# Conductance Spectroscopy of Majorana Zero Modes in Superconductor-Magnetic Insulator Nanowire Hybrid Systems


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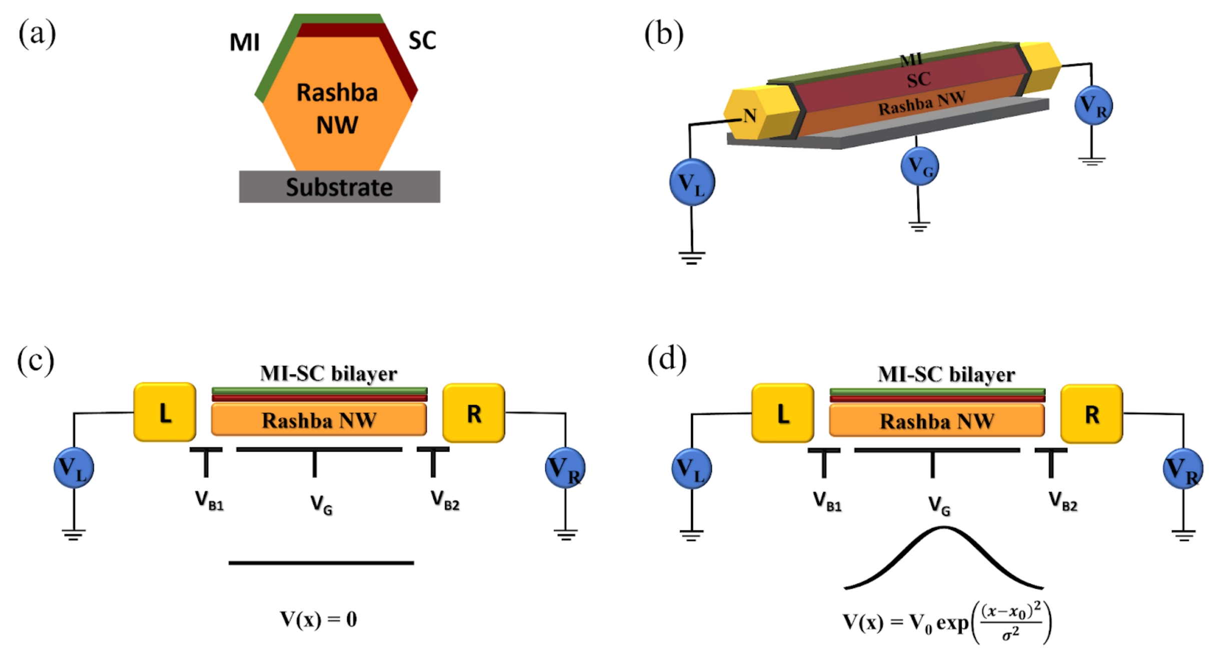
## Motivation & Objective

- Recently, there has been an interest in superconductor-magnetic insulator hybrid Rashba nanowire setups for potentially hosting Majorana zero modes (MZMs) at smaller external Zeeman fields.
- Large magnetic fields could potentially destroy superconductivity and there are practical issues regarding magnetic field alignment in nanowire networks.
- We develop a detailed quantum transport analysis for the hybrid system and uncover signatures of potential topological MZMs.
- We measure the nonlocal conductance in the 3-terminal device since the zero bias peak in the local conductance fails to distinguish between true MZMs and quasi-MZMs.

## Device Schematics




R.Singh and B. Muralidharan, ArXiv:2203.08413 (2022).

