

The algorithm can be used to explicitly demonstrate this from the edge structure and counting.

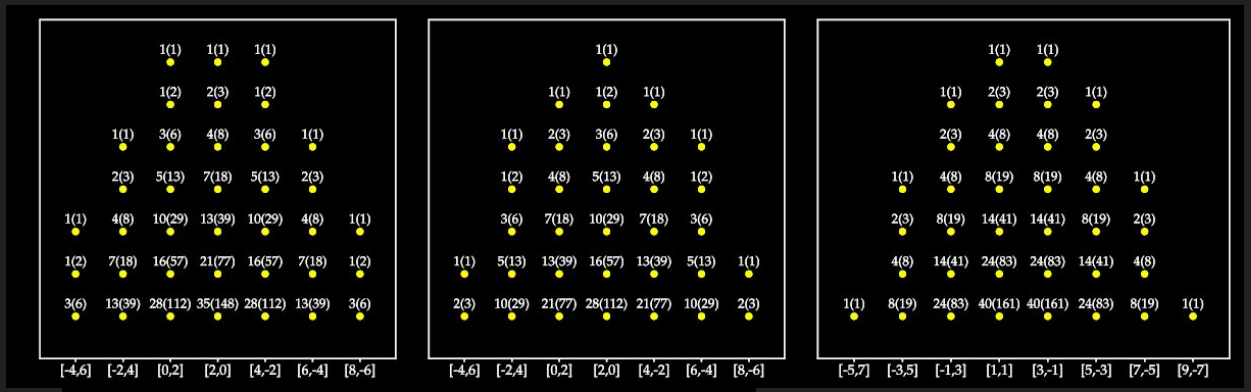
ES and edge counting in parton states

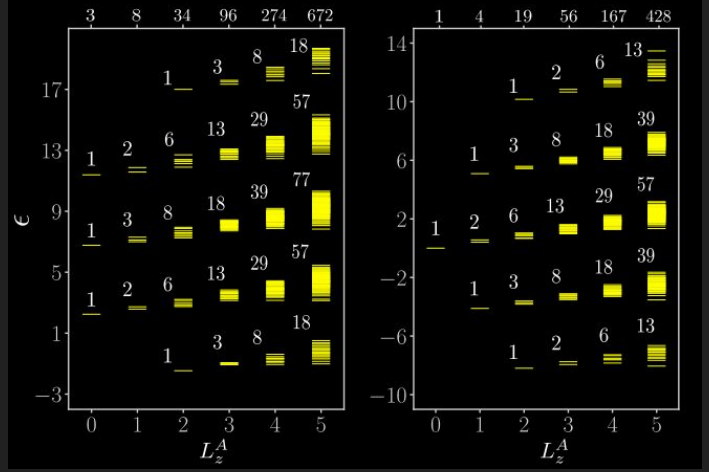
Representations of su(n)_k algebra →

System with even N

System with odd N

System with 1 bulk QH





Entanglement spectra in systems of size ~100 →

Bottomline: The counting are exactly identical to each other

More details, more cases, see Anand, Patil, Balram, Sreejith PRB 2022

Exact Hamiltonians for FQH states

Laughlin state at ⅓ is the densest exact ground state of V1 pseudopotential. The Pfaffian state at filling fraction 1/2 is the densest exact ground state of the 3 body short range interaction. (Laughlin 1983, Haldane 1983, Read Rezayi 1999, Simon Rezayi Cooper 2007)

$$\psi_{\frac{1}{3}} = \phi_1^3 \qquad \psi_{\frac{n}{2pn+1}} = \mathcal{P}_{\mathrm{LLL}} \phi_1^{2p} \phi_n$$

In general such pseudopotential interactions are not known for Jain sequence states that describe the other states in the LLL.

Unprojected Jain states are good approximations to the projected states. These can sometimes be written as the densest exact ground state of simple interactions. For instance the unprojected 2/5 Jain state (Trugman, Kivelson 1985)

$$\psi_{\frac{n}{2pn+1}} = \phi_1^{2p} \phi_n \qquad \nabla^2 \delta(\vec{r})$$

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$$\psi_{\frac{n}{2pn+1}} = \phi_1^{2p} \phi_n$$

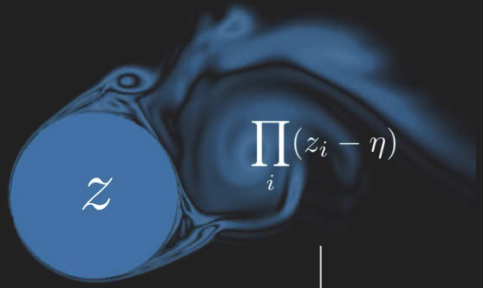
Rotational, translation symmetry allows us to write the interaction Hamiltonian as sums of relative angular momentum channels.

To specify the interaction, we just have to mention the energy cost of two particles to be at relative momenta L=0,1,2,3….

$$\nabla^2 \delta(\vec{r})$$

Gapped states: Composite fermion wavefunctions

$$KE + \frac{1}{r}$$



In the ground state, every electron is in the proximity of *2p* number of vortices of other electrons.

Statistically, |wavefunction| is higher when the electron coordinate is in the proximity of "zeros" of other electrons. Within the space of such correlated states, interactions are unimportant.
⇒ *These composite fermions are weakly interacting*

Statistically the electrons feel a reduced effective field

$$B^* = B - 2p\rho$$

$$\mathcal{P}_{\mathrm{LLL}} \prod_{i<j=1}^{N} (z_i - z_j)^{2p} \times \Psi_n$$

$$\nu = \frac{n}{2pn+1}$$

Example: Laughlin ⅓ state corresponds to 2p=2 fluxes attached to electrons. These objects fill one LL ie *n=1.*

$$\prod_{i<j=1}^{N} (z_i - z_j)^2 \times \prod_{i<j=1}^{N} (z_i - z_j)$$

Jain PRL 1989
Jain Kivelson Trivedi PRL 1990

Pairwise relative momenta

$$\prod_{i<j=1}^{N} (z_i - z_j)^2 \times \prod_{i<j=1}^{N} (z_i - z_j) = \prod_{i<j=1}^{N} (z_i - z_j)^3$$

More generally many particle states

$$\psi(z_1, z_2, z_3 \dots)$$

Pairs of particles could be in various possible states with various probabilities

Any two particle state can be uniquely decomposed in the following form

$$\sum_{M_{\mathrm{COM}}, m_{\mathrm{rel}}} (z_1 + z_2)^{M_{\mathrm{COM}}} (z_1 - z_2)^{m_{\mathrm{rel}}}$$

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No two particles have a relative angular momentum of 1 (0,2).

So it is the ground state of V1 pseudopotential - which penalizes the particles if they are in rel momentum of 1.

Particles could go into other relative momentum states - but leads to expansion of droplet

Angular momentum constraints, root partitions

The pseudopotential interactions specify the constraints on the allowed relative angular momentum between clusters of particles in the states.

