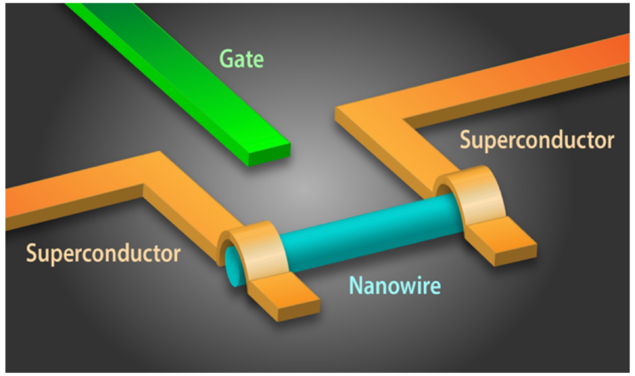
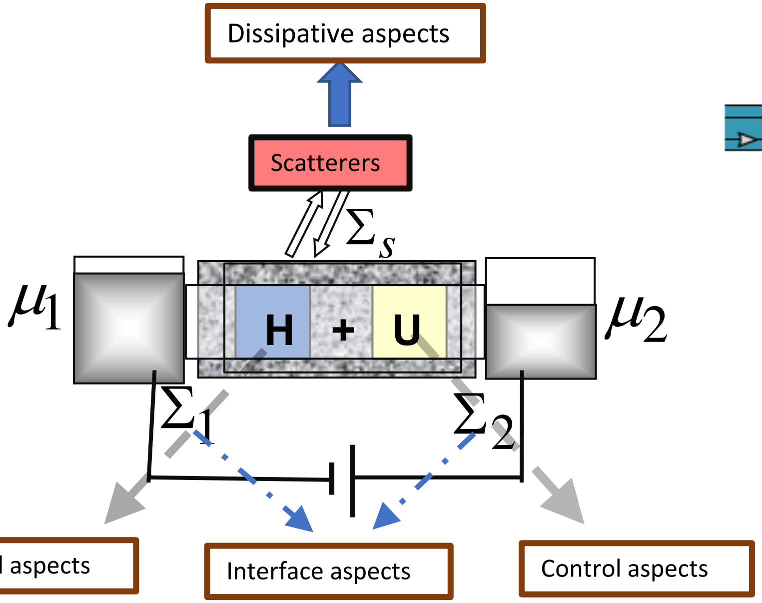
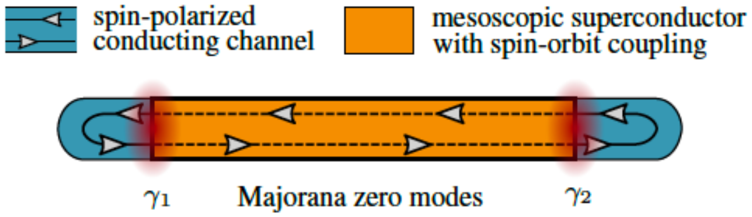
Understanding conductance spectroscopy of Majorana zero modes

https://twitter.com/quantumtranspo1

http://cnqt-group.org







PI: Prof. Bhaskaran Muralidharan
Email: bm@ee.iitb.ac.in
Professor
Department of EE
IIT Bombay

Current flow at the nanoscale

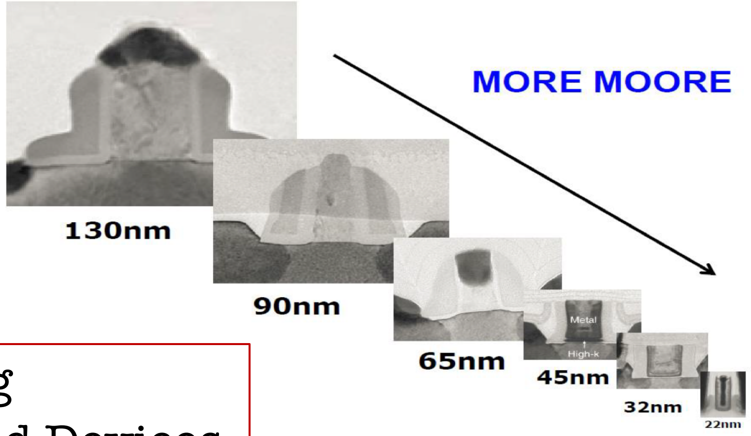
CTQM2022

IIT Bombay

Quantum Information Computing Science & Technology (QuICST), IIT Bombay

- 15+ associated faculty members
- Projects worth INR 28 Cr underway
- State-of-the-art fabrication and measurement infrastructure
- Inter-disciplinary dual degree (B Tech. – M Tech.) program

Prelude : "Beyond Moore" Nanoelectronics



1. Quantum
2. Neuromorphic

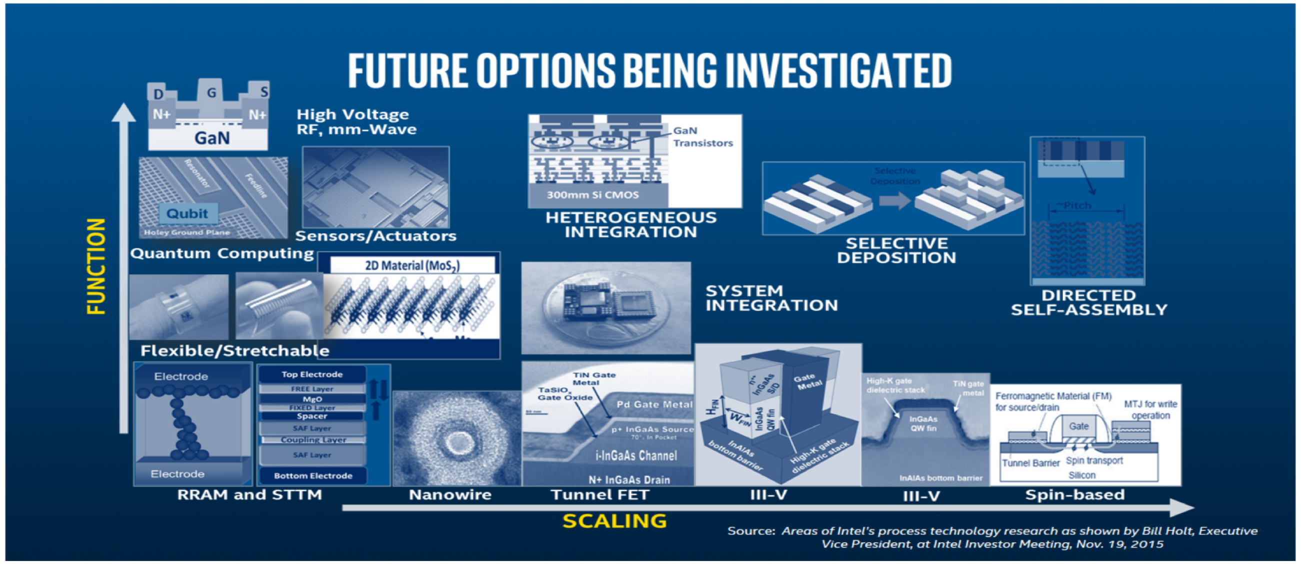
http://cnqt-group.org

What next ? After Ultimate Scaling?

Device Modeling

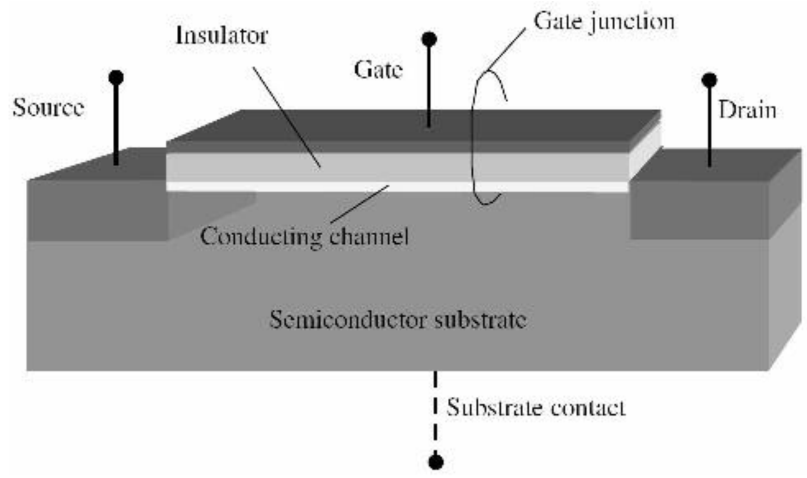
1. Quantum Materials and Devices
2. Topological Quantum Devices