# Keldysh Nonequilibirum Green's Function Technique

## The SC-MI bilayer

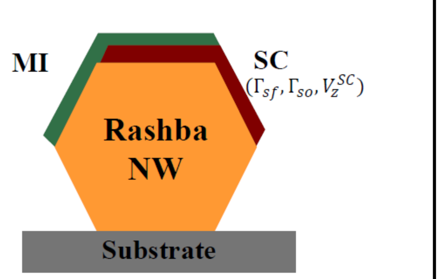
The superconducting gap equation and the Usadel equation are solved self-consistently to get the self-consistent value of the superconducting gap

**SUPERCONDUCTING GAP EQUATION**

$$\Delta \log\left(\frac{T}{T_{c0}}\right) = 2\pi T \sum_{\omega_n>0}\left(\frac{1}{4}\mathrm{Tr}(\hat{\tau}_x\check{g}) - \frac{\Delta}{\omega_n}\right)$$

**USADEL EQUATION**

$$D\nabla\cdot(\check{g}\nabla\check{g}) - \left[\omega_n\hat{\tau}_z + iV_Z^{SC}\cdot\hat{\sigma}\hat{\tau}_z + \Delta\hat{\tau}_x + \check{\Sigma}, \check{g}\right] = 0$$




**Effective of MI:**
Effective Zeeman field in SC, Rashba NW

**Diffusive Superconductor:**
Spin-Flip scattering off magnetic impurities ($\Gamma_{sf}$)
Spin-Orbit scattering ($\Gamma_{so}$)

**Magnetic Field in Superconductor:**

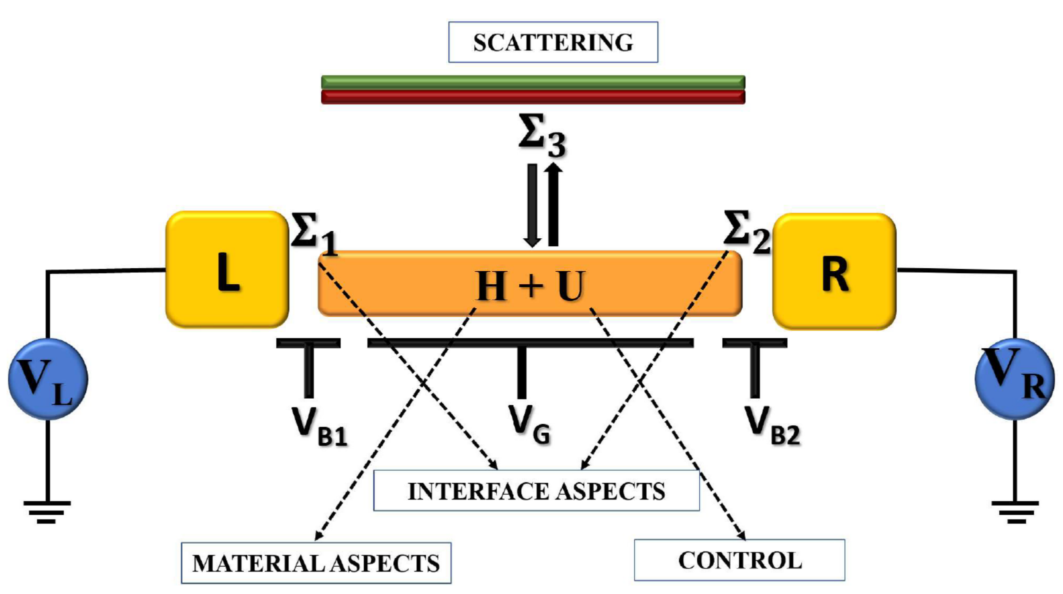
$$V_Z^{SC} = V_0^{SC} + g_{SC}V_Z^{ext}$$

**Self energy of SC-MI Bilayer:**

$$\check{\Sigma}'(\omega) = -i\gamma\hat{\tau}_z\check{g}(\omega_n)\big|_{\omega_n\to -i\omega}$$

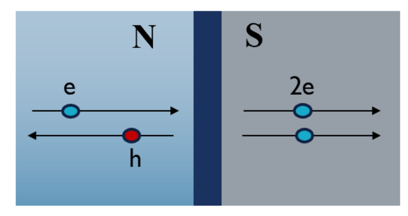
With this value of Δ, we can calculate the quasiclassical Green's function, for the bilayer and find its self-energy.