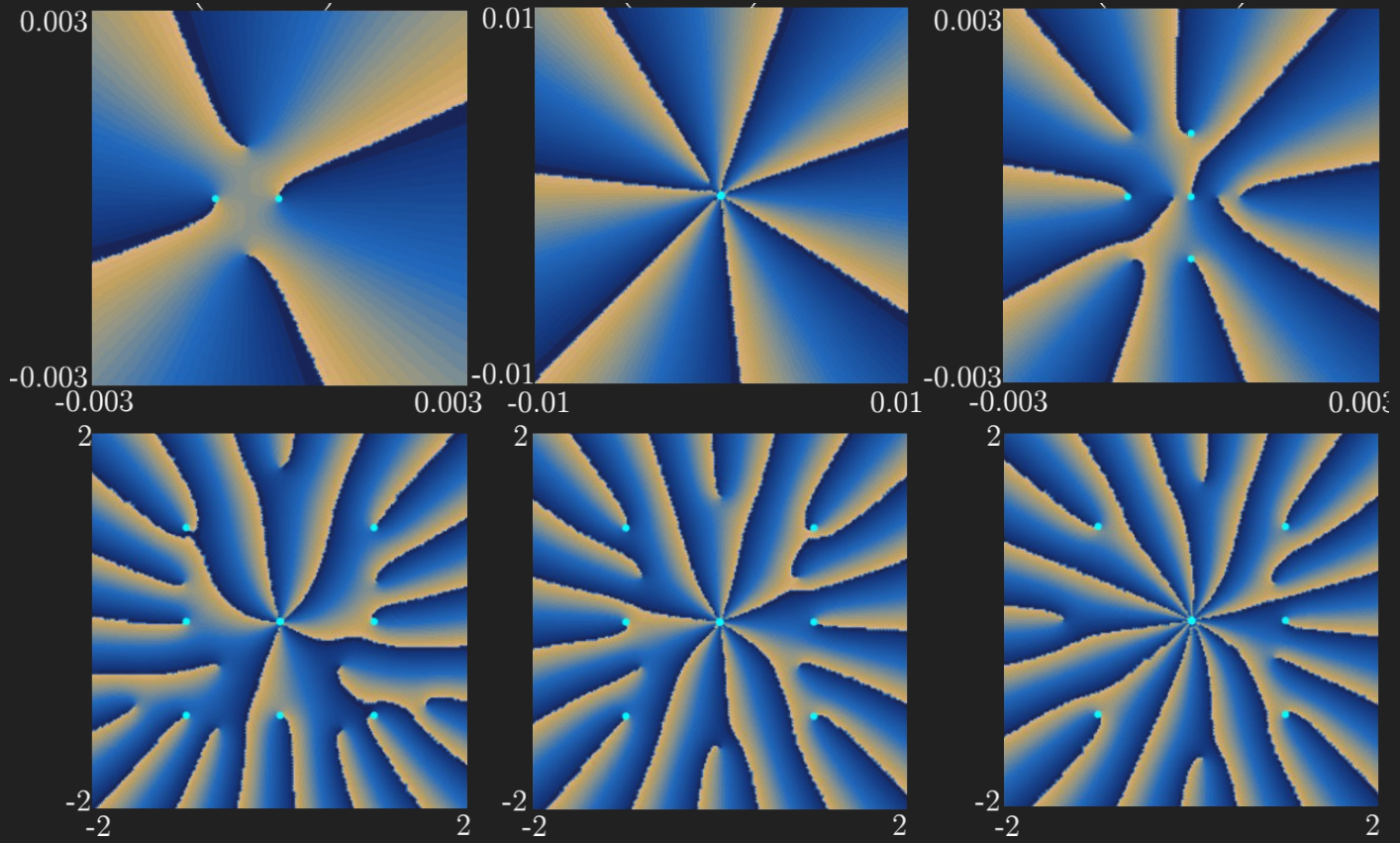
Another way to enumerate constraints is by specifying the root partition → Describes the number of zeros seen by a particle in a cluster of *n* other particles.

Root partition of the Laughlin state 0,3,6,9... ⇒ the wave function vanishes as (z1-z2)^3 when two particles are brought together ⇒ Relative angular momenta 1 is disallowed ⇒ Laughlin state should be a zero energy state of the V1 interaction. Similar idea for the Pfaffian.

For the bosonic Jain ⅔ state (as an example) the root partition can be constructed from the polynomial structure of the wavefunction. 0,2,4,5,7,8,10,11… ⇒ Analysis does not yield a unique zero energy state.

Bernevig Haldane 2007, 2008; Wen, Wang 2008; Simon, Rezayi, Cooper 2007; Sreejith, Fremling, Jeon, Jain (2018); Rodriguez *et al* (2012)

Root partitions, clustering of zeroes



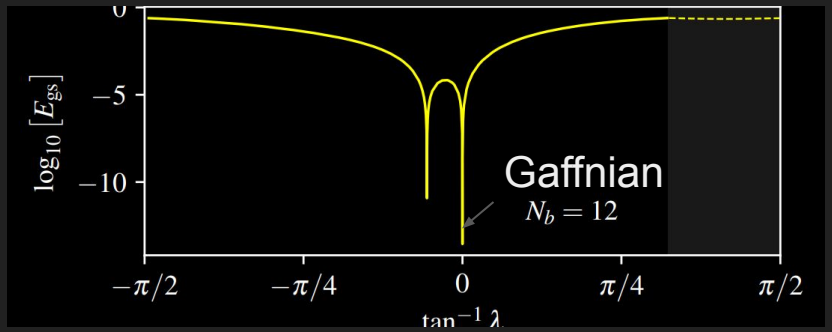
Sreejith Fremling Gunsang Jain PRB 2018

Angular momentum constraints, root partitions

For the bosonic Jain ⅔ state (as an example) the root partition can be constructed from the polynomial structure of the wavefunction. 0,2,4,5,7,8,10,11… ⇒ inferred H does not yield the right 2/3 zero energy state

We considered Hamiltonians with 2,3,4 body interactions - whose pseudopotentials are infrared from the root partition - more complex interactions are not uniquely determined. Addition of further constraints beyond what is implied by the root partition also does not yield the desired Hamiltonian.

(Sreejith, Jeon, Fremling, Jain, 2018)



Moreover there are more than 1 states with the same root partition (Regnault, Bernevig, Haldane PRL 2009).

Exact eigenstates of simple Hamiltonians

$$H = \sum_{i<j} \mathcal{P}^{ij}_{L=1}$$ Laughlin + QHs

$$H = \sum_{i<j<k} \mathcal{P}^{ijk}_{L=3}$$ Moore Read + QHs

$$H = \sum_{i<j} \nabla^2 \delta(r_i - r_j)$$ Jain CF state @ 2/5

- Only the zero energy GS and zero energy QH states can be written down
- A different model for every FQH state. Each model works for only one FQH state
- Conventional approach → cyclotron gap > interactions ⇒ Project into one/few LLs.

Laughlin 1983, Haldane 1983, Moore Read 1991, Greiter Wen Wilczek 1991, Simon Rezayi Cooper 2007, Trugman Kivelson 1985, Rezayi, MacDonald 1991, Jain, Kivelson, Trivedi 1990, Bandhopadhyay et al 2018, 2020, Sreejith, Fremling, Jeon, Jain 2018

Motivation

$$H = \sum_{i<j} \mathcal{P}^{ij}_{L=1}$$ Laughlin + QHs

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Features of the model
(PRL 126, 136601 (2021))

- Model is defined in the unconventional limit of very strong interactions (relative to the cyclotron energy)
- Strong interaction defines a highly constrained Hilbert space
- All low energy states - GS, QH, QP, neutral modes can be written down exactly
- A single model *H* works for multiple FQH states
(for all spin polarized Jain states at filling n/(2pn+1))
- Frustration free

Laughlin 1983, Haldane 1983, Moore Read 1991, Greiter Wen Wilczek 1991, Simon Rezayi Cooper 2007, Trugman Kivelson 1985, Rezayi, MacDonald 1991, Jain, Kivelson, Trivedi 1990, Bandhopadhyay et al 2018, 2020, Sreejith, Fremling, Jeon, Jain 2018

CF wavefunctions

Our construction is motivated by the structure of the CF wavefunction.