Beyond Moore

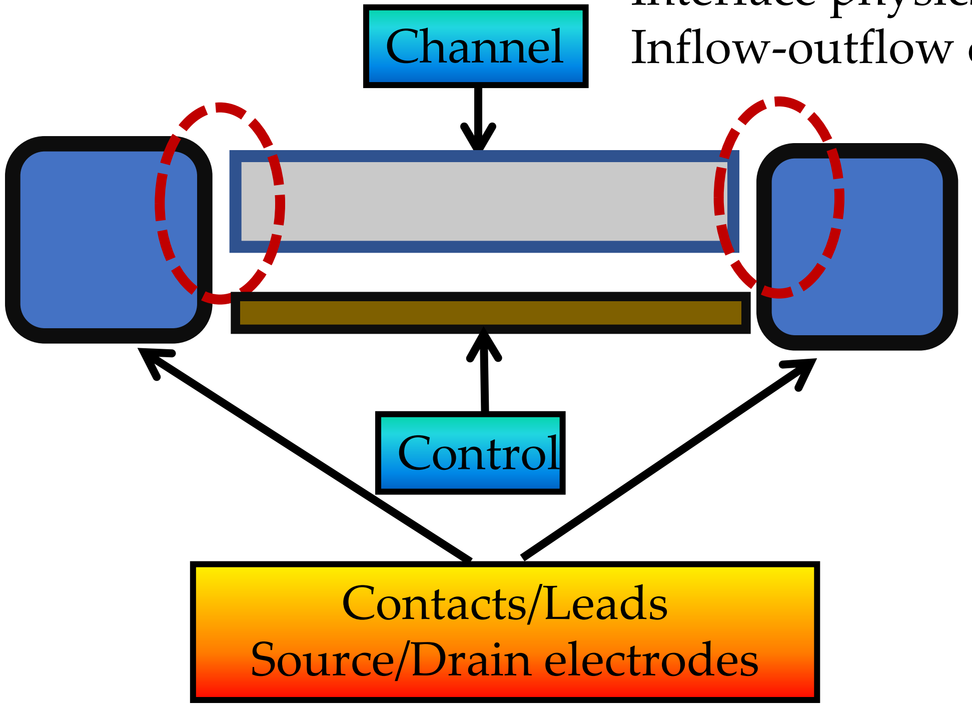


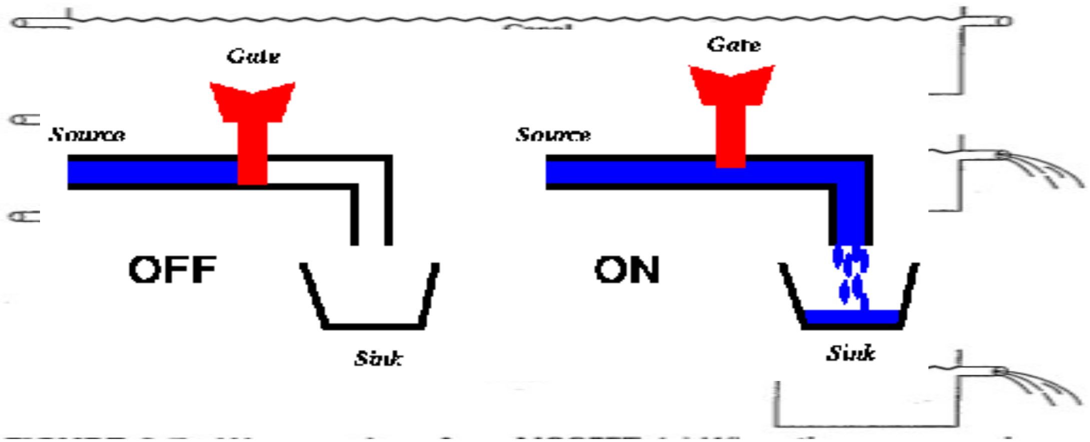
Current flow at the nanoscale

Anatomy of a Nano-Device



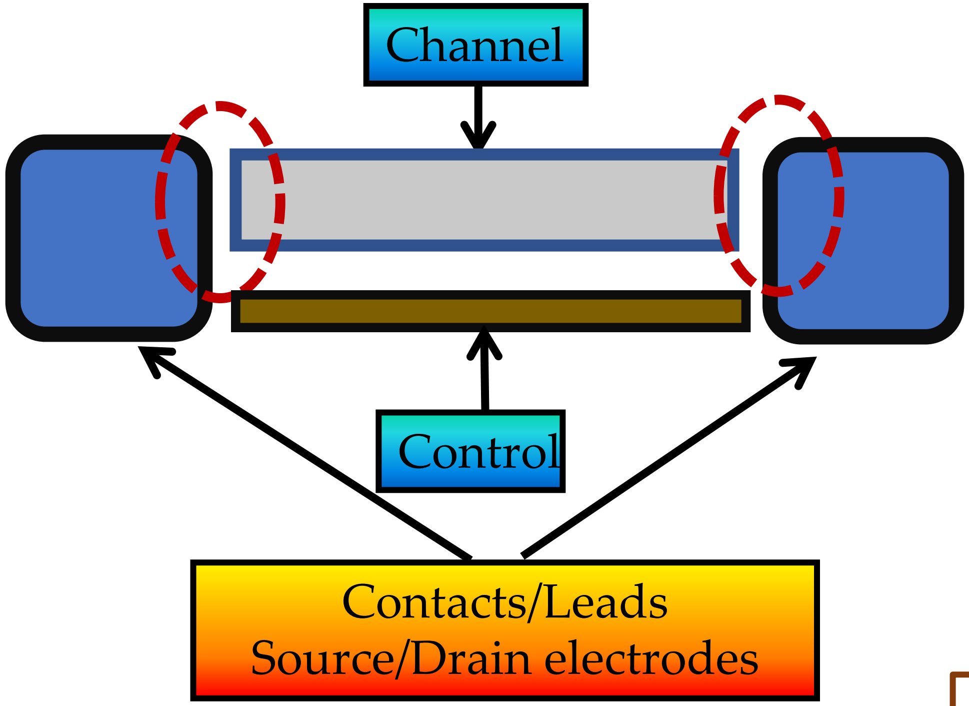
Material Science
Interface physics
Inflow-outflow dynamics

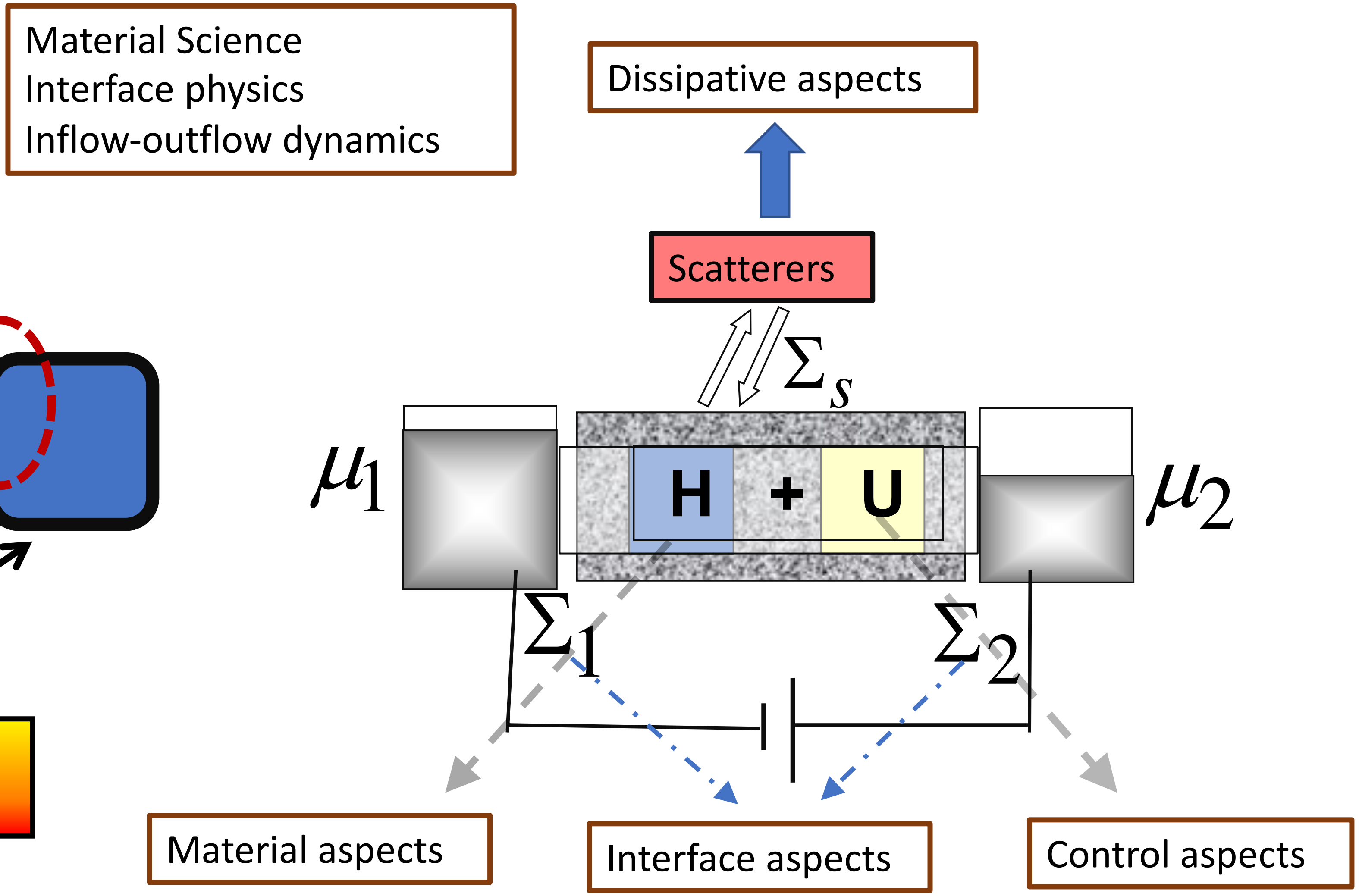




Datta, Lessons from Nanoelectronics, (2012)
Feynman, Lectures on Computation, (1998)

Anatomy of a Nano-Device: Non-equilibrium Green's function formalism

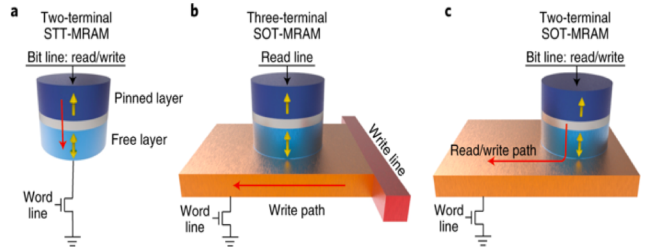




Datta, Quantum Transport, Cambridge Univ Press (2005)

"Beyond Moore" Device Research Highlights

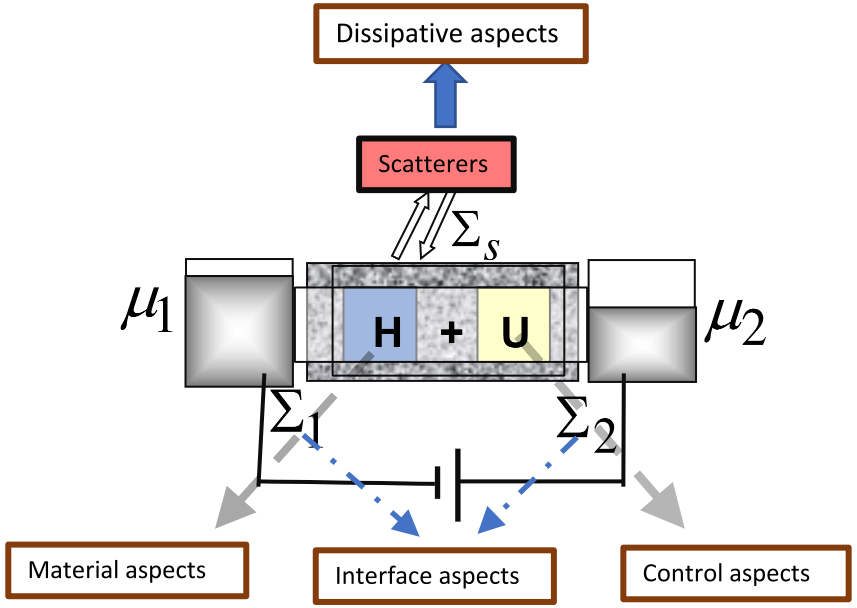
Spintronics: MRAM



Recent Publications
IEEE Trans.Elec Dev., 63, 4527-4563, (2016).
Phys. Rev. Applied, 8, 064014, (2017).
Appl. Phys. Lett., 112, 192404, (2018).
Phys. Rev. Applied, 12, 024038, (2019).
IEEE Trans. Nano., 19, 469, (2020).

