Non-Abelian 2/5 quantum Hall state in the lowest Landau level

Sahana Das[1*], Sudipto Das[1], Sutirtha Mukherjee[2], and Sudhansu S. Mandal[1]

[1]Indian Institute of Technology, Kharagpur 721302, India
[2]Korea Institute for Advanced Study, Seoul 02455, Korea
*Email: sahanadas1993@gmail.com

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Introduction

✎ Current carrying 2D electron gas in strong perpendicular magnetic field and low temperature

✎ Hall conductivity as well as resistivity quantized
$\sigma_H^{-1} = \rho_{xy} = \frac{1}{\nu}\frac{h}{e^2}$
$\nu \rightarrow$ filling factor, N/N_ϕ

✎ Hamiltonian :

$$H = \sum_j^N \frac{1}{2m}\left[\frac{h}{i}\vec{\nabla}_j + e\vec{A}(\vec{r_j})\right]^2 + \frac{e^2}{\epsilon}\sum_{j<k}^N \frac{1}{|\vec{r_j} - \vec{r_k}|}$$

✎ Partially filled Landau level $\rightarrow$ **fractional quantum Hall effect**
$\Rightarrow$ degeneracy lifted by Coulomb interaction
$\Rightarrow$ **not exactly solvable**

Conclusion

✍ The gapless Gaffnian state is reconstructed in a new way considering it as a paired state of inter-flavored CFs. Linear combination of this Gaffnian wavefunction with the minimally modified Gaffnian wavefunction can describe the 2/5 state.

✍ The high overlap of our hybridized wavefunction with the model potential H_α near 2-body Coulomb potential suggests our state to be gapped[3].

✍ The high overlap of two quasi-hole Hybrid wavefunction near Gaffnian limit indicates non-Abelian nature of our state.

✍ Composite fermionic wavefunction
$\longrightarrow$ **Abelian quasi-particle statistics**

Our hybrid state is the paired state of CFs with dissimilar flavors $\longrightarrow$ **non-Abelian quasi-particle statistics**
$\Longrightarrow$ due to very similar energy of two states, there could be spontaneous possession of one of these topological orders based on the external conditions.

References

[1] Simon *et. al.*, Phys. Rev. B **75**, 075317 (2007).

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Outlook

➡ Calculation of non-abelian phase for the hybrid wavefunction is important for confirming non-abelian state.

➡ The exact pseudopotential of the Hybrid state must be near Coulomb limit possibly having non-Abelian quasi-particle statistics. It is very interesting to achieve such pseudopotential in a realistic system for realizing non-Abelian statistics in the lowest Landau level.

➡ The Conformal Field Theory for the proposed wavefunction can provide topological data of the state.

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Theories for 2/5 state

Composite Fermion (CF) wavefunction

✰ Mean-field approximation
✰ The CF wave function with the filling factor $\nu = 2/5$ can be written as,

$$\Psi_{2/5} = \mathcal{P}_{\rm LLL}\Phi_2 \prod_{j<k}(z_j - z_k)^2 = \mathcal{P}_{\rm LLL}\Phi_2 \prod_{j<k} U_{j,k}^2$$

✰ Gapped 2/5 FQHE
✰ **Abelian** quasi-particle(anyon) statistics

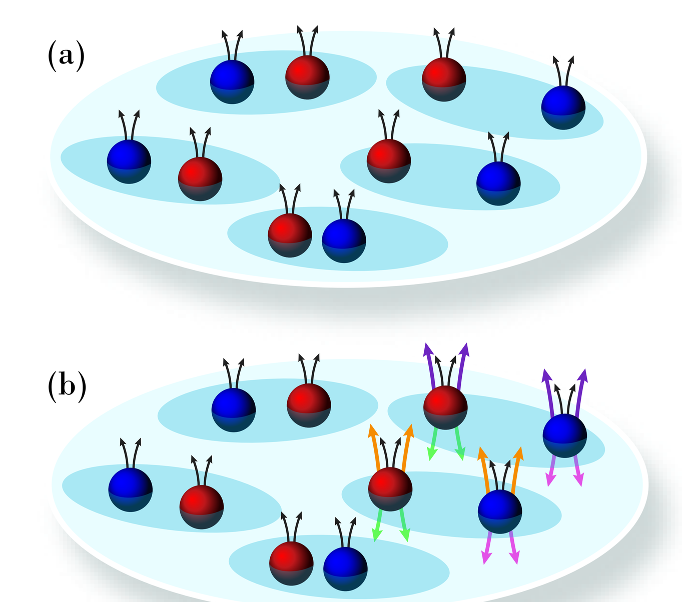
Gaffnian wavefunction[1]

