Topological Random Fractals

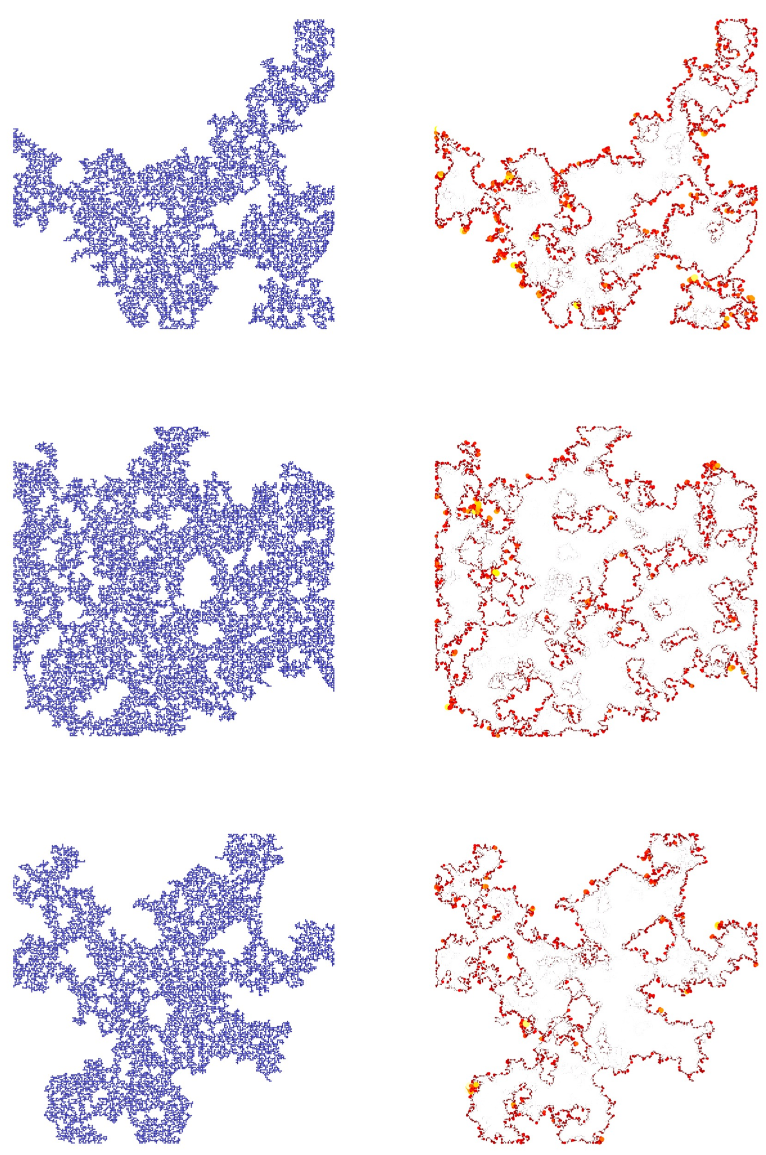
Moein N. Ivaki, Isac Sahlberg, Kim Pöyhönen and Teemu Ojanen

Computational Physics Laboratory, Physics Unit, Faculty of Engineering and Natural Sciences,
Tampere University, P.O. Box 692, FI-33014 Tampere, Finland and
Helsinki Institute of Physics P.O. Box 64, FI-00014, Finland

Tampereen yliopisto Tampere University

Abstract

In this work we generalize the notion of topological electronic states to random lattices in **non-integer dimensions**. By considering a *class D tight-binding model on critical percolating clusters* resulting from a two-dimensional site percolation process, we demonstrate that these topological random fractals exhibit the hallmarks of topological insulators while hosting a gapless spectrum.



arXiv:2112.08824

Main findings

Our large-scale numerical studies reveal that topological random fractals:

- Display a ***robust mobility gap***,
- Support ***quantized conductance.***
- Represent a ***well-defined thermodynamic phase of matter***.