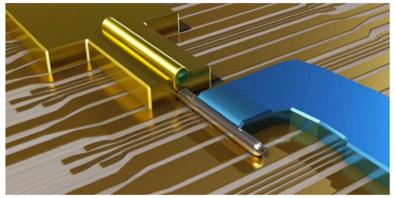
- Spin filtering devices
- STT-MRAM
- Toward Neuromorphic

- 2D topological spintronics
- Materials -> Devices -> Functionalities



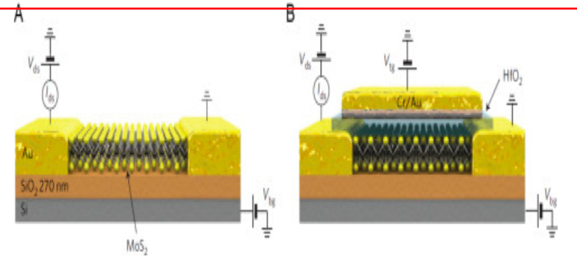
Unified Quantum Device Simulation Platform

Topological hybrid quantum systems



Recent Publications
Phys. Rev. B, 98, 125417, (2018).
Phys. Rev B, 100, 155431, (2019)
Phys. Rev. Research 2, 043430, (2020).
Phys. Rev B,103,165432,(2021).
Phys Rev B, 105, L161403, (2022)

2D Quantum Materials and Devices



Recent Publications
Phys. Rev. Applied, 10, 014022, (2018).
Phys. Rev. B, 99, 075415, (2019).
Phys. Rev. B (Rapid Comm), 100, 081403, (2019).
Phys. Rev. Materials, 3, 124005, (2019).
Phys. Rev. Research, 2, 043041, (2020)
npj 2D materials., 6, 19, (2022)

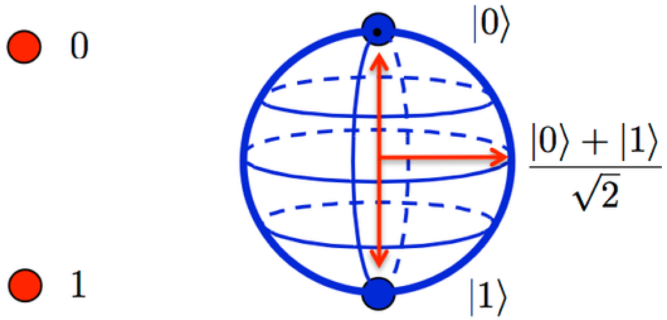
- Quantum Hall hybrid systems
- Straintronics
- Topotronics

- 1-D Majorana devices
- Topological vs trivial
- Entropic signatures
- Magnetic insulator hybrids

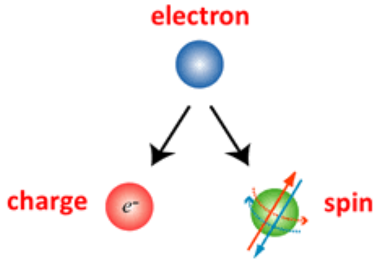
Why Topological Quantum Materials?

Beyond Moore: Quantum Computation

ISSUE: Qubit stability (Decoherence)

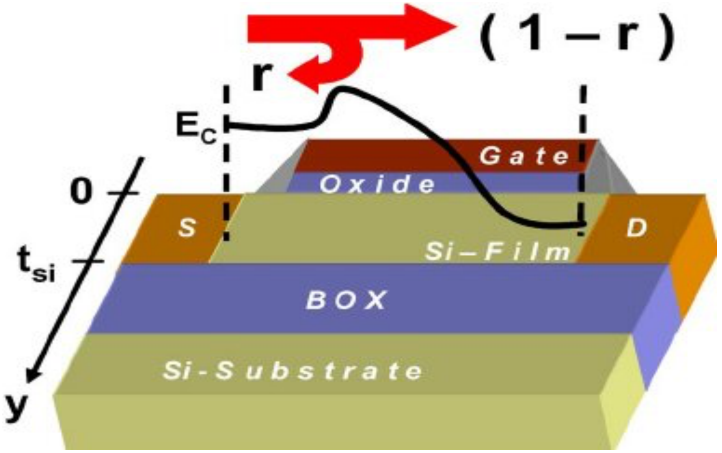


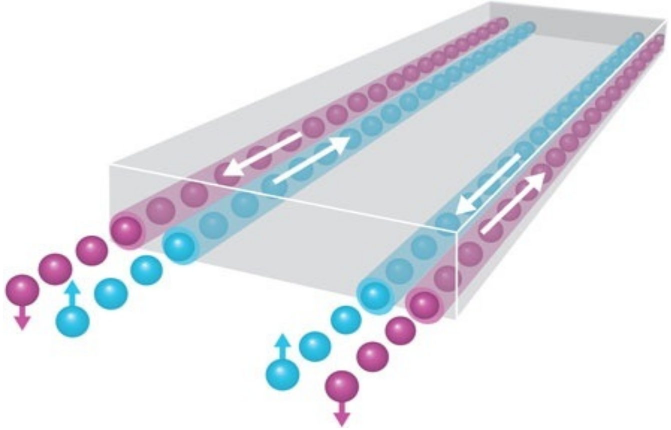
Classical Bit **Qubit**



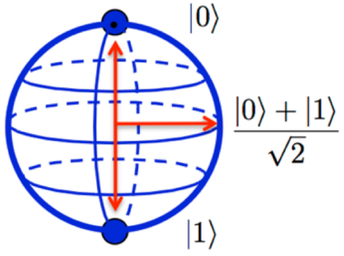
Beyond Moore: Binary and non-binary Logic

ISSUE: Power Dissipation

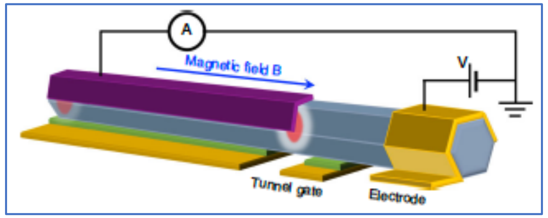


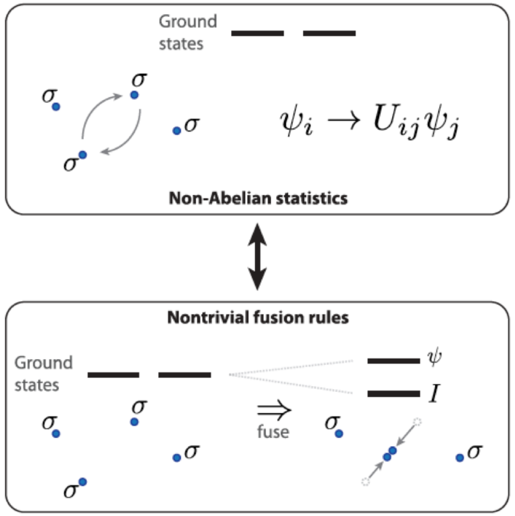


Prelude : Topological Quantum Computing



Classical Bit **Qubit**





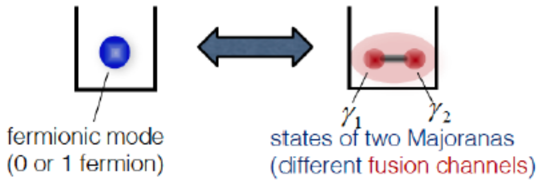
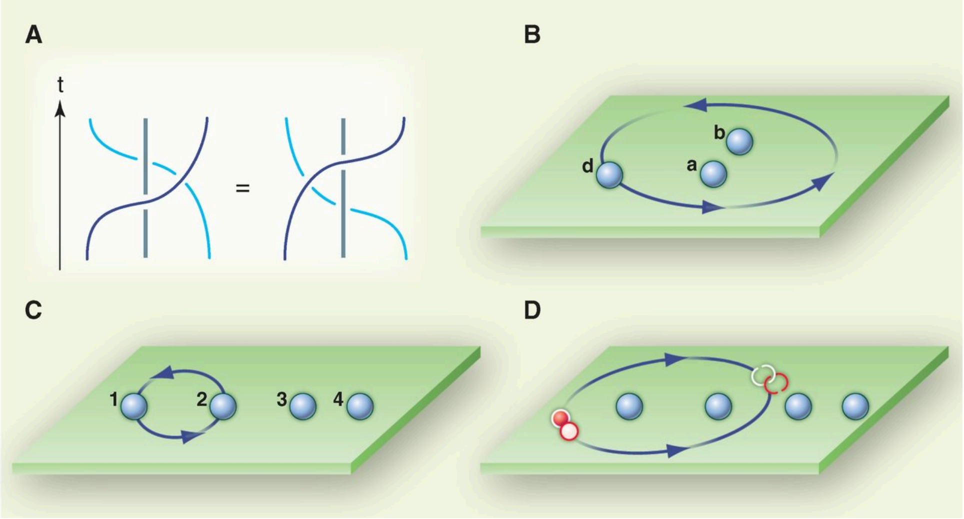
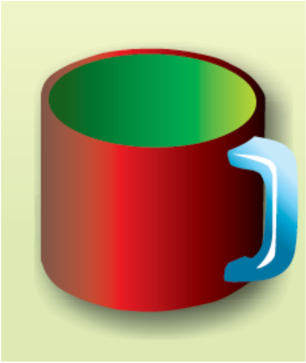
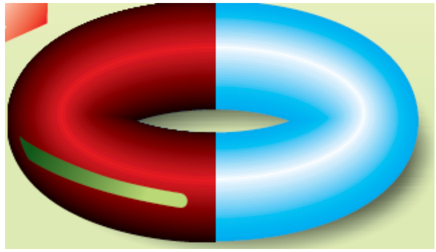


