Boiling quantum vacuum: Thermal manifestation of entanglement

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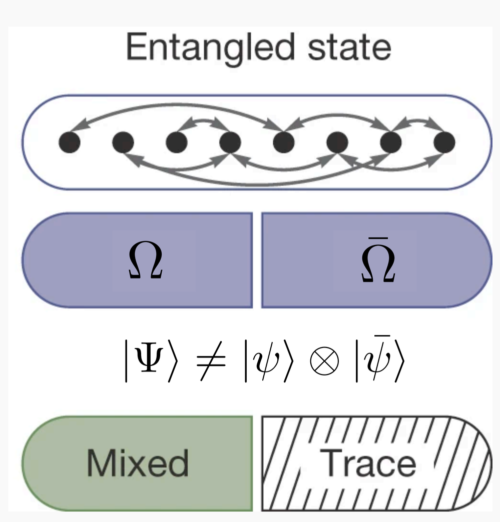
Introduction

Entanglement expresses the inherent non-locality of quantum systems that prohibits an independent description of the state of particles when they are parts of an entangled multi-particle state. On a fundamental level, entanglement is now considered a key unifying concept in underpinning diverse phenomena from quantum information science, and condensed matter systems to black hole physics.

A well-known example is provided by the Hawking and Unruh effects where the emergence of finite temperature in black holes and accelerated vacua is attributed to entanglement with an unobservable region beyond the horizon and Rindler wedge. So the key motivation and eventually result of this work is to find experimentally accessible systems in which such a temperature-entanglement map can be realized.



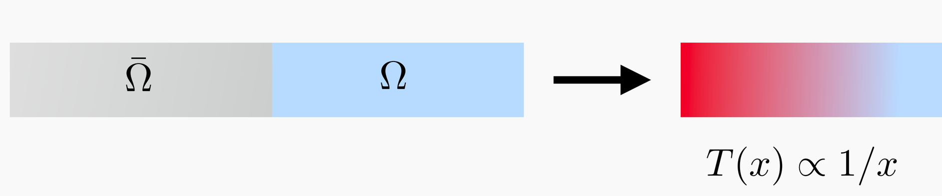
In manybody systems, a typical way of quantifying entanglement is provided through the bipartitioning, and then the reduced density matrix of one subsystem,

$$\rho_\Omega = \mathrm{Tr}_{\bar{\Omega}}\rho_{\mathrm{full}} = \mathrm{Tr}_{\bar{\Omega}}|\Psi\rangle\langle\Psi|$$

The positive-definiteness of the reduced density matrix enables us to define the entanglement Hamiltonian through:

$$\rho_\Omega = \frac{1}{\mathcal{Z}}e^{-H_E}$$

This form might suggest some sort of connection to thermal density matrices. So far, such a connection has remained a purely theoretical enterprise since the resulting temperature strongly varies with spatial coordinates in the light of the *Bisognano-Wichmann theorem* and entanglement area-law behavior.