$$G^R(\omega) = (\omega - H - \Sigma_1 - \Sigma_2 - \Sigma_3)^{-1}$$




$$I_1^{op}(\omega) = \frac{e}{h}\left[G^R(\omega)\Sigma_1^< - \Sigma_1^< G^A(\omega)\right. \left. G^<(\omega)\Sigma_1^A(\omega) - \Sigma_1^R(\omega)G^<(\omega)\right]$$

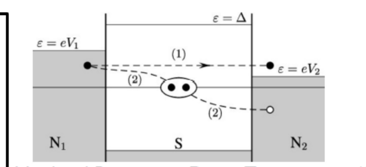
## Local and Nonlocal Conductance




Andreev Reflections : Local Process

**Transport in NSN Devices:**
- Andreev Reflections
- Direct Transmission
- Crossed Andreev Reflections

We also have to consider the current flowing into the SC-MI bilayer




Nonlocal Processes : Direct Transmission (1) Crossed Andreev Reflections (2)
Mikhail S. Kalenkov and Andrei D. Zaikin (Physical review. B, Condensed matter 75(17))

**Conductance Matrix**

$$[G] = \begin{pmatrix} G_{LL} & G_{LR} \\ G_{RL} & G_{RR} \end{pmatrix} = \begin{pmatrix} \left.\frac{\partial I_L}{\partial V_L}\right|_{V_R=0} & \left.\frac{\partial I_L}{\partial V_R}\right|_{V_L=0} \\ \left.\frac{\partial I_R}{\partial V_L}\right|_{V_R=0} & \left.\frac{\partial I_R}{\partial V_R}\right|_{V_L=0} \end{pmatrix}$$

**Local conductance signatures:**
Don't probe bulk gap, Trivial local zero energy states can mimic MZMs

**Nonlocal conductance signatures:**
Show bulk gap closing and reopening, better indicator in presence of disorder

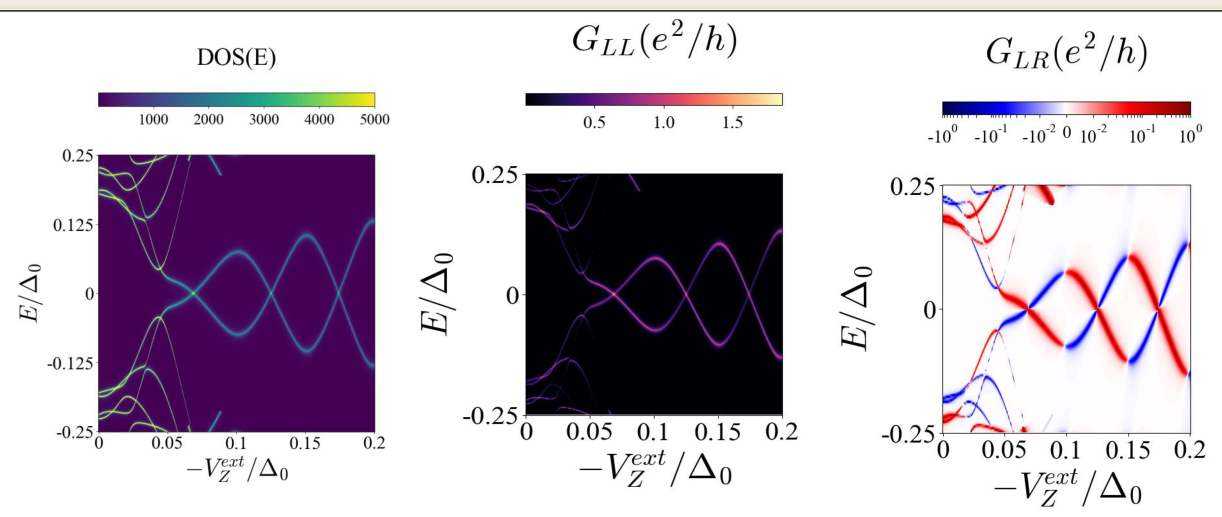
**Local Conductance Signature**

$$G_{LL}(V)|_{T\to 0} \equiv \frac{e^2}{h}\left[T_A(E=eV) + T_A(E=-eV) + T_{CAR}(E=eV) + T_D(E=eV) + G'_{LL}(V)\right]$$