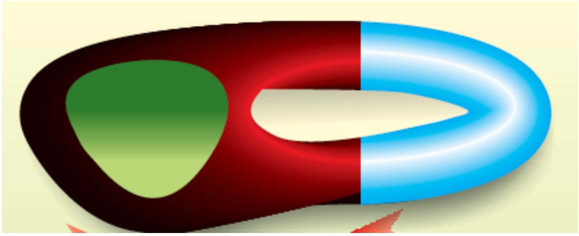
Table 4.1 Anyonic quantum computation

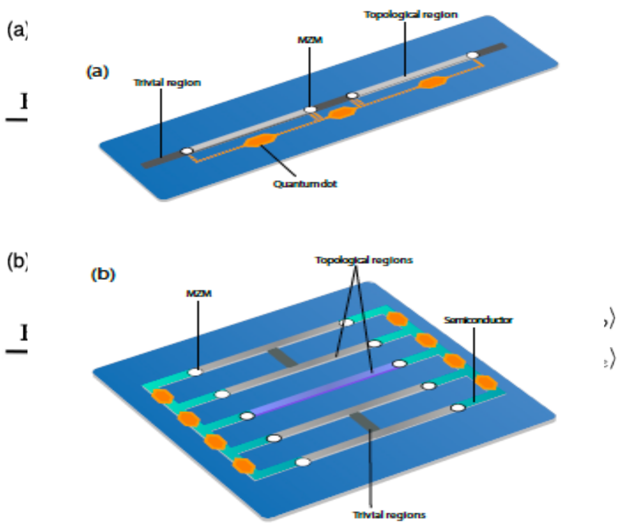
Quantum computation		Anyonic manipulation
State initialisation	→	Create and arrange anyons
Quantum gates	→	Braid anyons
State measurement	→	Detect anyonic charge





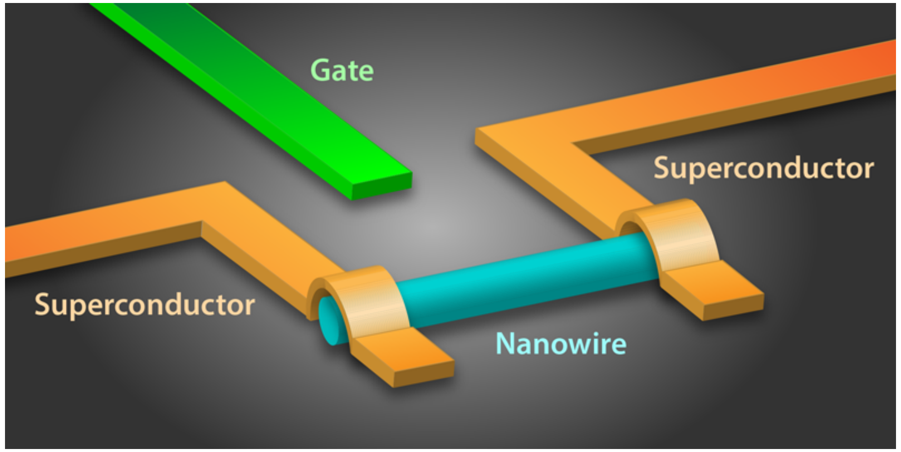


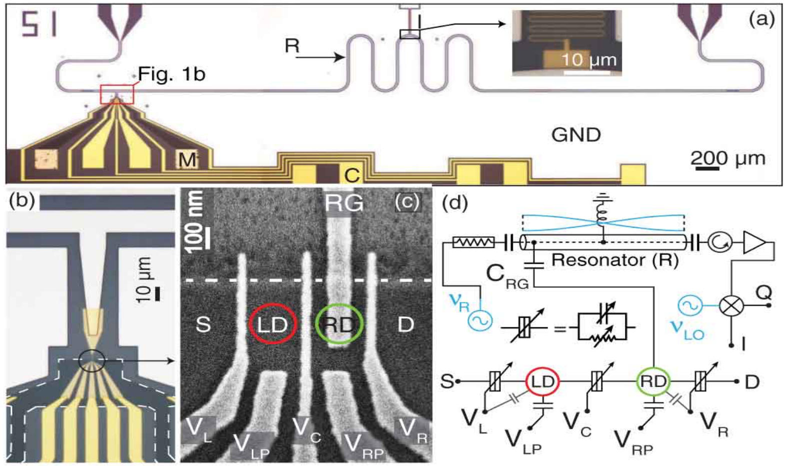




Current flow at the nanoscale

Hybrid Quantum Systems





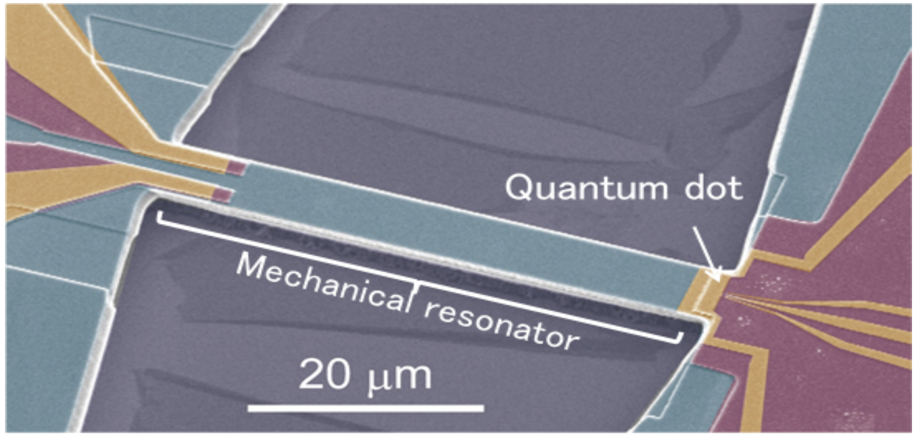
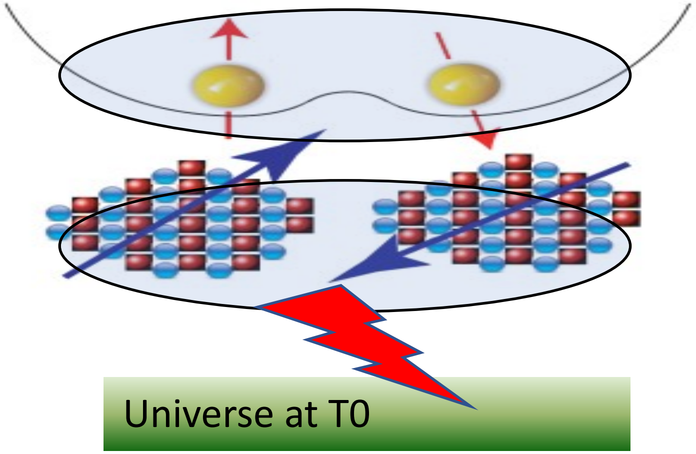
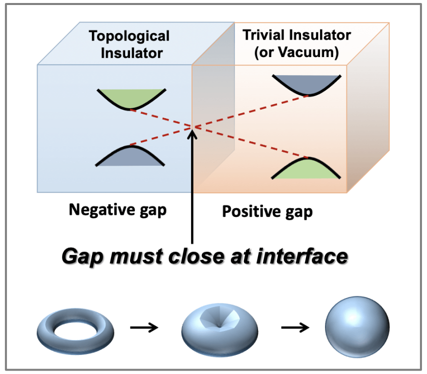
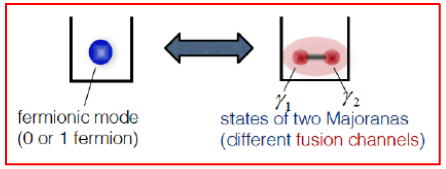


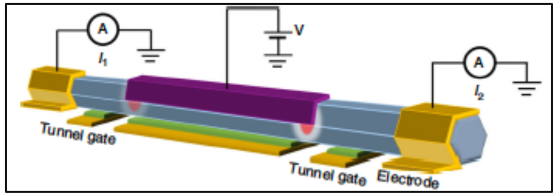
Fig: Fabricated mechanical resonator and quantum dot hybrid device. The resonator motion can be detected through the change in the conductance of quantum dot.

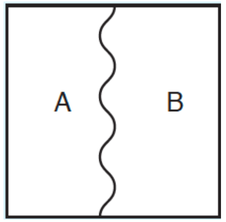


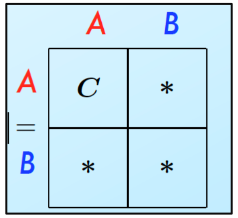
What is this tutorial all about??