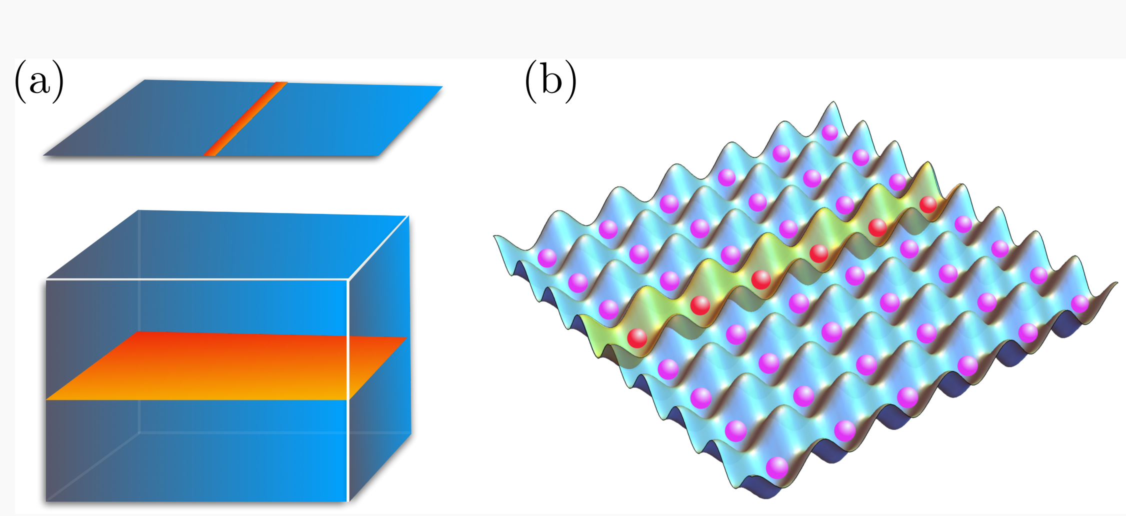


The main result in a nutshell

We show how thermal density matrices emerge from vacuum entanglement in an unambiguous and experimentally realizable way in lower-dimensional subsystems embedded in a D-dimensional gapped fermionic vacuum (Fig. below). The generic reduced density matrix of such subsystems reads:

$$\rho_{D_s} = \frac{e^{-\sum_{k_s}\beta(\mathbf{k}_s)\hat{d}^\dagger_{\mathbf{k}_s}H_{D_s}(\mathbf{k}_s)\hat{d}_{\mathbf{k}_s}}}{\mathcal{Z}}$$

We particularly prove this result for Dirac systems, where the resulting temperature has a weak momentum-dependent and the effective subsystem Hamiltonian is a lower-dimensional Dirac model induced by the parent system.



Entanglement-temperature map?

For free fermionic models, all the information encoded in a reduced density matrix is identical to those in correlations:

$$\mathcal{C}^{\alpha\alpha'}_{\mathbf{x},\mathbf{x}'} = \langle \hat{c}^\dagger_{\mathbf{x}\alpha}\hat{c}_{\mathbf{x}'\alpha'}\rangle^*$$

$$\rho_\Omega \longleftrightarrow \mathcal{C}(\text{correlation martix})$$

For a D_s-dimensional subsystem we find

$$\mathcal{C}^{\alpha\alpha'}_{\mathbf{x},\mathbf{x}'} = \frac{1}{L^{D_s}}\sum_{\mathbf{k}_s} e^{-i\mathbf{k}_s\cdot(\mathbf{x}-\mathbf{x}')}\langle\alpha|\hat{\mathcal{C}}^{\mathrm{sub}}(\mathbf{k}_s)|\alpha'\rangle$$

with $\hat{\mathcal{C}}^{\mathrm{sub}}(\mathbf{k}_s) = \frac{1}{L^{D-D_s}}\sum_{\text{filled } \nu,\mathbf{k}_\perp}|\psi_{\nu\mathbf{k}}\rangle\langle\psi_{\nu\mathbf{k}}|$ $\xrightarrow[\text{mapping?}]{\text{Thermal}}$ $\hat{\mathcal{C}}^{\mathrm{th}}(\mathbf{k}_s) = \sum_\nu |\phi_{\nu\mathbf{k}_s}\rangle\langle\phi_{\nu\mathbf{k}_s}|n_F(\omega_{\nu\mathbf{k}_s})$

Example: 2D Chern model $H = \mathbf{d}_2(\mathbf{k})\cdot\sigma$

$$\mathbf{d}_2(\mathbf{k}) = (\sin k_x, t_y\sin k_y, m - \cos k_x - t_y\cos k_y)$$

1D subsystem:

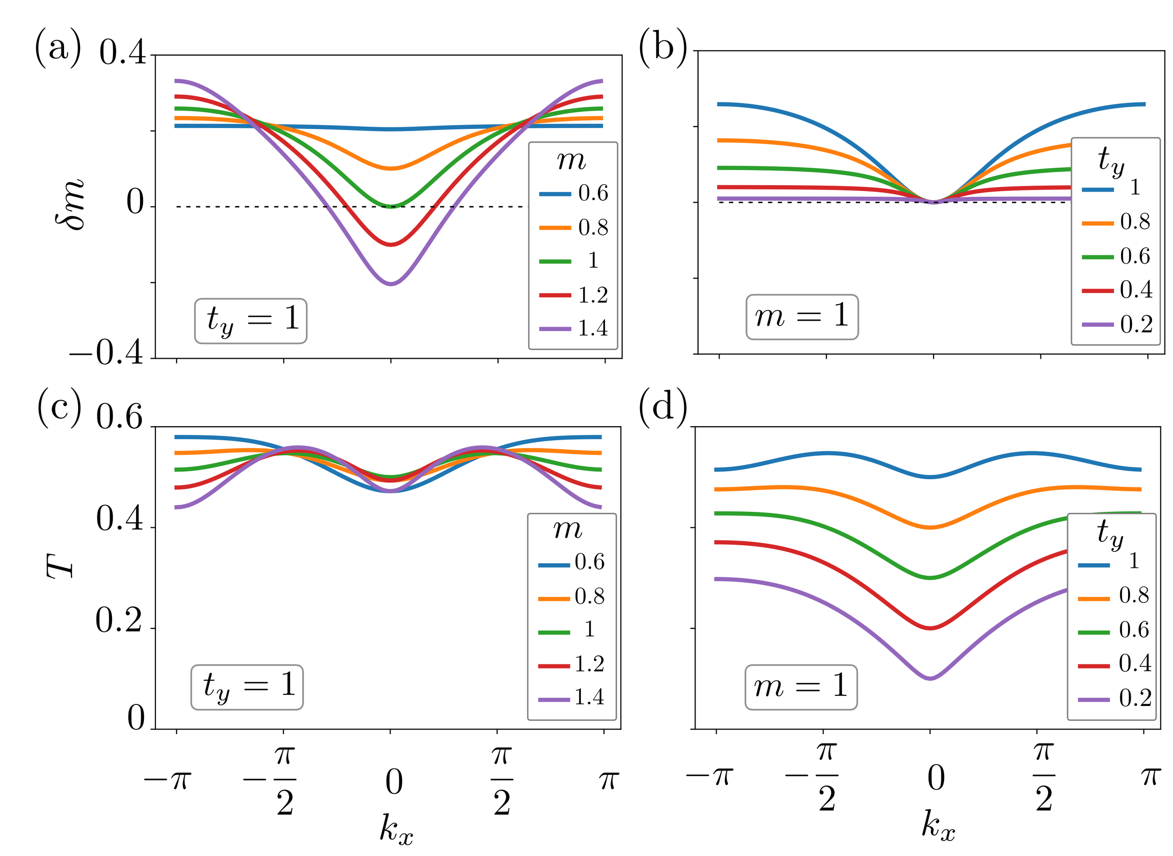
$$\mathcal{C}(k_x) = \frac{1}{2L}\sum_{k_y}\Big(\mathbb{1} - \frac{\mathbf{d}_2(\mathbf{k})}{|\mathbf{d}_2(\mathbf{k})|}\cdot\boldsymbol{\sigma}\Big) = \frac{1}{2}\big[\mathbb{1} - \mathcal{F}(k_x)\mathbf{d}_1(k_x)\cdot\boldsymbol{\sigma}\big]$$

$$\mathcal{F}(k_x) = \frac{1}{L}\sum_{k_y}\frac{1}{|\mathbf{d}_{2D}(\mathbf{k})|},$$