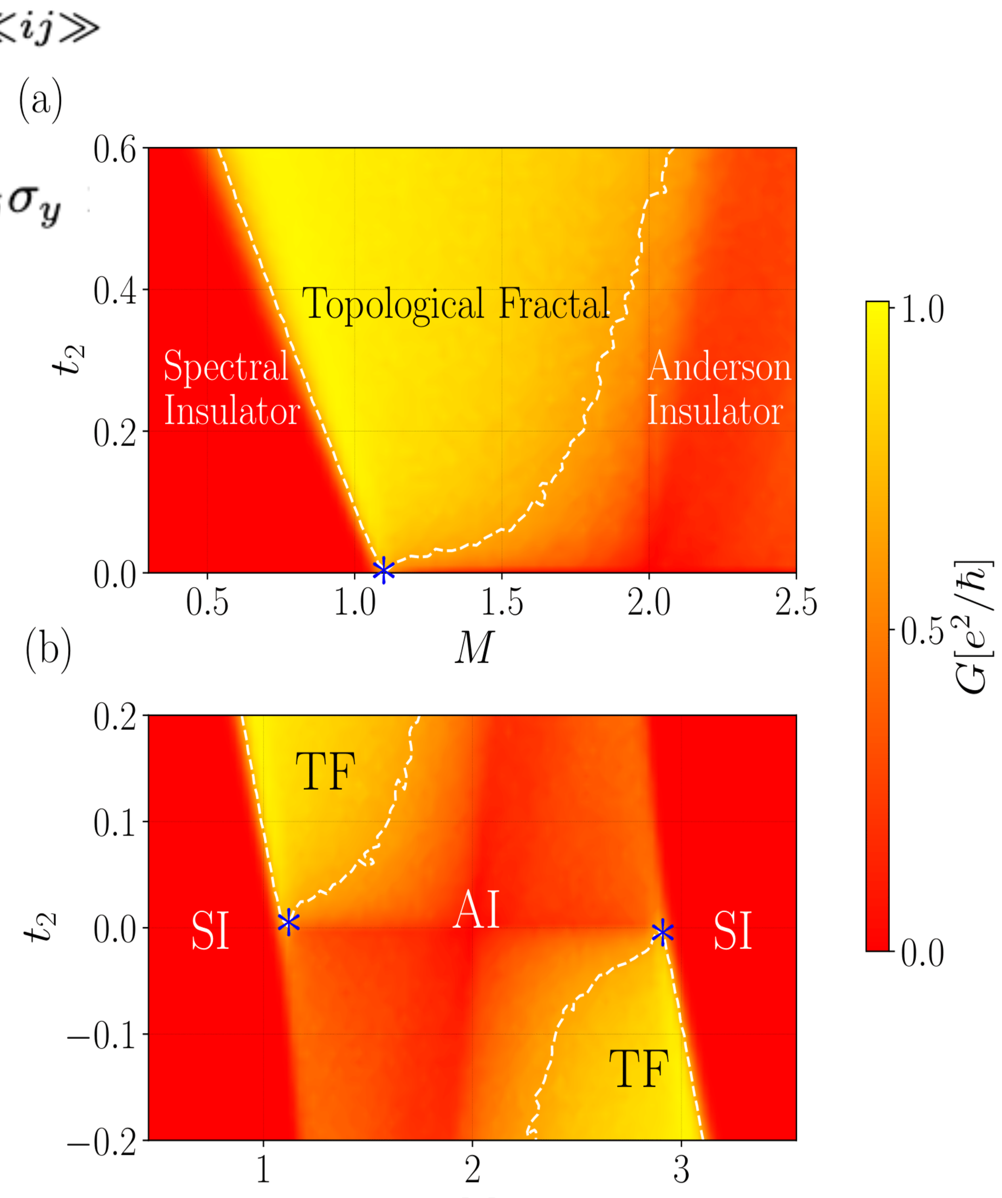
The finite-size scaling analysis further suggests that the ***critical properties are not consistent with the expectations of class D* systems in two dimensions**, hinting to the nontrivial relationship between fractal and integer-dimensional topological states.