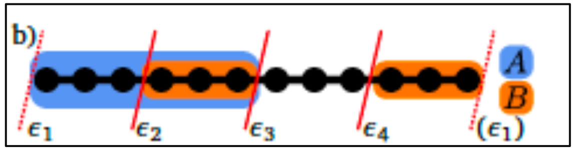








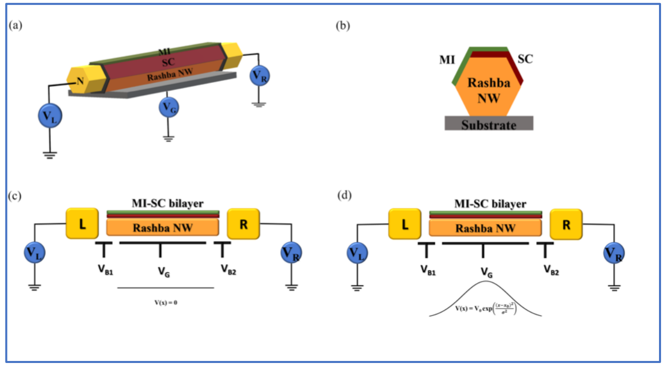


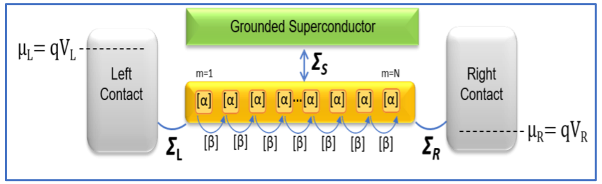


Topology and gap closure
Connection to Conductance

Transport spectroscopy
Detection of MZMs
Quantum transport
Gap closing and false postives

Quantum Materials
Entanglement Entropy

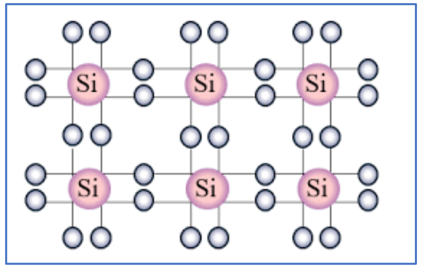


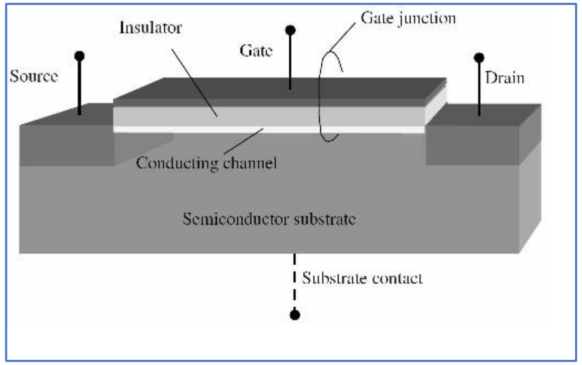


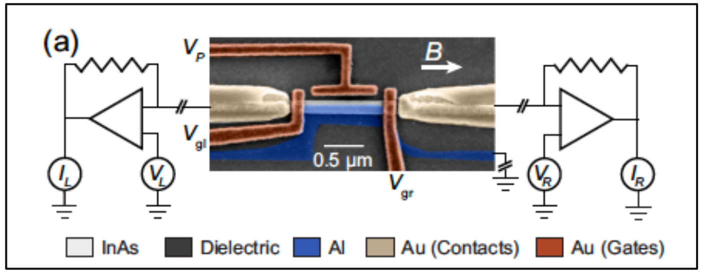
Advanced Device Modeling
Rashba nanowire systems
Dephasing
Magnetic Insulator hybrids

JPCM, 33, 365301, (2021)
PRB, 103, 165432, (2021)
PRB (L), 105,161403, (2022)
ArXiv: 2203.08413 (2022)

Moving on…







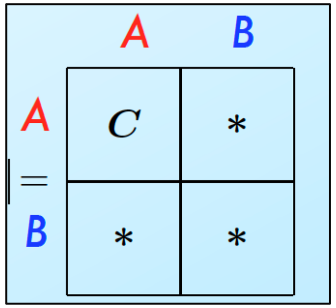
The un-avoidable crossing

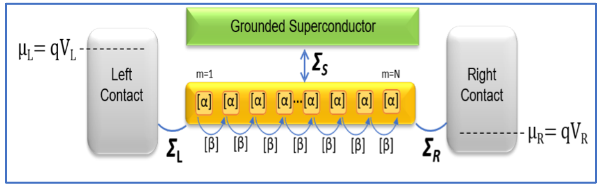
Part I : Materials to devices FAQ

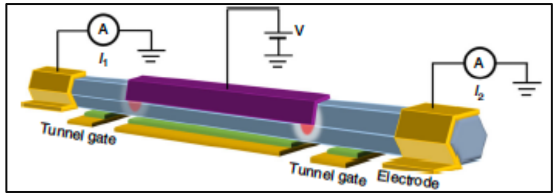
- Materials and structures – quantum effects
- Quantum transport basics

Part II : Transport Spectroscopy

- Andreev bound states
- Detecting Majoranas via quantization
- Issues with conductance spectroscopy





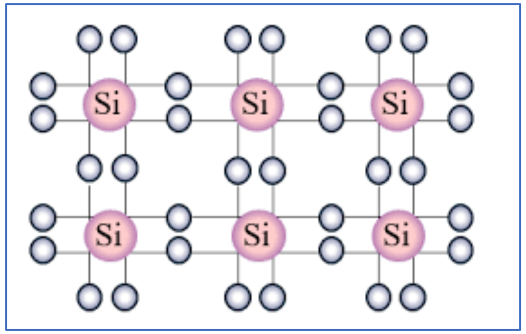


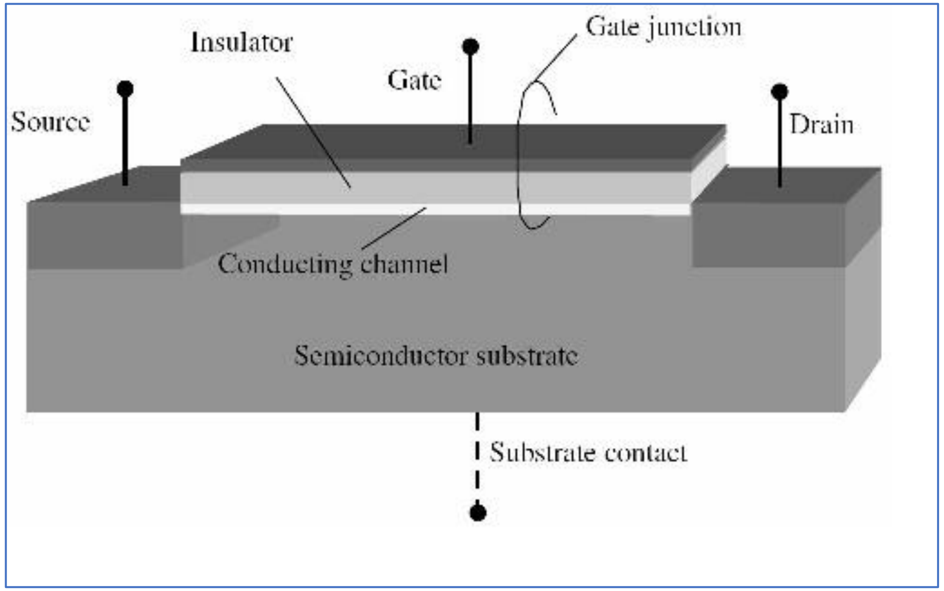
Part III : Beyond transport spectroscopy

- Topological states and von Neumann entropy
- How Majorana bound states can be identified
- Outlook and device modeling

Part I : Materials to devices FAQ

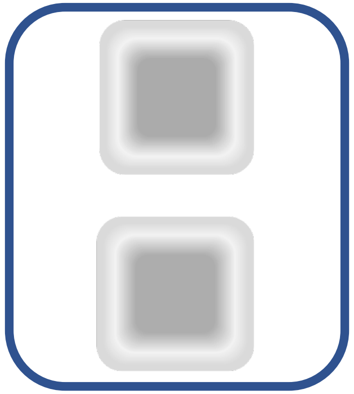
- Materials and structures – quantum effects
- What is a topological quantum material?
- Detection of such states – conductance quantum



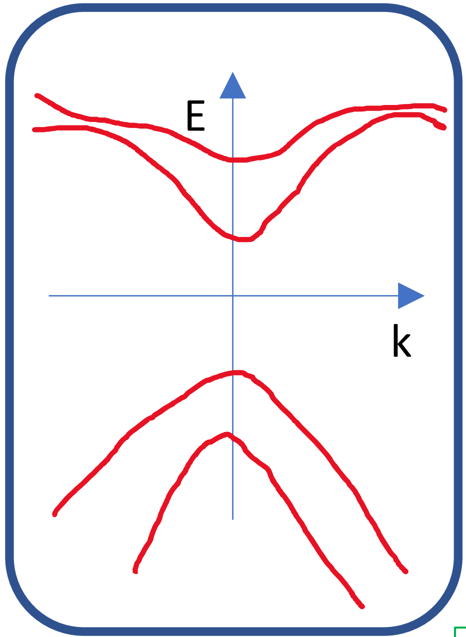


Materials to Devices:: FAQs

Energy Bands



Dispersion Relation
Bandstructure



Fill in the blanks here

$$m^* = \frac{1}{\hbar^2}\frac{\partial^2 E}{\partial k^2}$$

$$\sigma = \frac{ne^2\tau}{m^*}$$

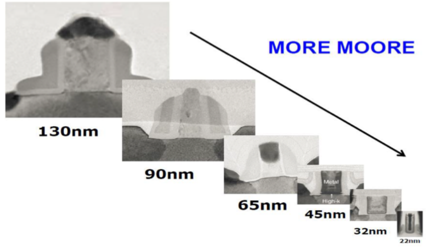
Apply
Newtons laws
F=m*dv/dt

Calculate::
1) Mobility
2) Conductivity
3) Currents

“quantum” ends here !

Materials to Devices:: Regimes

How do the quantum aspects just disappear??
Size/Dimension does Matter!