✰ Conformal Field Theory (CFT) approach
✰ Exact zero energy ground-state of a 3-body Hamiltonian
✰ High overlap with the exact Coulomb ground-state
✰ Non-unitary CFT $\rightarrow$ gapless state
$\rightarrow$ **quantum critical state**
✰ **Non-Abelian** quasi-particle (anyon) statistics

Our proposed wavefunction

Generalization of CF wavefunction for 2/5 state[2]:

$$\Psi_{\rm CF}^{2/5} = \prod_{i<j\le N} U_{i,j}^2 \mathcal{A}\left[\left(\prod_{k<l\le N/2} U_{k,l}U_{k+N/2,l+N/2}\right)\prod_{m\le N/2}\left(\sum_{n\neq m}\frac{1}{U_{m,n}}\right)\right]$$



Gaffnian (Gf) wavefunction:

$$\Psi_{\rm Gf}^{2/5} = \prod_{i<j\le N} U_{i,j}^2 \mathcal{A}\left[\left(\prod_{k<l\le N/2} U_{k,l}U_{k+N/2,l+N/2}\right)\prod_{m\le N/2}\frac{1}{U_{m,m+N/2}}\right]$$

Modified-Gaffnian (m-Gf) wavefunction:

$$\Psi_{\rm m-Gf}^{2/5} = \prod_{i<j\le N} U_{i,j}^2 \mathcal{A}\left[\left(\prod_{k<l\le N/2} U_{k,l}U_{k+N/2,l+N/2}\right)\prod_{m\le N/2}\frac{1}{U_{m,m+N/2}}\left(\frac{U^2_{l_1,l_1+N/2}U^2_{l_2,l_2+N/2}}{U^2_{l_1,l_2}U^2_{l_1+N/2,l_2+N/2}}\right)_{l_1<l_2\in N/2}\right]$$

Our proposed wavefunction: $\Psi_{\rm Hybrid}^{2/5} = c_1\Psi_{\rm Gf}^{2/5} + c_2\Psi_{\rm m-Gf}^{2/5}$

Results

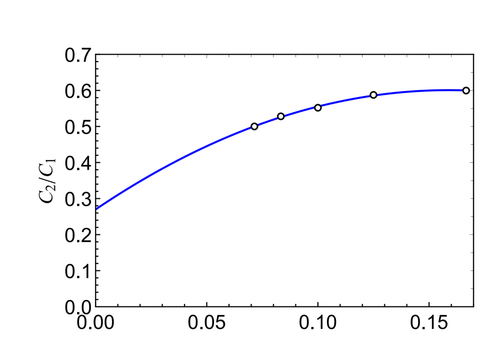
Overlap & Energy

N	$\langle\Psi_{\rm ex}^{2/5}\vert\Psi_{\rm CF}^{2/5}\rangle$	$\langle\Psi_{\rm ex}^{2/5}\vert\Psi_{\rm Gf}^{2/5}\rangle$	$\langle\Psi_{\rm ex}^{2/5}\vert\Psi_{\rm m-Gf}^{2/5}\rangle$	$\langle\Psi_{\rm ex}^{2/5}\vert\Psi_{\rm Hybrid}^{2/5}\rangle$
6	0.99968(0)	0.9883947	0.9676911	0.999831
8	0.99937(1)	0.9771221	0.9327855	0.999709
10	0.99777(2)	0.9715446	0.9098972	0.997357
12	0.99686(2)	0.9643(5)	0.8788(16)	0.99550(4)
14	0.99577(3)	0.9585(5)	0.8467(18)	0.99299(8)

➡ Overlap of $\Psi_{\rm Hybrid}^{2/5}$ with $\Psi_{\rm ex}^{2/5}$ is very high and similar to that of $\Psi_{\rm CF}^{2/5}$