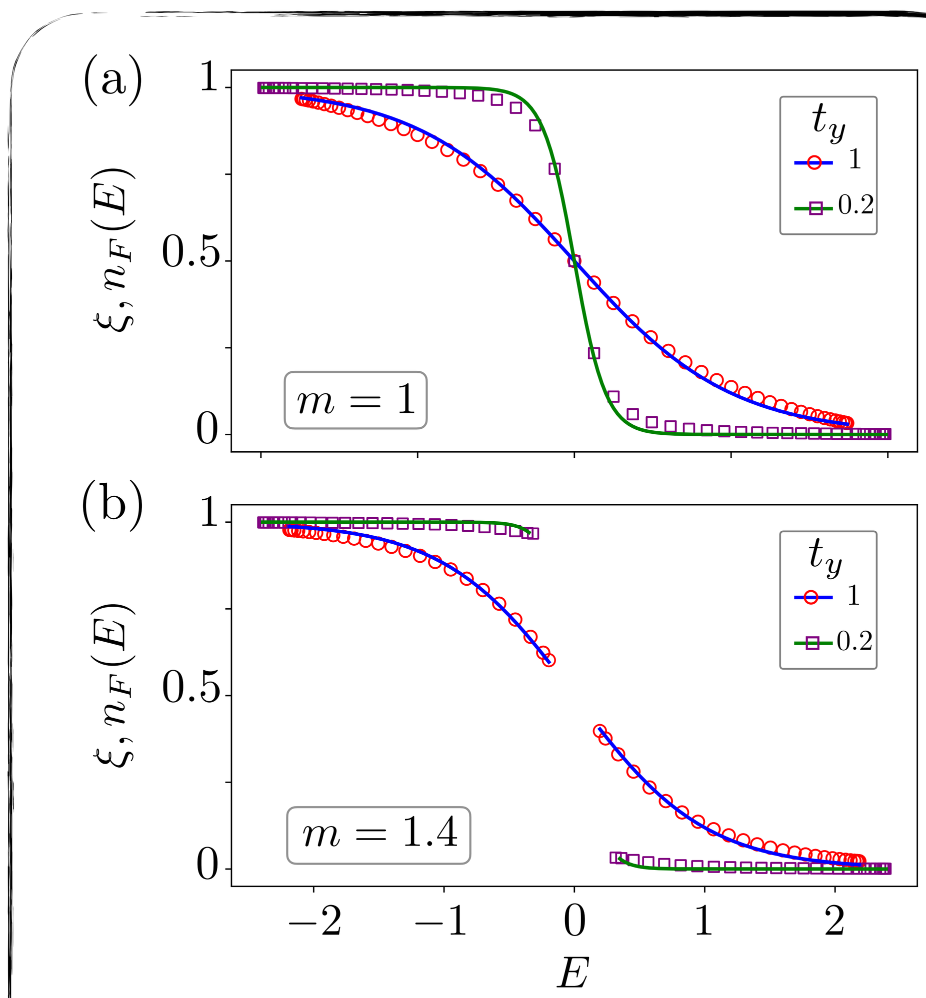
$$\mathbf{d}_{1D}(k_x) = \big[t_x\sin k_x, 0, m - \delta m(k_x) - t_x\cos k_x\big],$$

$$\delta m(k_x) = \frac{1}{\mathcal{F}(k_x)}\frac{t_y}{L}\sum_{k_y}\frac{\cos k_y}{|\mathbf{d}_{2D}(\mathbf{k})|}.$$

$$T(k_x) = \frac{|\mathbf{d}_{1D}|}{2\,\mathrm{arctanh}(|\mathbf{d}_{1D}|\mathcal{F})} \quad \text{(Temperature)}$$