CF wf describing an incompressible state at $\nu = \dfrac{n}{2pn+1}$

$$\prod_{i<j=1}^{N} (z_i - z_j)^{2p} \times \Psi_n$$

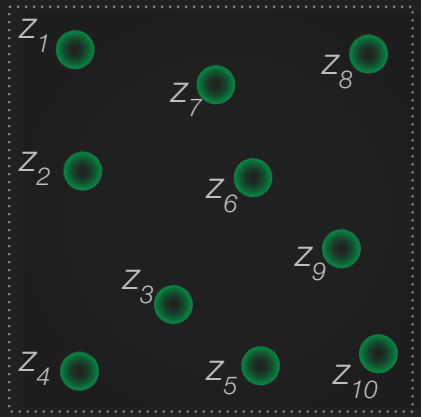
Attaches *2p* vortices to each electron

IQH state of CF at an effective *B*

Jain 1989

Correlations in two particle sectors

$$\prod_{i<j=1}^{N} (z_i - z_j)^{2p} \times \Psi_n$$



Pairs of electrons in this correlated state occur in various possible relative angular momentum channels

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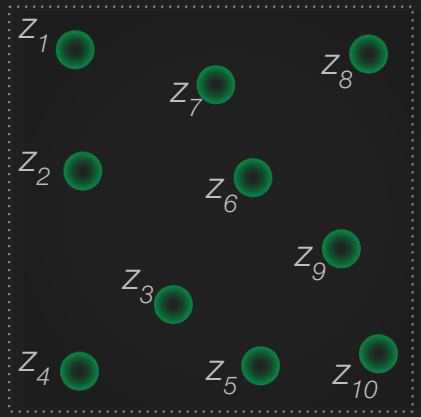

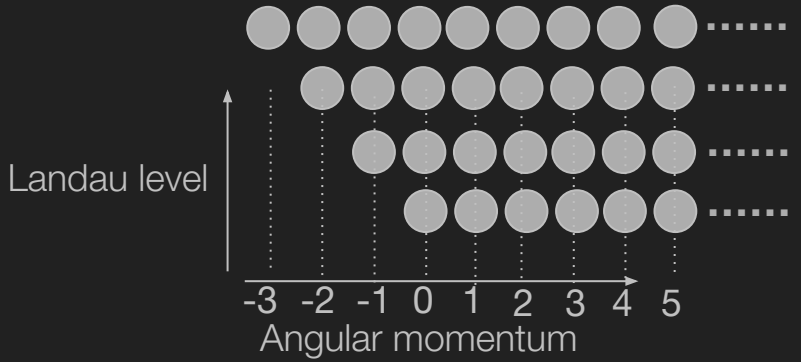

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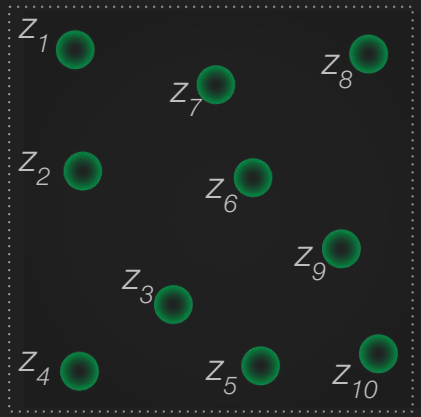

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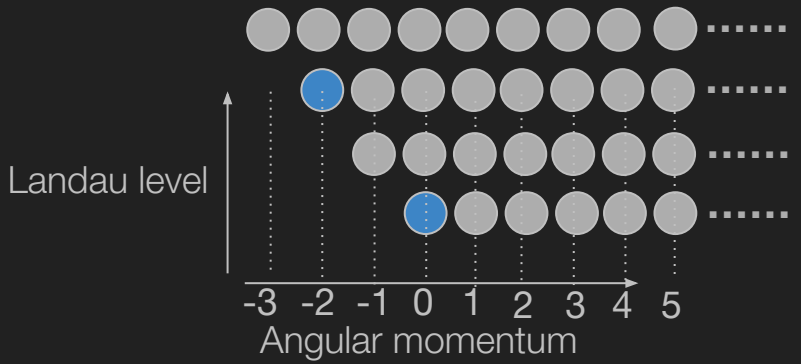
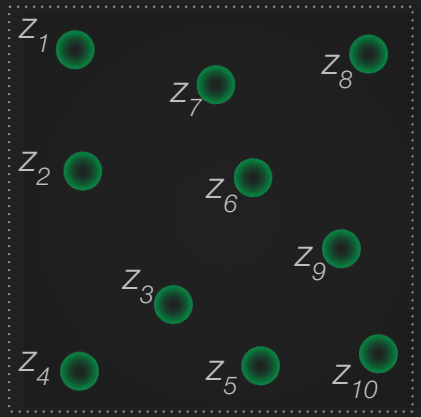


In the IQH state Ψ_1, minimum relative angular momentum is ***-n-m***

Correlations in two particle sectors

$$\prod_{i<j=1}^{N} (z_i - z_j)^{2p} \times \Psi_n$$



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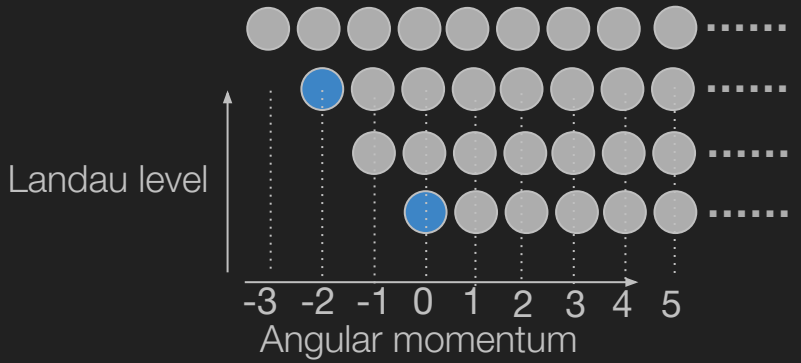
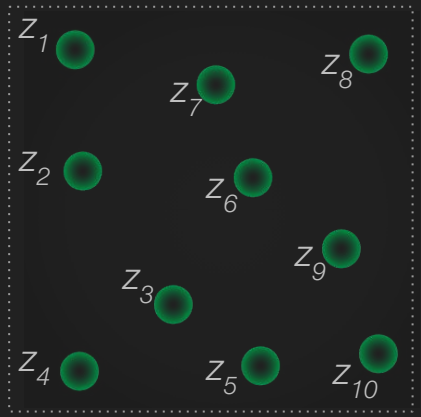