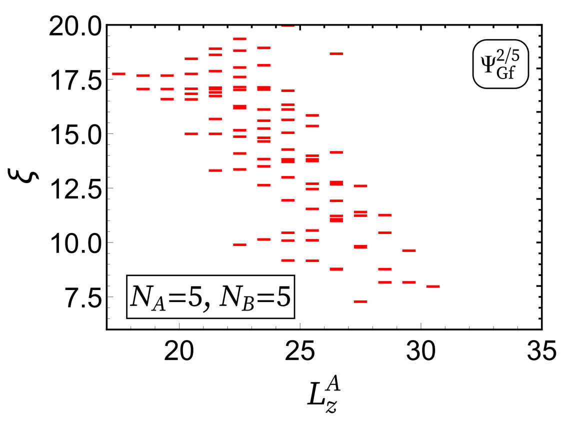
N	$E_{\rm ex}$	$E_{\rm CF}$	$E_{\rm Gf}$	$E_{\rm Hybrid}$
6	-0.50040	-0.5004 (2)	-0.4994 (2)	-0.5004 (2)
8	-0.48024	-0.4802 (3)	-0.4788 (4)	-0.4802 (3)
10	-0.46945	-0.4693 (5)	-0.4680 (6)	-0.4693 (5)
12	-0.46265	-0.4625 (7)	-0.4611 (9)	-0.4625 (7)
14	-0.45799	-0.458 (2)	-0.457 (2)	-0.458 (2)

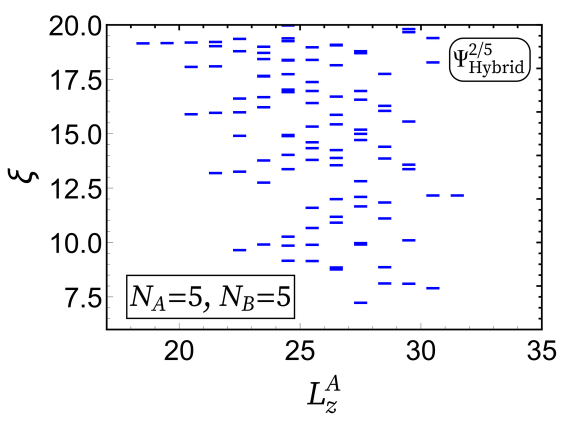
➡ Energy of our Hybrid state is very similar to that of CF and very close to the exact state

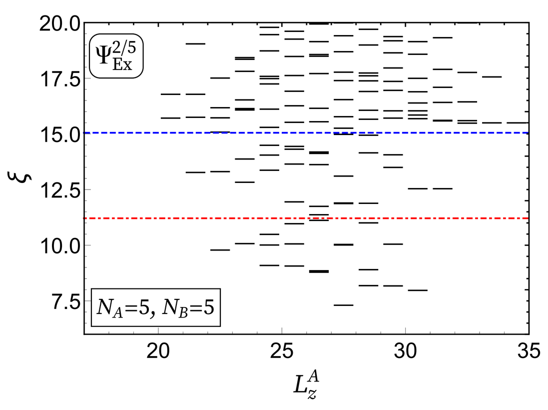


➡ Finite value of ratio at $N \rightarrow \infty \Rightarrow$ both $\Psi_{\rm Gf}^{2/5}$ and $\Psi_{\rm m\text{-}Gf}^{2/5}$ sustain