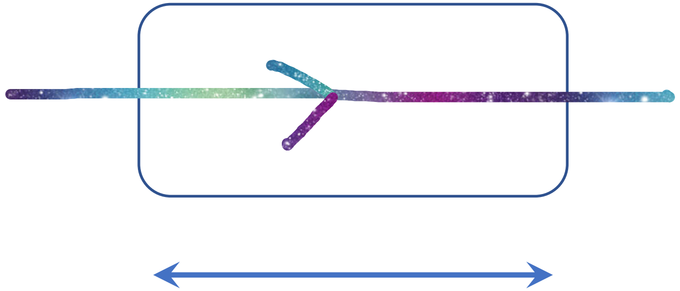


Diffusive Limit

$L \gg \lambda, \lambda_\varphi$

Standard device physics

Ballistic Limit

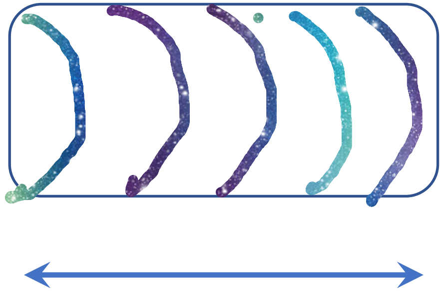


L<<λ

Conductance quantization

Ohm's law NOT VALID !

Quantum Limit

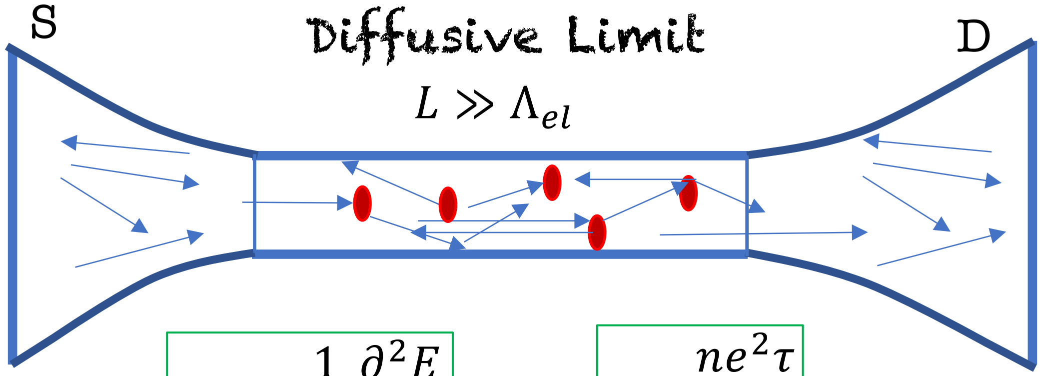


L<<λ, λ_φ

Quantum effects seem over device length scales

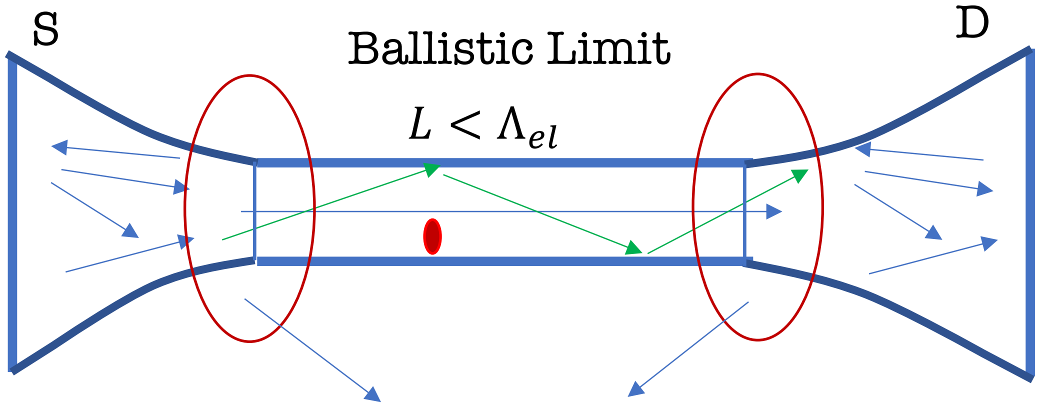
Conductance fluctuations!

What makes a device?



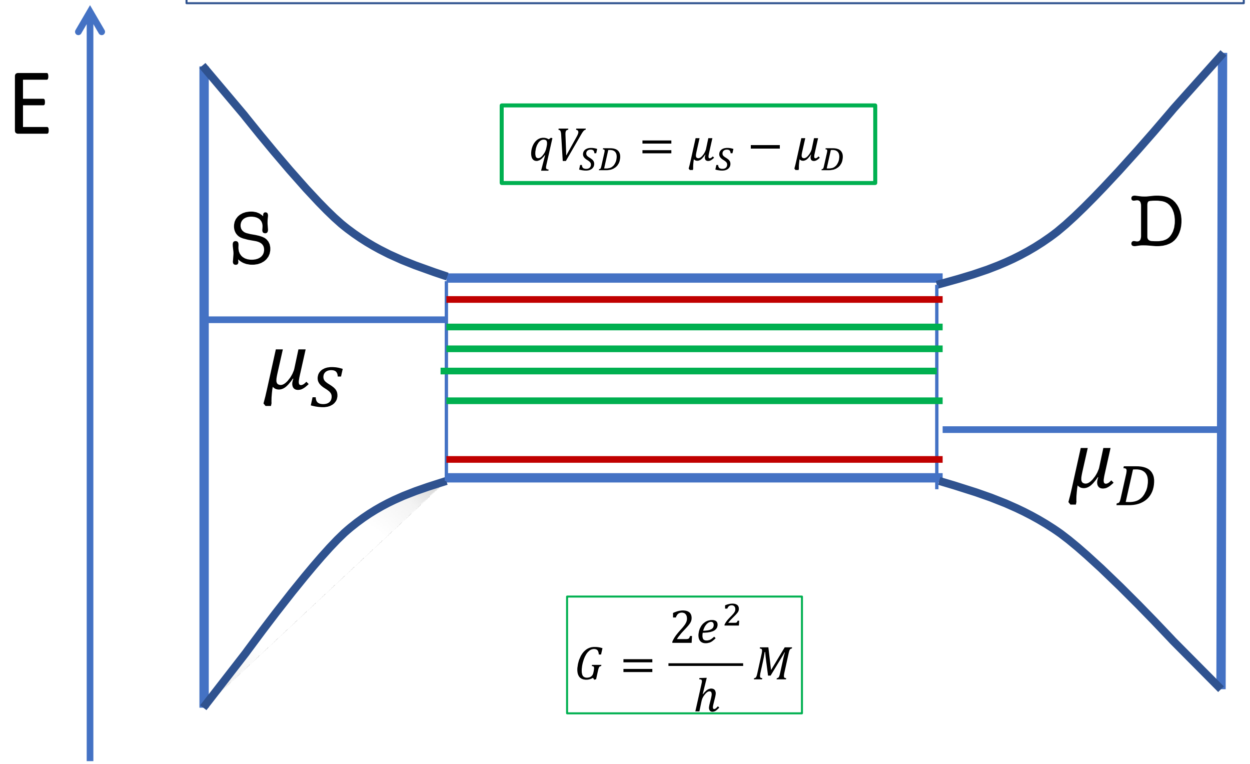
$$m^* = \frac{1}{\hbar^2}\frac{\partial^2 E}{\partial k^2}$$

$$\sigma = \frac{ne^2\tau}{m^*}$$



$$G = \frac{2e^2}{h}M$$

In the mode-space picture- Energy freeways



$$qV_{SD} = \mu_S - \mu_D$$

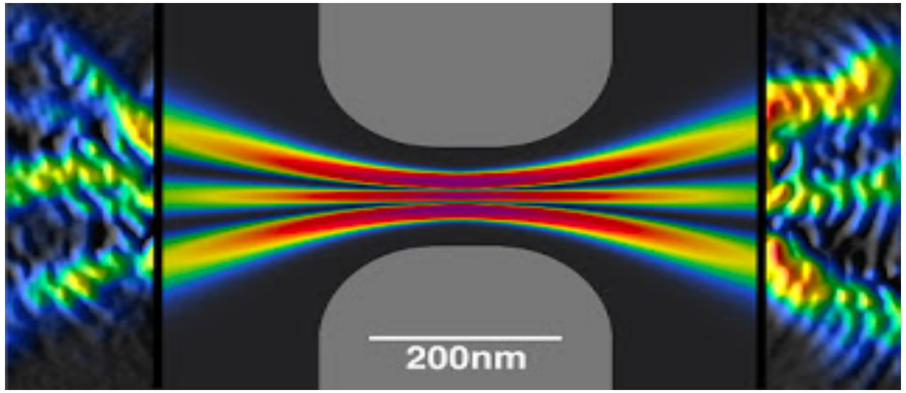
$$G = \frac{2e^2}{h}M$$

$$I = \frac{e}{h}\int_{-\infty}^{\infty} dE\,T(E)[f(E-\mu_S) - f(E-\mu_D)]$$

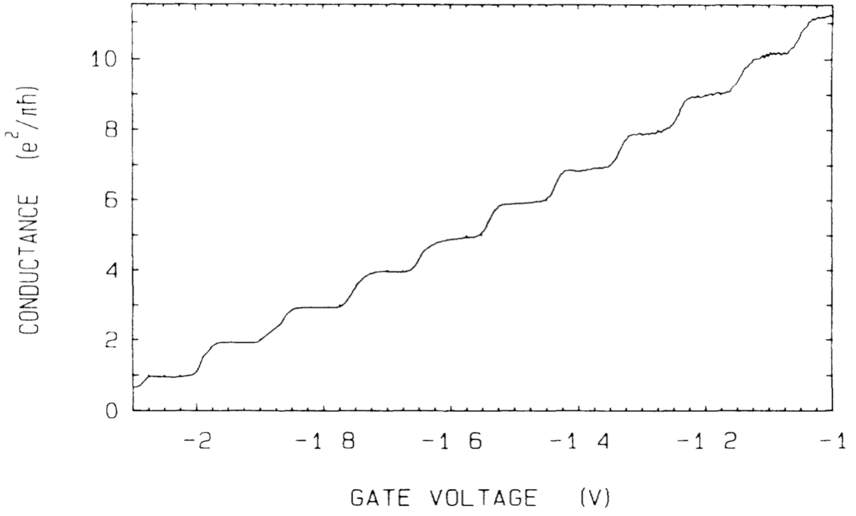
Quantized conductance at the interface!