In the IQH state Ψ_1, minimum relative angular momentum is ***-n-m***

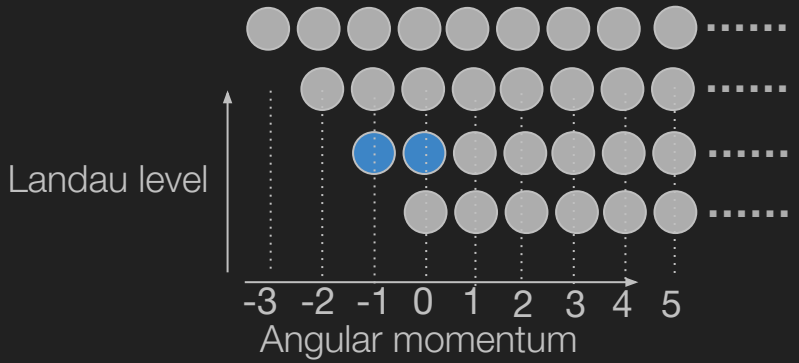
Flux attachment increments this to ***-n-m+2p***

Correlations in two particle sectors

$$\prod_{i<j=1}^{N} (z_i - z_j)^{2p} \times \Psi_n$$



Pairs of electrons in this correlated state occur in various possible relative angular momentum channels

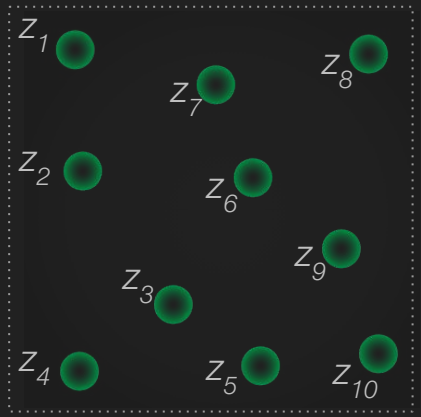


In the IQH state Ψ_1, minimum relative angular momentum is ***-n-n+1***

Flux attachment increments this to ***-n-n+1+2p***

Correlations in two particle sectors

$$\prod_{i<j=1}^{N} (z_i - z_j)^{2p} \times \Psi_n$$



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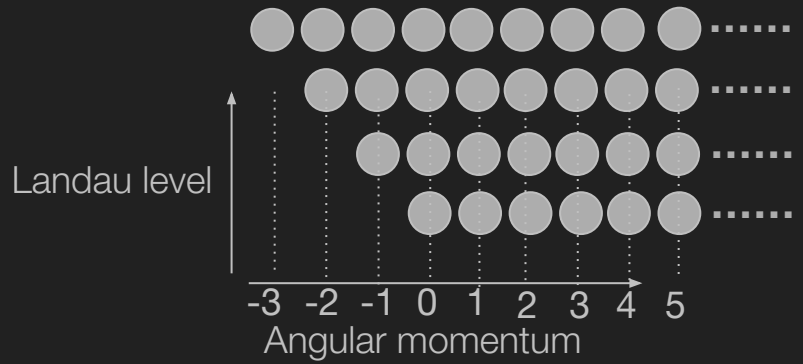

Summary: For two particles in LLs ***n*** and ***m,*** relative momenta are lower bound by

$$-n - m + \delta_{nm} + 2p$$

Correlations in two particle sectors

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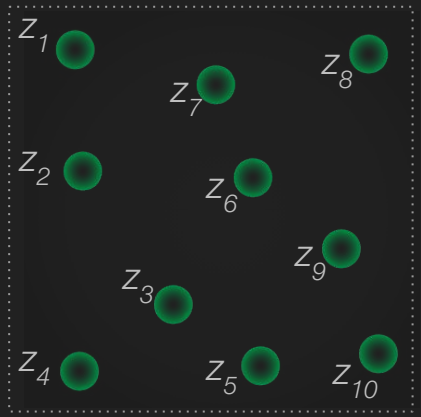

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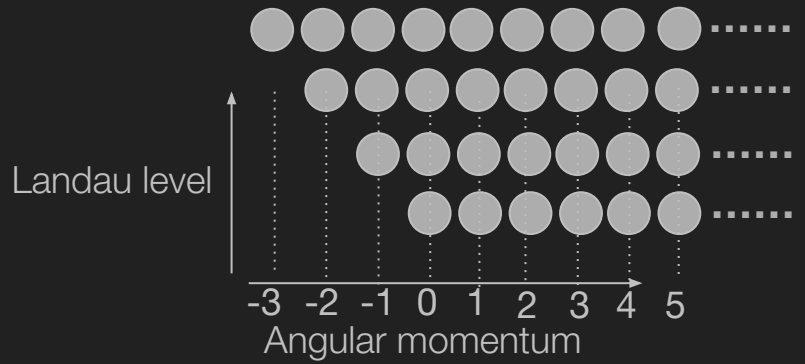


Summary: For two particles in LLs ***n*** and ***m***, relative momenta are lower bound by

$$-n - m + \delta_{nm} + 2p$$

Can this be used to construct a parent Hamiltonian by projecting out the forbidden sectors ?

No. Multiplication by Jastrow factor does not preserve the LL indices.

Jastrow factor of guiding center coordinates

Consider a slightly different wavefunction

$$\prod_{i<j=1}^{N} (z_i - z_j)^{2p} \times \Psi_n \quad \text{replace with} \quad \prod_{i<j=1}^{N} (\hat{Z}_i - \hat{Z}_j)^{2p} \times \Psi_n$$

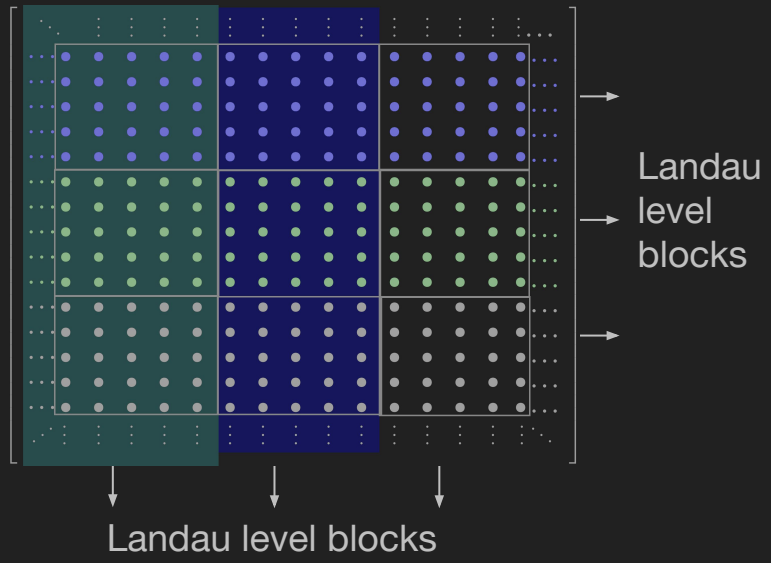
Slater determinant
Fermions occupying arbitrary single particle KE eigenstates

-Raises relative angular momenta
-But preserves LL labels

$$\hat{Z} = \hat{\pi} - (\hat{z} \times \vec{r})$$
$$\sim \hat{z} - \imath(\hat{\pi}_x - \imath\hat{\pi}_y)$$