Entanglement Spectra



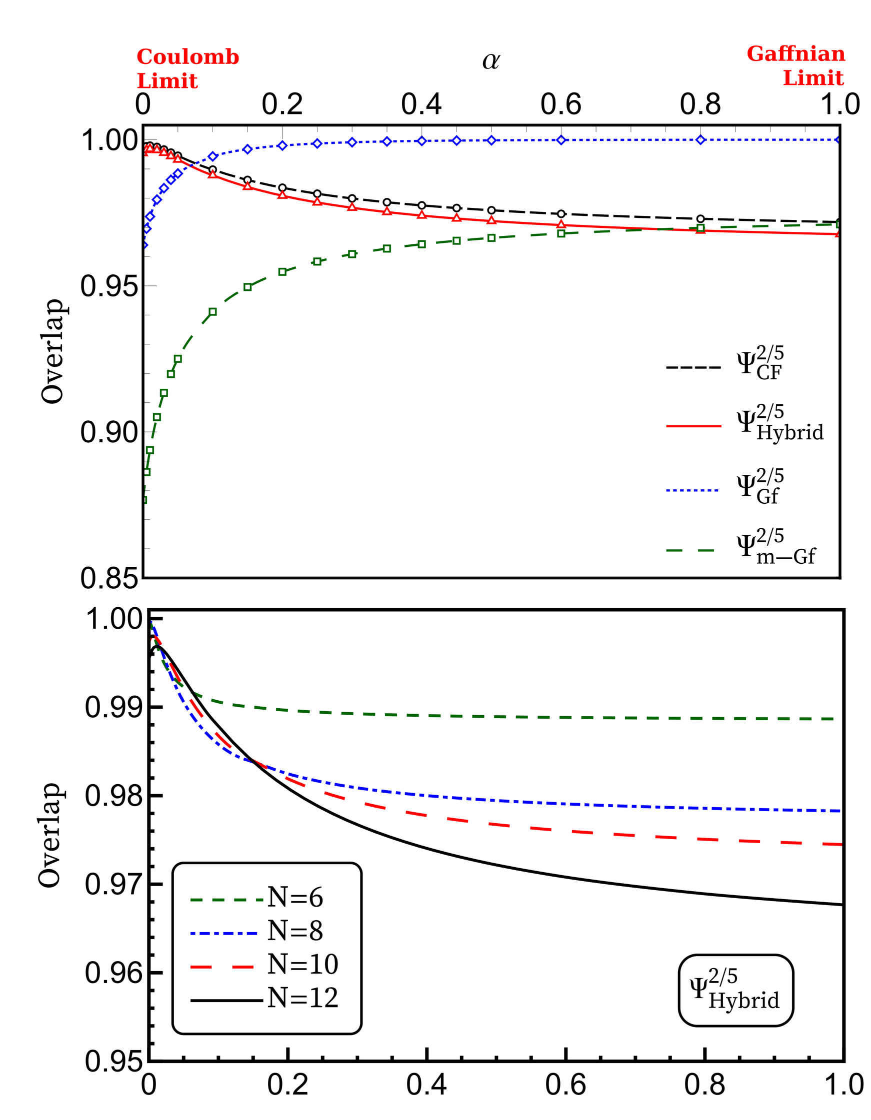




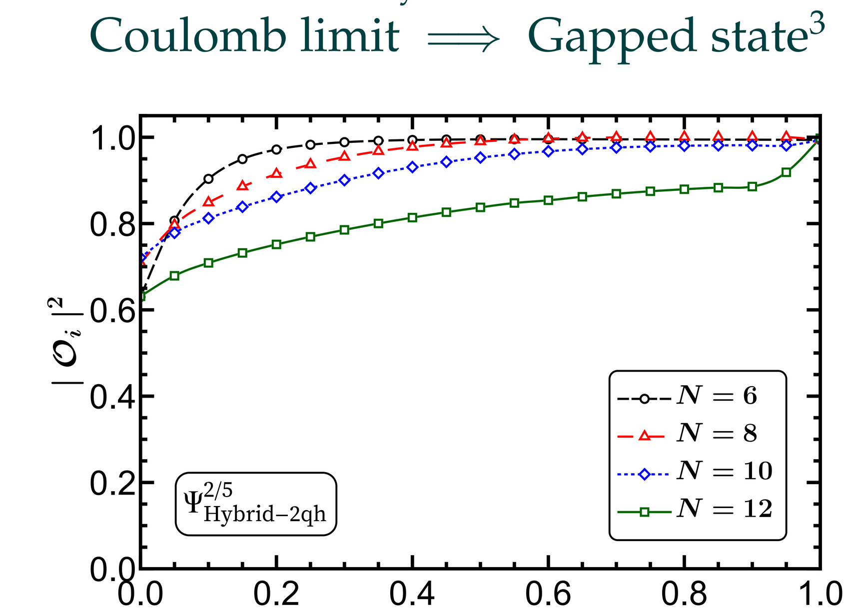
➡ $\Psi_{\rm Hybrid}^{2/5}$ reproduces the ES up to sufficiently higher energy $\Longrightarrow$ similar topological order as the exact state

Overlap with the model potential

$$H_\alpha = \alpha V_{Gf} + (1-\alpha)V_C$$



➡ Overlap of $\Psi_{\rm Hybrid}^{2/5}$ is maximised near Coulomb limit $\Longrightarrow$ Gapped state[3]



➡ Squared overlap of the $\Psi_{\rm Hybrid\text{-}2qh}^{2/5}$ is maximised near Gaffnian limit $\Longrightarrow$ non-Abelian statistics