Model and phase diagram

$$\mathbb{H} = (2-M)\sum_i \boldsymbol{c}_i^\dagger \sigma_z \boldsymbol{c}_i - \frac{t}{2}\sum_{<ij>} \boldsymbol{c}_i^\dagger \eta_{ij} \boldsymbol{c}_j - \frac{t_2}{2}\sum_{\ll ij \gg} \boldsymbol{c}_i^\dagger \eta_{ij} \boldsymbol{c}_j + \text{h.c.},$$

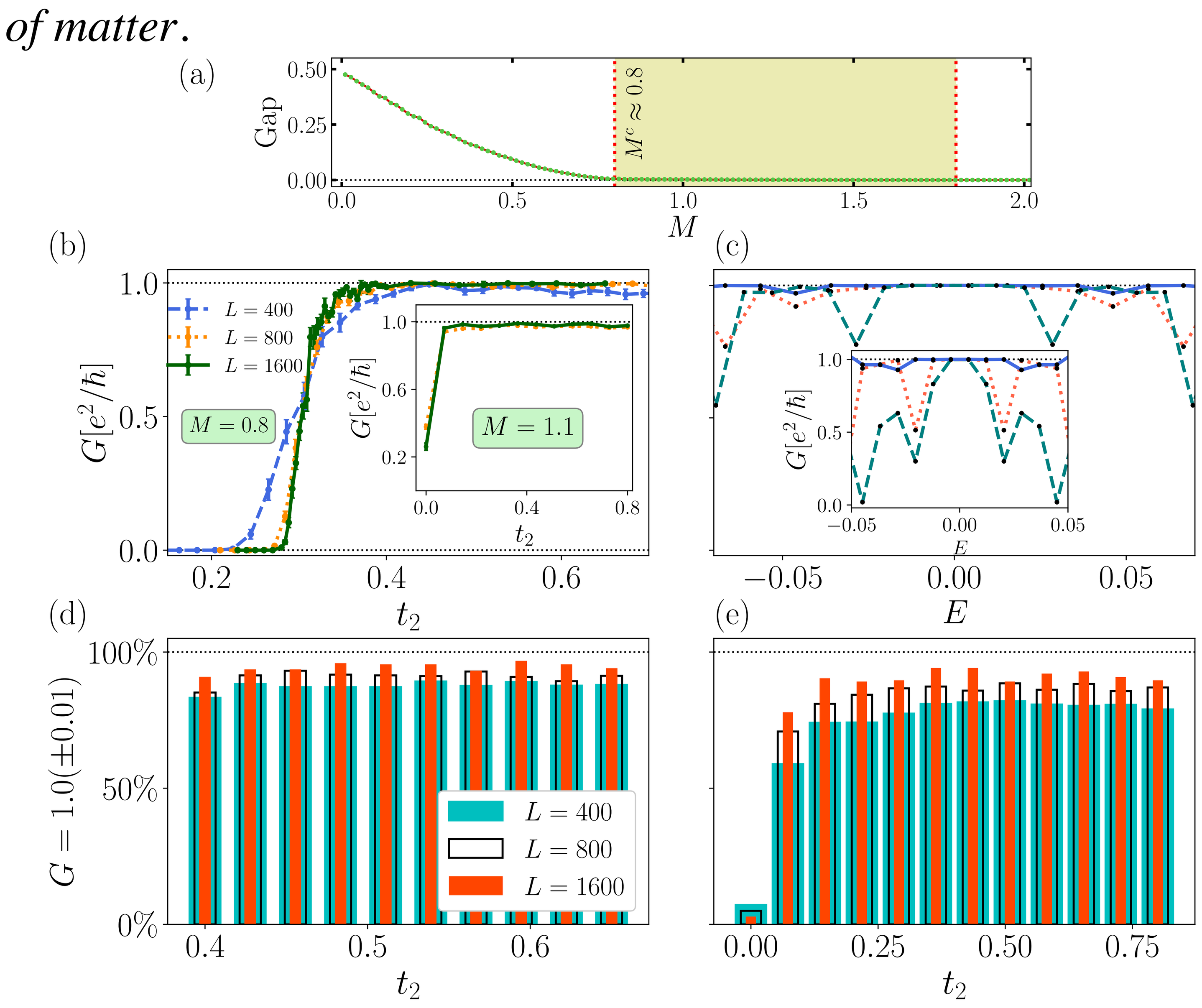
$$\eta_{ij} = \sigma_z + i\cos\theta_{ij}\sigma_x + i\sin\theta_{ij}\sigma_y$$

- This model **breaks *TR*** and **satisfies *PH*** symmetries, hence belonging to the **class *D***.
- On a square lattice in the clean limit, the model supports topological phases with **nonzero Chern number**.