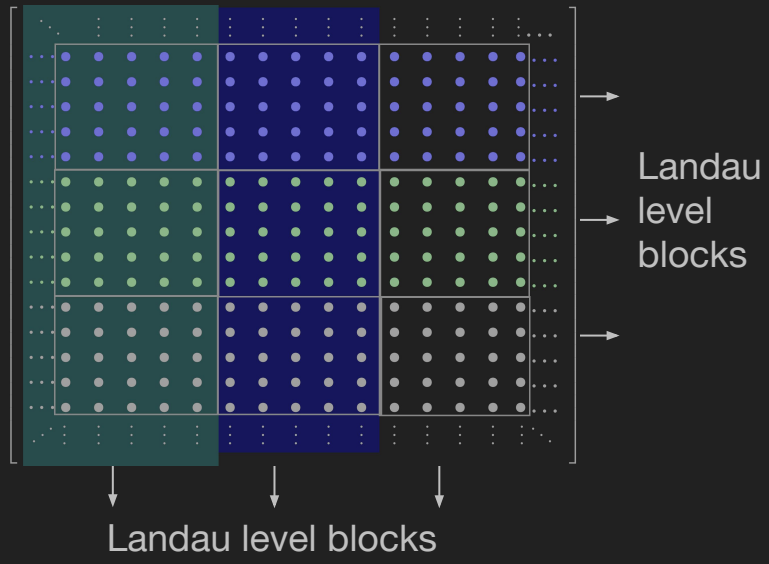
Guiding center coordinates

Position operator scatters between LLs

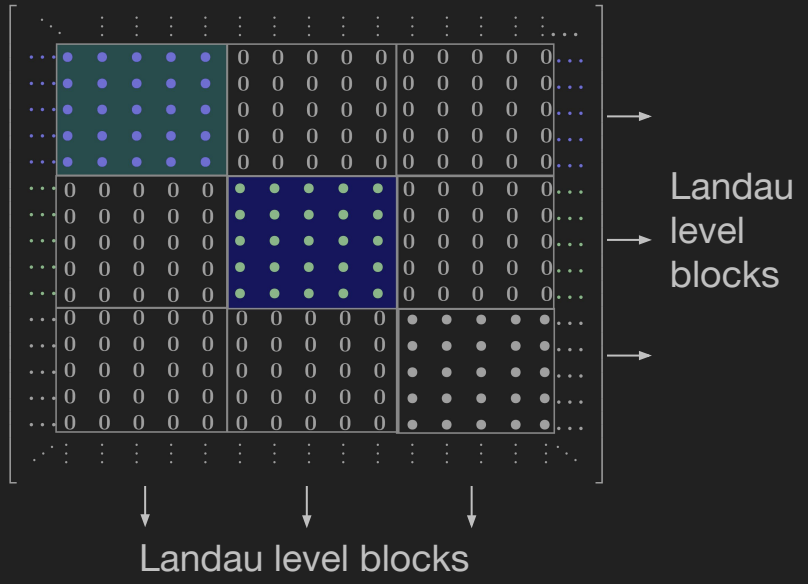


Guiding center coordinates

Position operator scatters between LLs



Guiding center coordinates do not scatter between LLs



Guiding center can be thought of as $\hat{Z} = \sum_{n=0}^{\infty} \hat{P}_{nthLL} \hat{z} \hat{P}_{nthLL}$

Guiding center coordinates

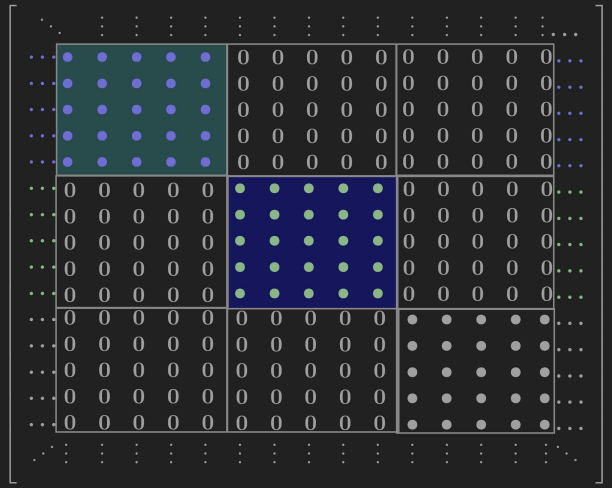
$$\hat{Z} = \hat{\pi} - (\hat{z} \times \vec{r})$$

$$\sim \hat{z} - \imath(\hat{\pi}_x - \imath\hat{\pi}_y)$$

[T , Z] = 0 ⇔ Preserves Landau level

[L , Z] = Z ⇔ changes momentum

Guiding center coordinates do not scatter between LLs



$$\prod_{i<j=1}^{N} (z_i - z_j)^{2p} \times \Psi_n \xrightarrow{\text{replace with}} \prod_{i<j=1}^{N} (\hat{Z}_i - \hat{Z}_j)^{2p} \times \Psi_n$$

For two CFs in LLs *n* and *m*, relative momentum in the state is lower bound by

$$-n - m + \delta_{nm} + 2p$$

Jastrow factor of guiding center coordinates

$$\prod_{i<j=1}^{N} (z_i - z_j)^{2p} \times \Psi_n \quad \text{replace with} \quad \prod_{i<j=1}^{N} (\hat{Z}_i - \hat{Z}_j)^{2p} \times \Psi_n$$

Slater determinant
Fermions occupying arbitrary single particle KE eigenstates

satisfies this property

-Raises relative angular momenta
-But preserves LL labels

For two particles in LLs ***n*** and ***m***, relative momenta in this state is lower bound by

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For two particles in LLs ***n*** and ***m***, relative momenta in this state is lower bound by

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The wavefunction is different from the Jain CF states → Jastrow factor is replaced with an operator.

Pseudopotential Hamiltonian

We can construct an interaction for which previously mentioned states are exact zero energy states

$$\hat{V} = \sum_{n \leq n' = 0}^{\infty} \sum_{M = -n-n'+\delta_{n,n'}}^{-n-n'+\delta_{n,n'}+2p-1} \mathcal{P}^{M}_{nn'}.$$

Sum over Landau levels of particle pairs

Sum over forbidden relative momentum channels

Projector onto relative momentum channels

Ground state of the model is closely related to that of a multilayer model. The excited states have a qualitatively different structure.

Related bilayer model for ⅖ states: Milovanovic, Papic (2010), Papic, Goerbig, Regnault, Milovanovic (2010)

Anand, Jain, Sreejith (2021)

Pseudopotential Hamiltonian

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Sum over Landau levels of particle pairs

Sum over forbidden relative momentum channels

Projector onto relative momentum channels

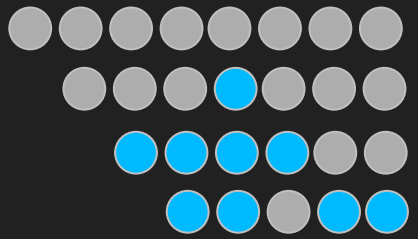
- Imposes an energy cost for pairs in the forbidden relative momentum sectors
- Includes intra LL and inter LL interactions
- Number of particles in each LL is conserved

Interaction Hamiltonian & its zero interaction energy space

$$\hat{V} = \sum_{n \leq n' = 0}^{\infty} \sum_{M=-n-n'+\delta_{n,n'}}^{-n-n'+\delta_{n,n'}+2p-1} \mathcal{P}^{M}_{nn'}.$$

$$\hat{V}\Phi = 0 \quad \text{for all } \Phi \text{ of the form } \prod_{i<j=1}^{N} (\hat{Z}_i - \hat{Z}_j)^{2p} \times \Psi_n$$

$$\prod_{i<j=1}^{N} (\hat{Z}_i - \hat{Z}_j)^{2p} \times$$



Interaction Hamiltonian & its zero interaction energy space

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These states are low energy states of a Hamiltonian $H = \hat{T} + \lambda \hat{V}$ in the limit $\lambda \to \infty$

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$$H\Phi = (T + \lambda V)\Phi = T\Phi = \prod (\hat{Z}_i - \hat{Z}_j)^{2p}\, T\Psi_n = E_n \Phi$$

Energy of the state is same as the KE of the slater determinant Ψ_n

Interaction Hamiltonian & its zero interaction energy space

$$\hat{V} = \sum_{n \leq n' = 0}^{\infty} \sum_{M=-n-n'+\delta_{n,n'}}^{-n-n'+\delta_{n,n'}+2p-1} \mathcal{P}_{nn'}^{M}.$$

We conjecture that these type of states exhaust the null space of ***V*** and are linearly independent.