Direct signature of thermal mapping

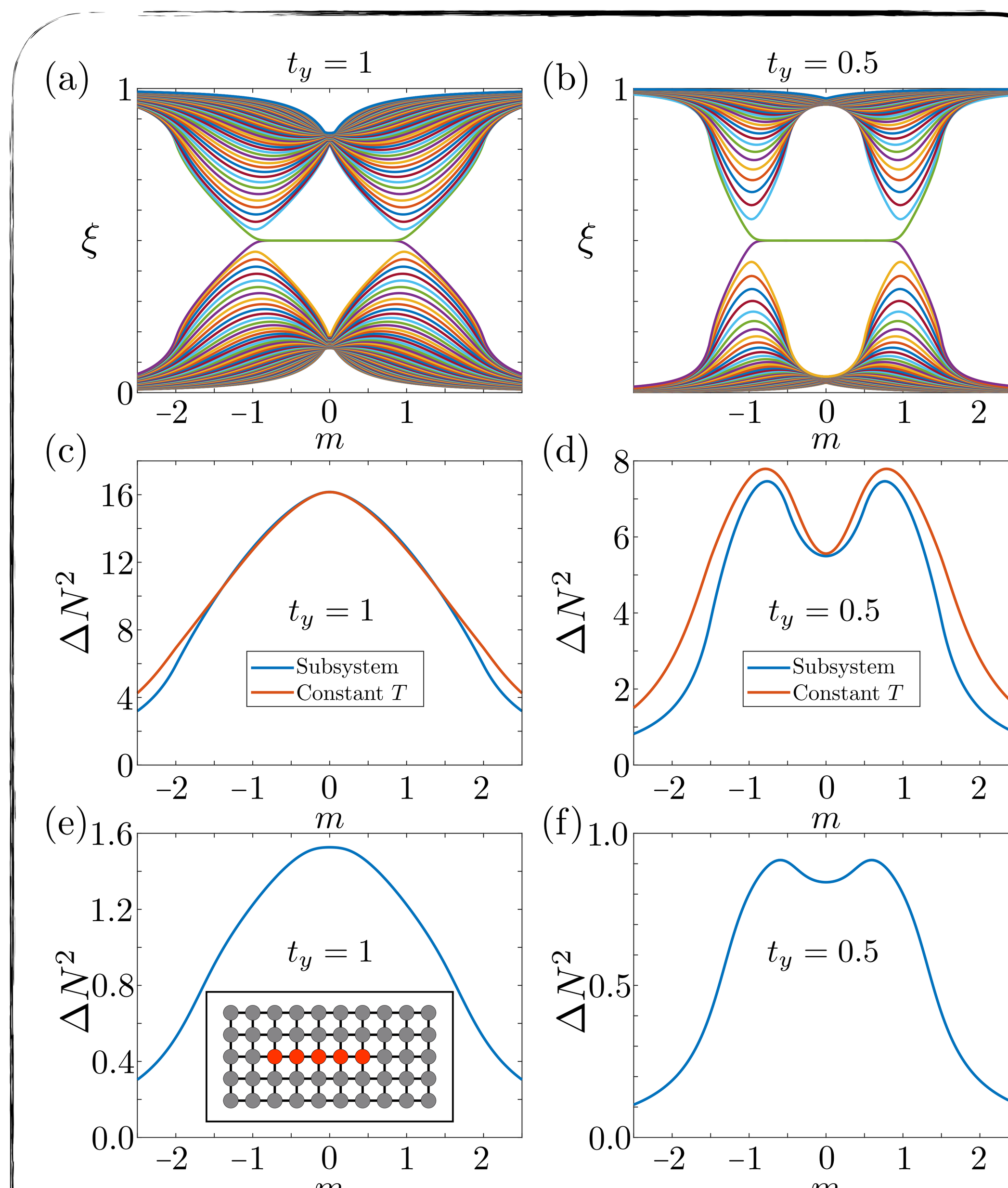


Energy-resolved distribution of the 1D subsystem in the 2D Chern model for two different m. Circles/ squares show the correlation matrix spectra for the 1D subsystem, whereas the solid lines correspond to a thermal 1D system and therefore they are identical to Fermi-Dirac distribution functions.

Above results show that the subsystem's density matrix almost perfectly resembles that of a genuinely thermal system subjected to a constant temperature.

The temperature is determined with t_y, thus it can be quite high.

Experimentally accessible consequences



Correlation matrix spectra & particle number variances of finite 1D subsystems of a 2D Chern model, and a corresponding genuinely thermal 1D system.

The fluctuation of particle number (or other conserved extensive quantities in other models) provides an experimental route to justify the thermal-like properties of the lower-dimensional subsystem.

Conclusions

- A generic framework for observation of a thermal state emerging from vacuum entanglement
- Lower-dimensional subsystems of a zero-temperature gapped state that look like hot gapless systems.
- Natural connections to the dimensional reduction picture and the periodic table of topological phases.
- Possible experimental tests in quantum simulators e.g. those based on ultracold atoms in optical lattices.

More information in "AG Moghaddam, K Pöyhönen, T Ojanen, arXiv:2204.01791 (to appear in *PRX Quantum*)"