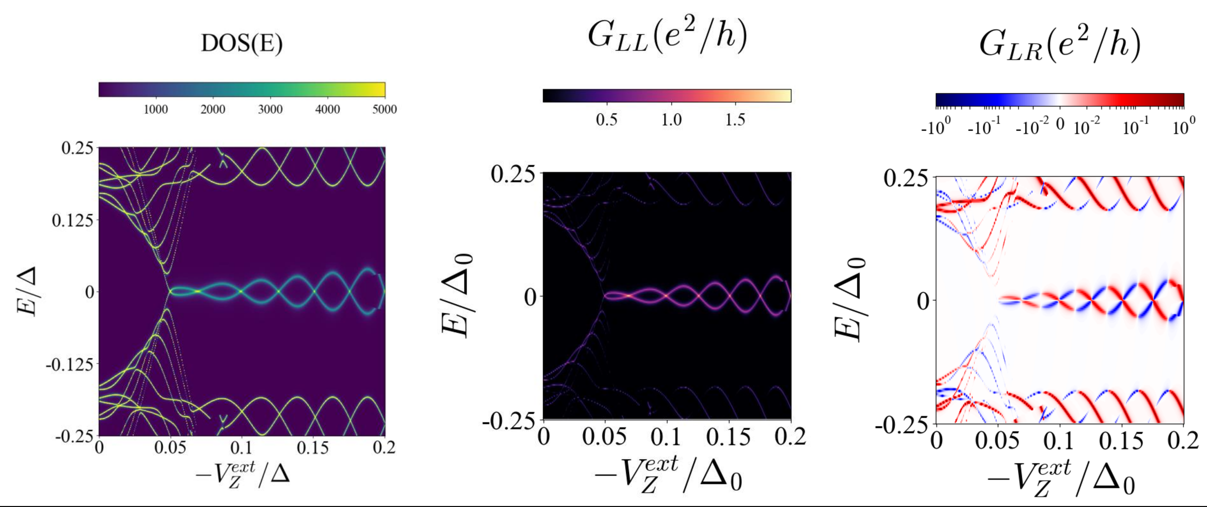
**Non-local Conductance Signature**

$$G_{LR}(V)|_{T\to 0} \equiv \frac{e^2}{h}\left[T_{CAR}(E=-eV) - T_D(E=eV)\right]$$