Conjecture tested and found to be exact in every one of ~200 systems studied.

$$\hat{V}\Phi = 0 \quad \text{for all } \Phi \text{ of the form } \prod_{i<j=1}^{N} (\hat{Z}_i - \hat{Z}_j)^{2p} \times \Psi_n$$

These states are low energy states of a Hamiltonian $H = \hat{T} + \lambda \hat{V}$ in the limit $\lambda \to \infty$

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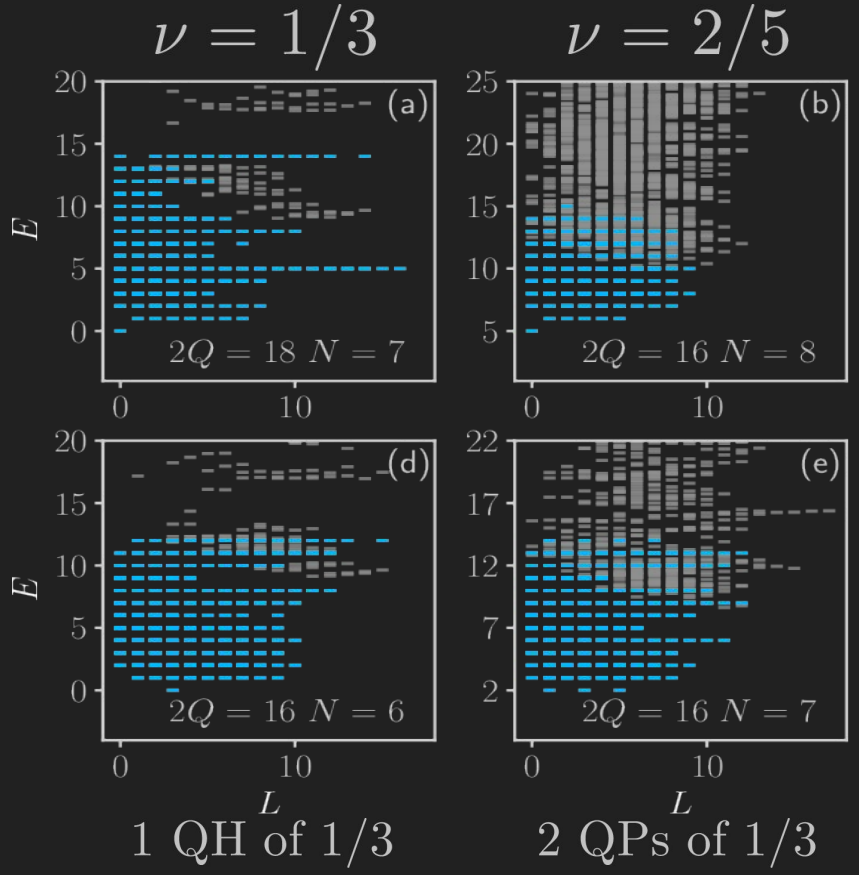
Spectrum of $H=T+V$



Blue color dashes = Null space of interaction *V*

→ Includes QP, QH, neutral modes etc

→ wavefunctions for all these states can be explicitly written down

→ Low energy quantum numbers same as
- those of non-interacting electrons in a reduced field **B***, and
- by implication same as those of Coulomb problem in LLL (Jain 1989)

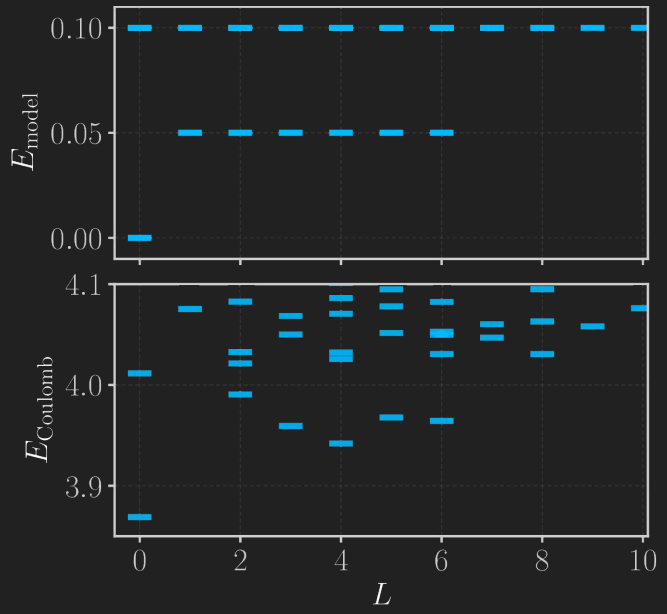
Grey color dashes = Finite interaction energy states

These will pushed to infinite energies if interaction strength is sent to infinity