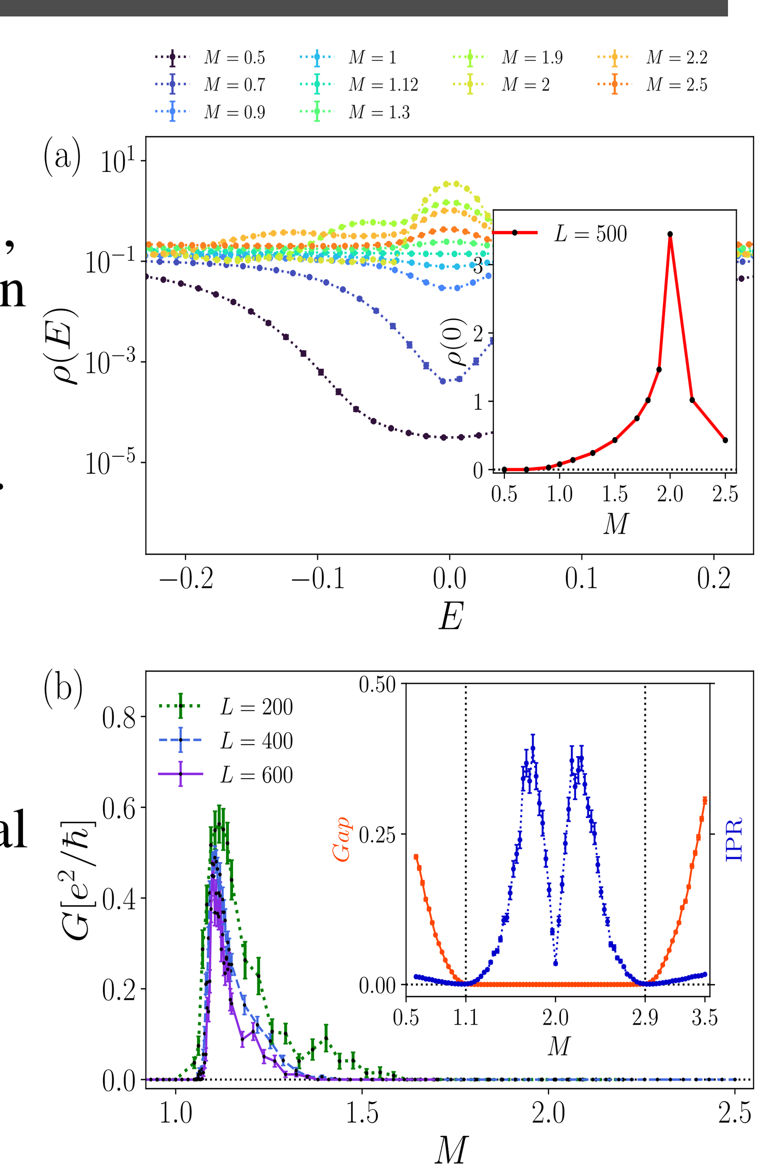
Properties of topological random fractal phase

- The gapped-gapless phase transition coincides with the trivial-topological fractal transition.
- If the energy is in the mobility gap, the conductance (for samples larger than the localization length) remains quantized.
- The quantization develops rapidly when moving from the tricritical point towards the topological phase**. Larger systems exhibit on average more precise quantization**, indicating that the random fractal phase is a well-defined *thermodynamic phase of matter*.



Spectral insulator-Anderson insulator transition

- The mid-spectrum DOS indicates the formation of a spectral gap for $M < 1.1$, while for $1.1 < M < 2.9$ the system is in a gapless AL phase. Both phases separated by the **tricritical metallic point** $(t_2^c, M_c) \approx (0, 1.1)$ are insulating.
- The inverse participation ratio in the region $1.1 < M < 2.9$ shows great enhancement around $E = 0$ everywhere in the AL regime, indicating that the spectrum is gapless but consists of trivial localized states.



Finite-size scaling of two-terminal conductance

Near the transition, one expects that the configuration-averaged conductance obeys a *single-parameter scaling* hypothesis in the large system limit. By calculating the conductance as a function of the second-nearest-neighbour hopping, we can show that the ***standard two-dimensional class D scaling does not match the numerical evidence.***