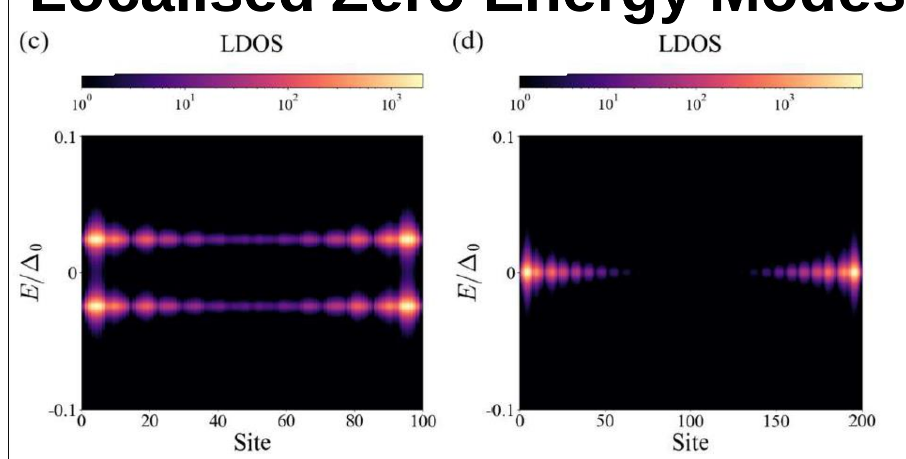
# Results and discussion

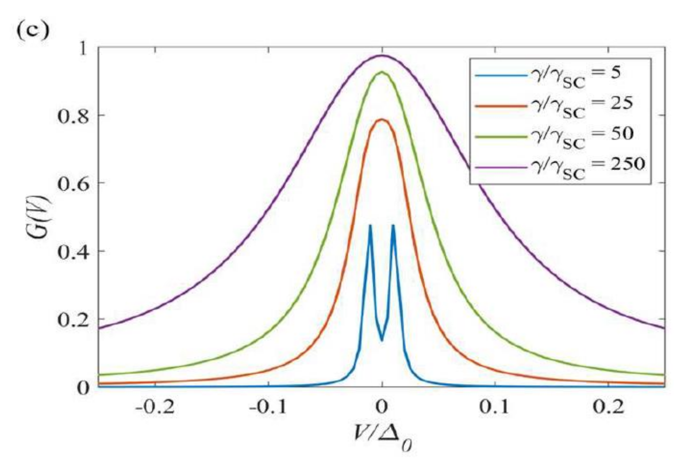



## Clean Nanowire

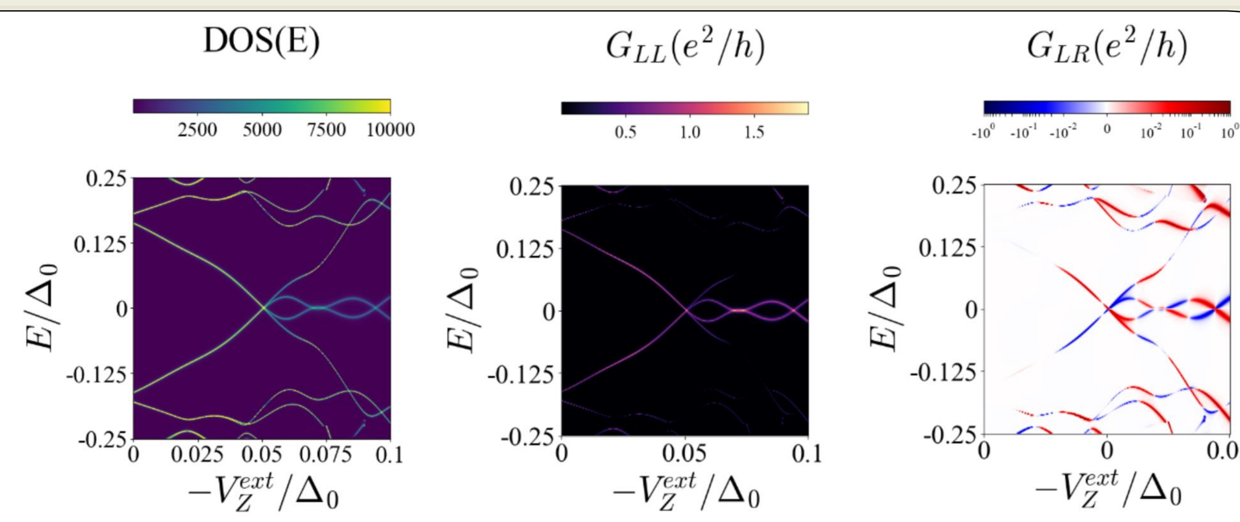



## Localised Zero Energy Modes

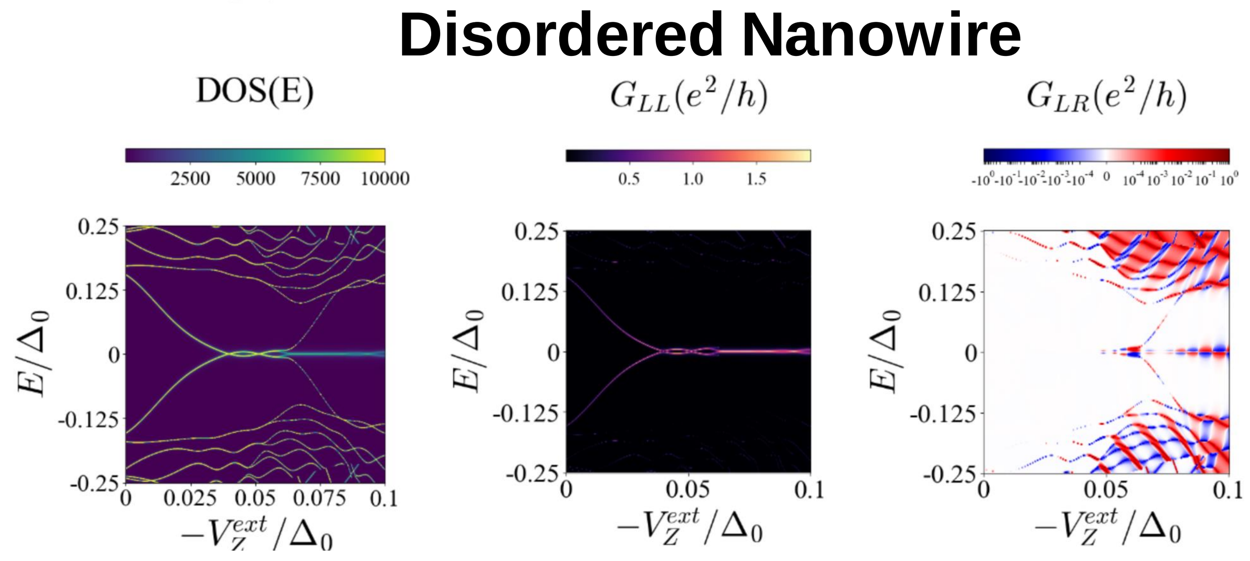



## Scaling of the ZBCP




## Disordered Nanowire




## Features observed

- As the coupling to the metallic contacts becomes much stronger than the coupling to the bilayer, the ZBCP asymptotically reaches the expected quantized value.
- For the system without disorder, the bulk gap closing and reopening is visible in the local and nonlocal conductance followed by the emergence of Majorana oscillations around zero energy. A finite low-bias non-local conductance only emerges after the topological transition (marked by closing, reopening of the gap).
- The low bias non-local conductance is rectifying in nature and switches sign as the voltage polarity is reversed. At the turning points, the non-local conductance vanishes.
- For the disordered case, we find signatures characteristic of a quasi MZM state, followed by a gap reopening signature and the emergence of a potential topological MZM.

## Analysis

- We see a ZBCP in the topological regime, which is split due to hybridization of the MZMs through the finite nanowire.
- The ZBCP is not exactly quantized due to the broadening induced by the SC-MI contact. When this broadening becomes negligible compared to that induced by the metallic contacts, it asymptotically reaches the quantized value.
- Since the zero modes appear after the gap closing and reopening in the clean nanowire, they are expected to be topological MZMs.
- The zero mode appearing in the disordered nanowire before the gap reopens is likely a quasi-MZM. A true MZM is formed at higher fields.
- There is a correspondence between non-local conductance and the BCS charges of the bound state at the leads. The vanishing of the nonlocal conductance at the turning points indicates that we have chargeless MZMs at the turning points, as expected.

## Conclusion

- The SC-MI hybrid nanowire exhibits transport signatures consistent with those expected from topological zero modes which are protected by a gap for the pristine and disordered wire.
- The local and nonlocal conductance can be used to identify MZMs in compliance with the topological gap protocol.
- When a smoothly varying potential is present, quasi MZMs may form with a premature gap closure in the conductance spectra.

## References


[1] R. Singh and B. Muralidharan, ArXiv:2203.08413 (2022).
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[6] Aghaee, Morteza,et. al., ArXiv:2207.02472 (2022).