Spectrum of ***H=T+V*** compared to LLL Coulomb

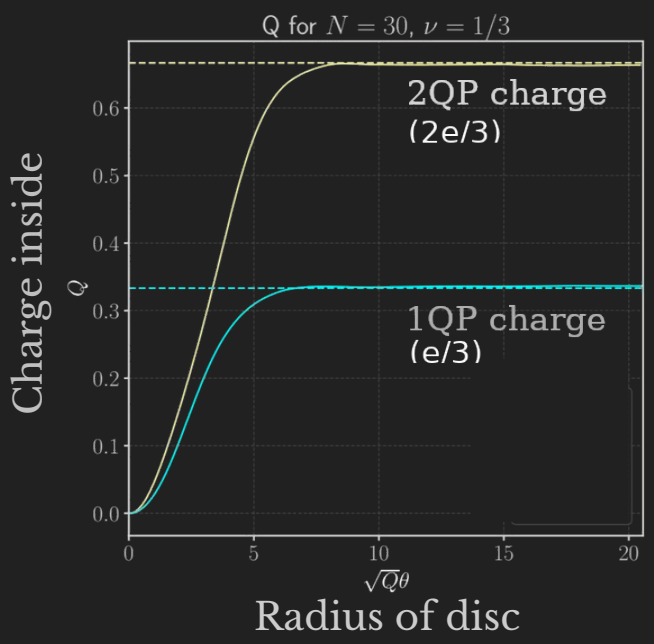
Model

LLL Coulomb

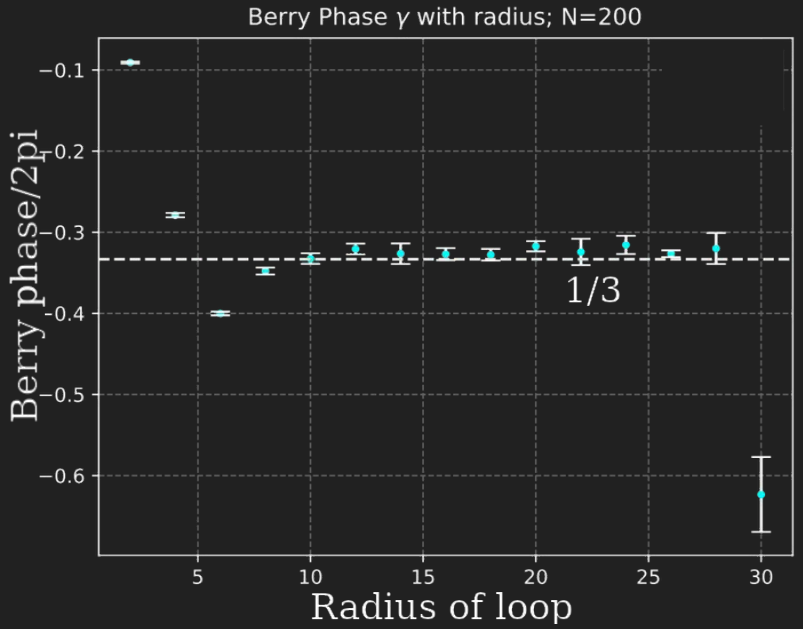


Low energy states of LLL Coulomb have the same quantum numbers as the low energy states of the model Hamiltonian.

Berry phase, charge of localized excitations



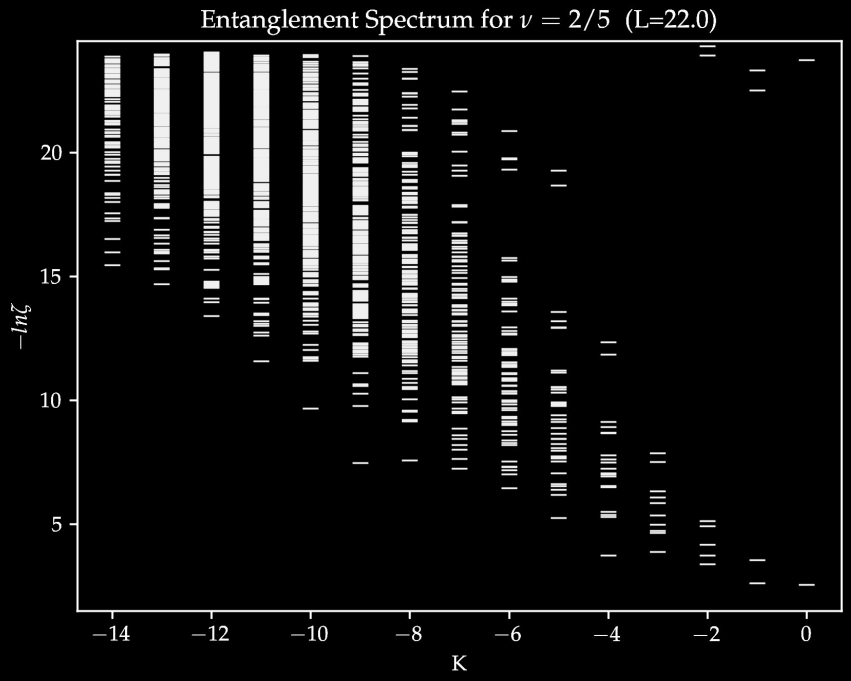
Charge inside a disc around 1 or 2 localized QPs of the model

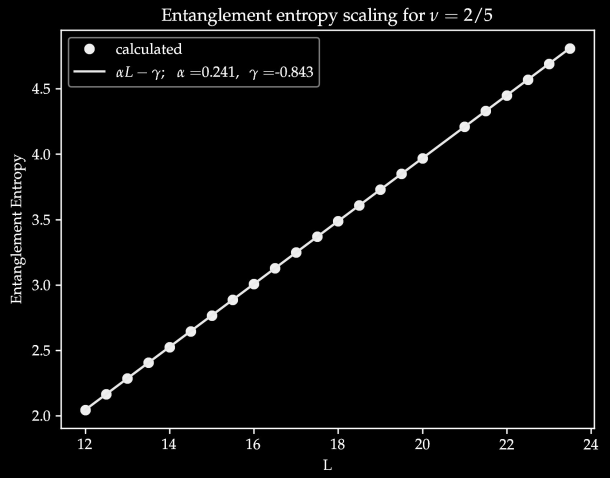


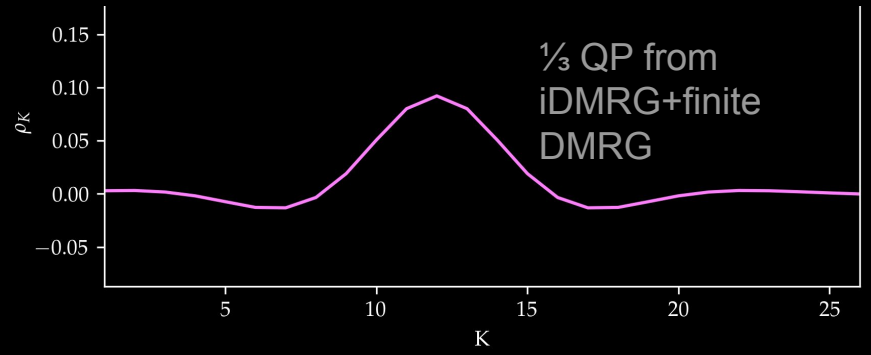
Berry phase from winding one localized QP around a QH of 1/3

A Anand,A Kolhatkar, unpublished

Infinite DMRG on cylinder







MESs generated by translates of

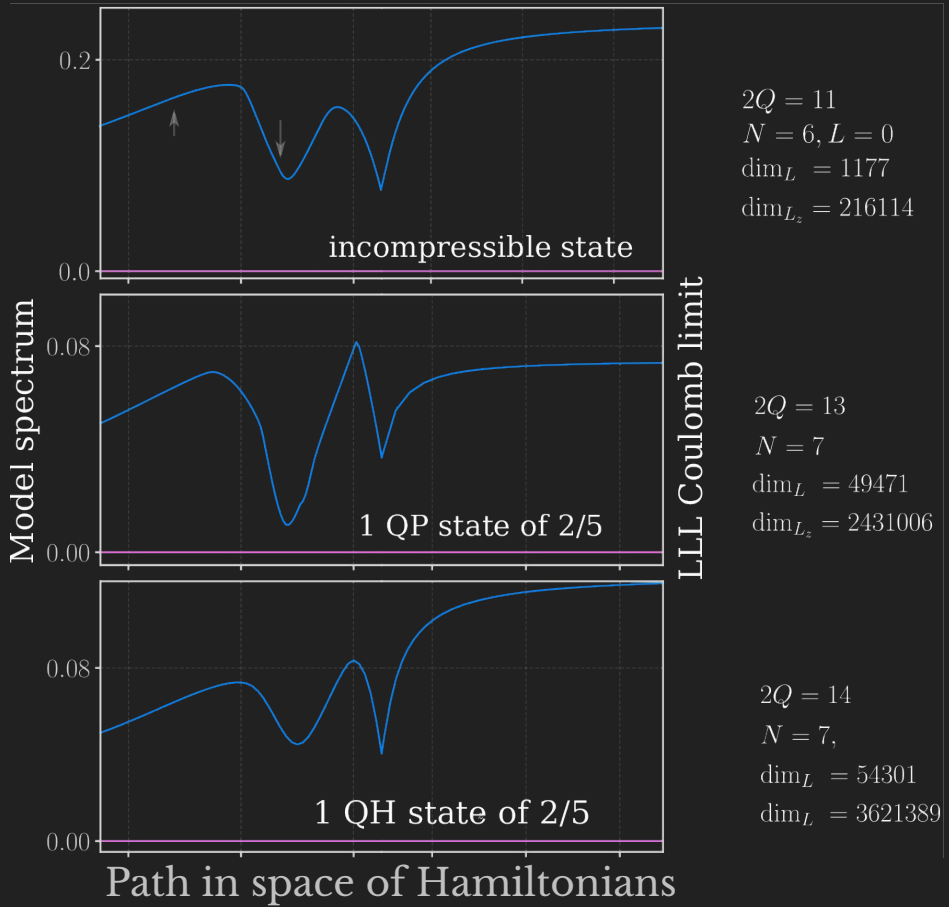
$$\frac{\ldots 001000010000100001000001\ldots}{\ldots 100001000010000100001000100\ldots}$$