Conductance quantization.. Standard confinement effect…and more?

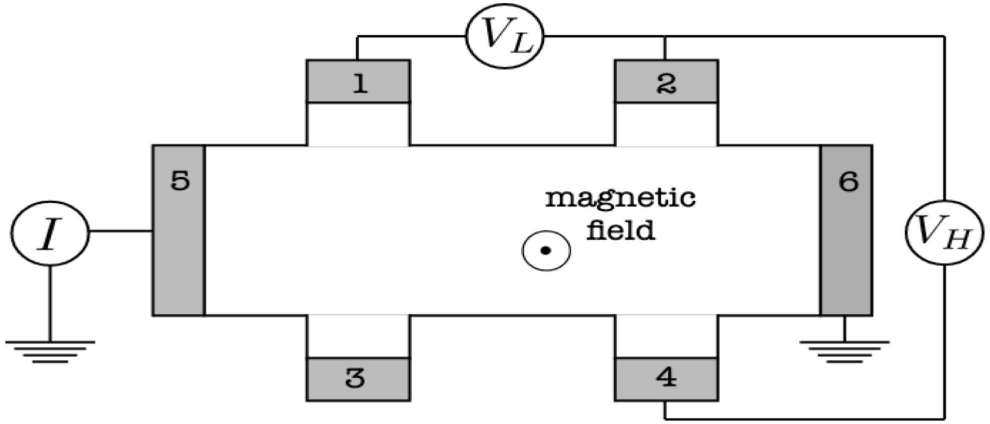
Standard confinement effect
Dies out due to impurities/defects etc.,



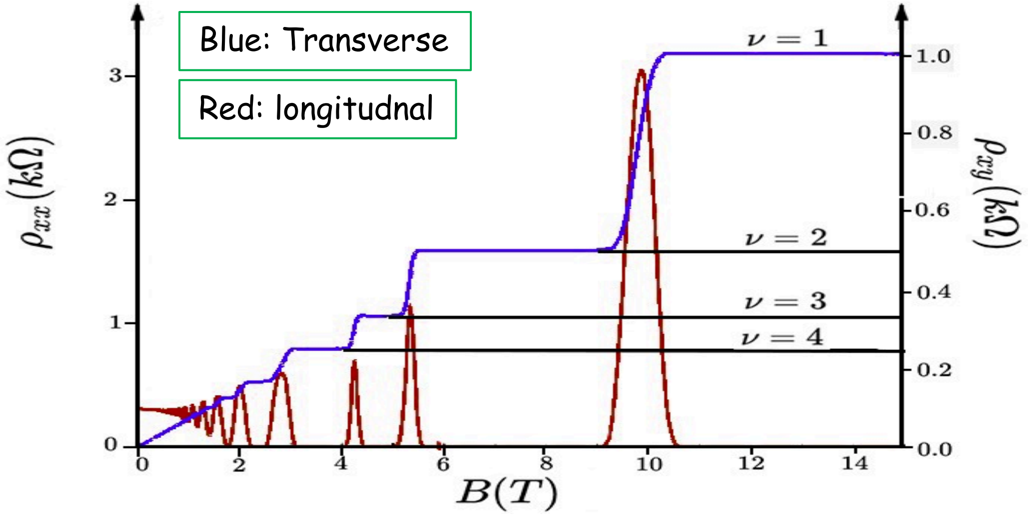
QPC: 1988 – Longitundnal conductance quantization
Confinement effect: Short channel + Clean channel



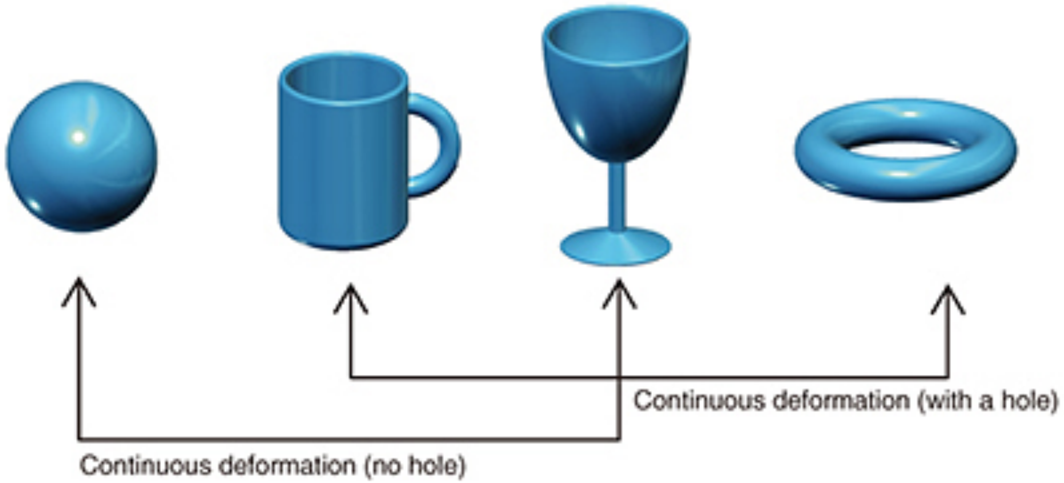
QH edge states are protected
despite impurities/defects etc.,

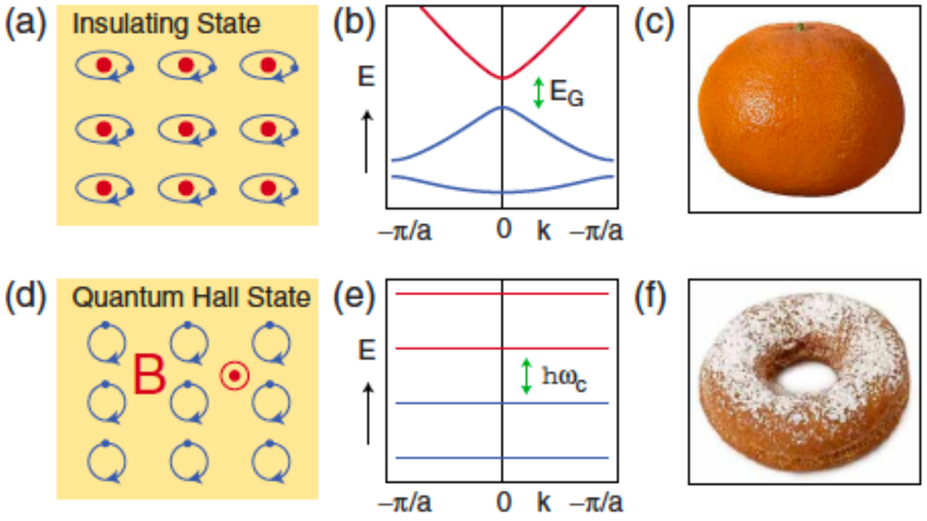


QHE: 1980 – Hall conductance quantization
Conduction via “edge states”: MOSFET – 2DEG



Topology- a way to classify - Quantum robustness





The Nobel Prize in Physics 2016

Ill: N. Elmehed. © Nobel Media 2016
David J. Thouless
Prize share: 1/2

Ill: N. Elmehed. © Nobel Media 2016
F. Duncan M. Haldane
Prize share: 1/4

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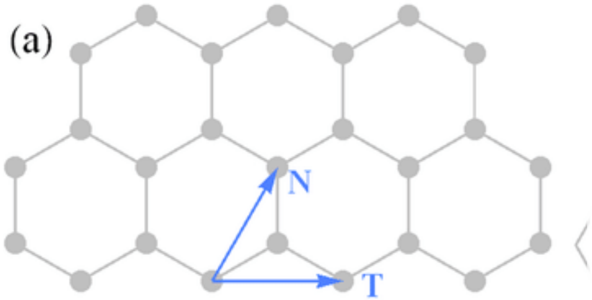
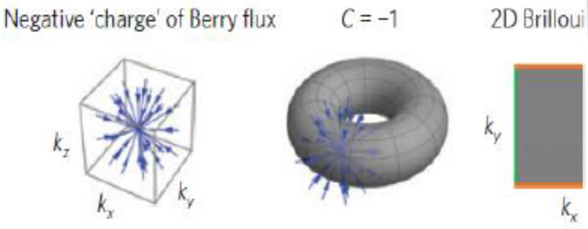
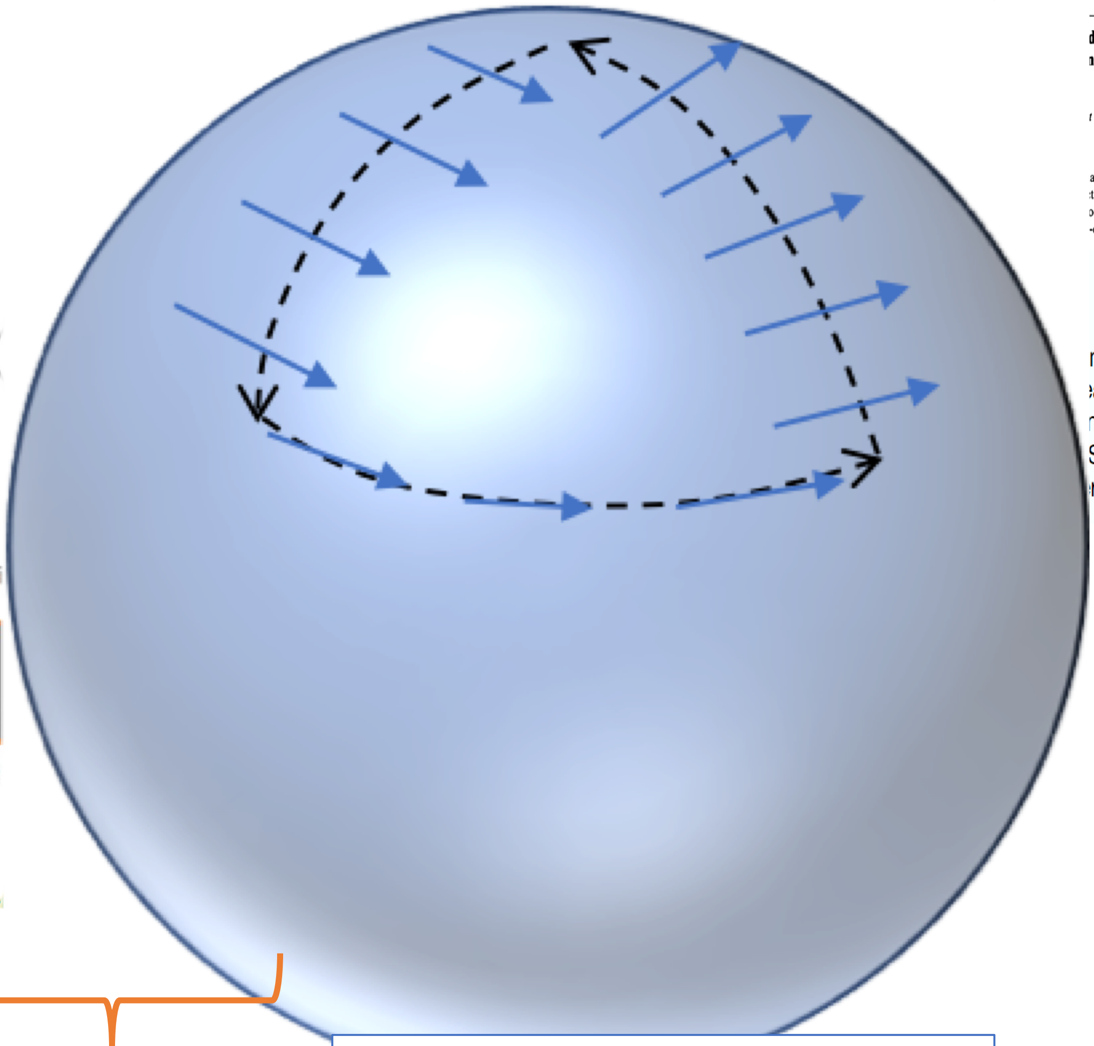
J. Michael Kosterlitz
Prize share: 1/4