Zalatel, Mong, Pollmann 2013

$$\frac{\ldots 000000000100000000000\ldots}{\ldots 100100100001001001001\ldots}$$

A Anand, Kolhatkar unpublished

Adiabatic continuity: From model to *LLL Coulomb*

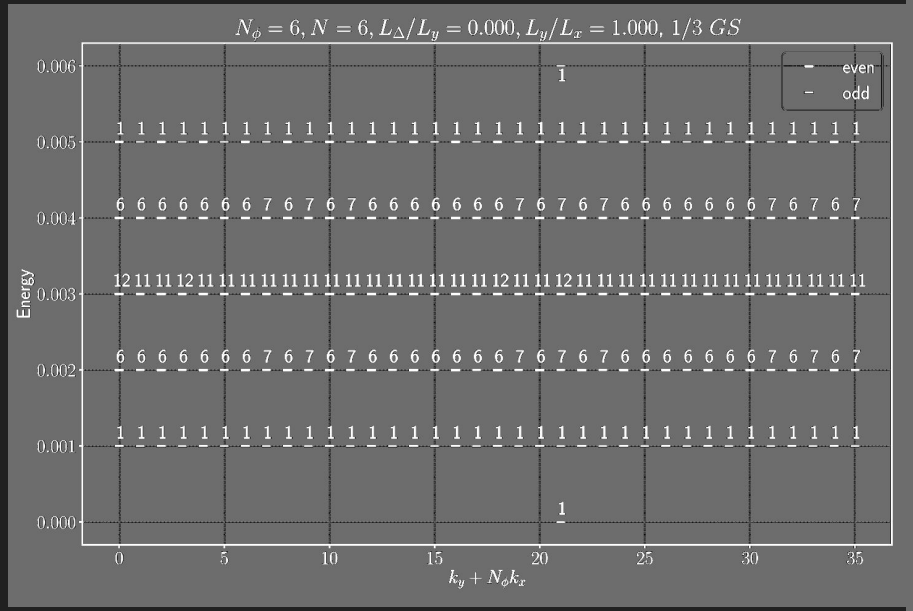


For all finite systems that we could study ground state, single quasihole state
and single quasiparticle state
of the model are adiabatically connected with corresponding LLL Coulomb states.

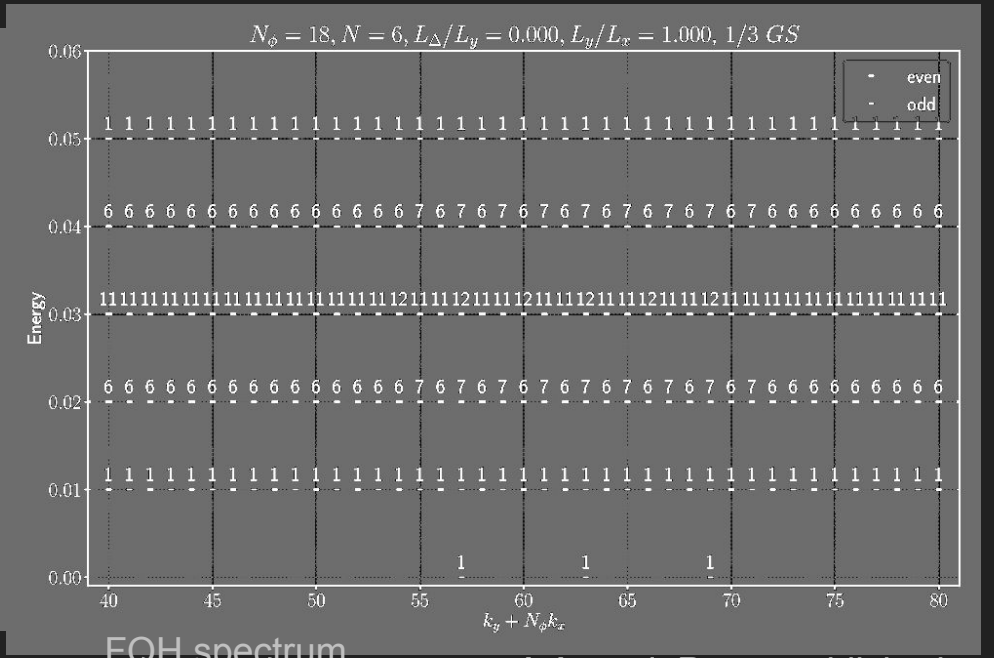
Generalization to the Torus geometry

Though the model was originally written in terms of angular momenta, the interaction is local.
So we expect that the model should be generalizable to geometries without any rotational symmetry.

Indeed this can be generalized to the torus geometry. *(Haldane, 1990; Haldane 1985)*



IQH spectrum



FQH spectrum

A Anand, Pu, unpublished

Conclusions

- An infinitely strongly interacting model with exactly solvable spectra - not just GS and QH but also QP, exciton states and all excited states can be constructed.
  - Gapped GS at Jain sequence *ν=n/(2pn+1)*
  - Low energy quantum numbers, charge, statistics, topological degeneracies, shifts of incompressible states all same as LLL Coulomb
  - Microscopic model with an exact mapping between FQHE of electrons at B and IQHE of CFs at B*

- An unusual limit: Large interaction compared to the cyclotron gap
  - Instead of the usual single particle constraint of restricting to LLL
  - This has a many particle constraint wherein the low energy Hilbert space is made of highly correlated many body states
  - KE splits this space to produce LLL Coulomb-like low energy spectra.

- Numerics clearly show that the low energy quantum numbers expected from the exact solutions hold in all geometries - torus, sphere, disk, cylinder.
  - Exact solutions exist only on systems with an open boundary - cylinder and disk.
  - Attempts at writing the solutions on closed manifolds fail → Wavefunctions “spill out of the Hilbert space” on the sphere *(Greiter 2011)*
  - For closed manifolds another form of solution can be obtained for all states with filling < ⅔

Anand, Jain, Sreejith, PRL 126, 136601 (2021), Anand, Pu (unpublished)

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