Quantum Topology Connection



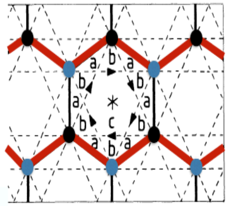




$$n = \frac{1}{2\pi i}\int_{BZ} d^2\mathbf{k}\cdot\langle\nabla_{\mathbf{k}} u$$

Berry Curvature

$$\sigma_{xy} = \frac{e^2}{\hbar}\int_{BZ}\frac{d\mathbf{k}}{(2\pi)^2}\Omega_{k_x,k_y} = n\frac{e^2}{h},$$



VOLUME 61, NUMBER 18 PHYSICAL REVIEW LETTERS 31 OCTOBER 1988

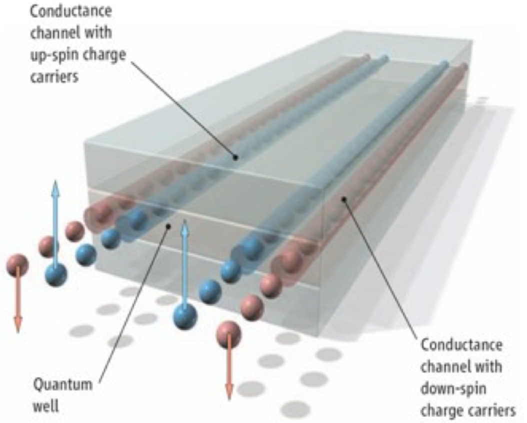
del for a Quantum Hall Effect without Landau Levels: ndensed-Matter Realization of the "Parity Anomaly"

F. D. M. Haldane
t of Physics, University of California, San Diego, La Jolla, California 92093

(Received 16 September 1987)

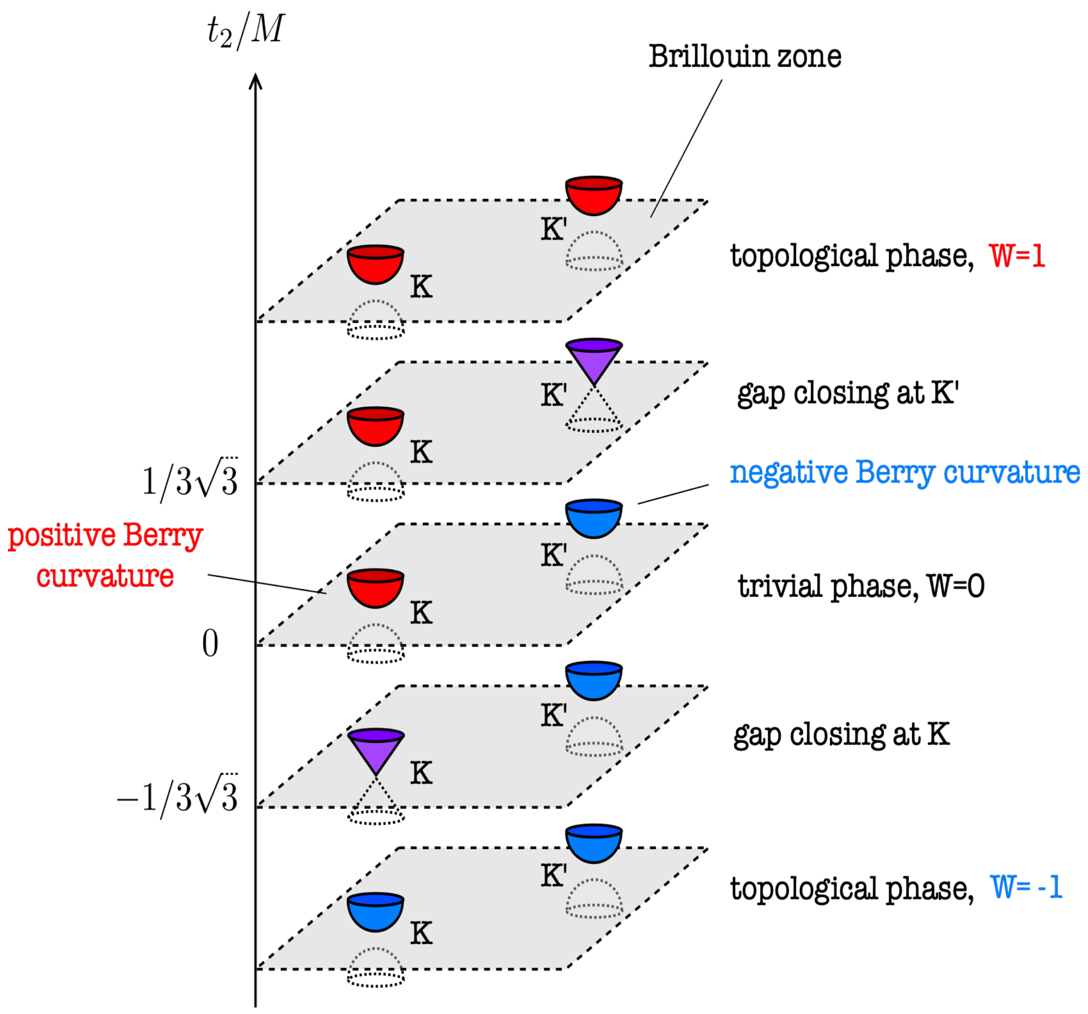
al condensed-matter lattice model is presented which exhibits a nonzero quantization tance σ^{xy} in the *absence* of an external magnetic field. Massless fermions *without* occur at critical values of the model parameters, and exhibit the so-called "parity -dimensional field theories.

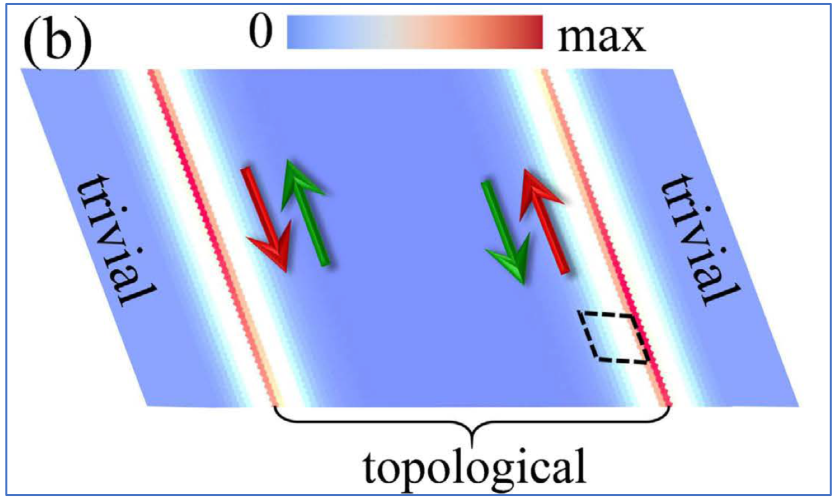
metal, inversion symmetry + time-reversal symmetry eate a degeneracy at isolated points in the Brillouin n top of valence band and bottom of conduction band S opens a gap, resulting in a 'Chern insulator' with σ_{xy} ro magnetic field.

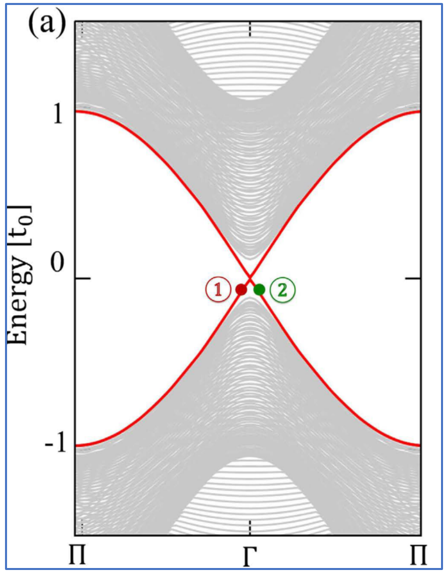


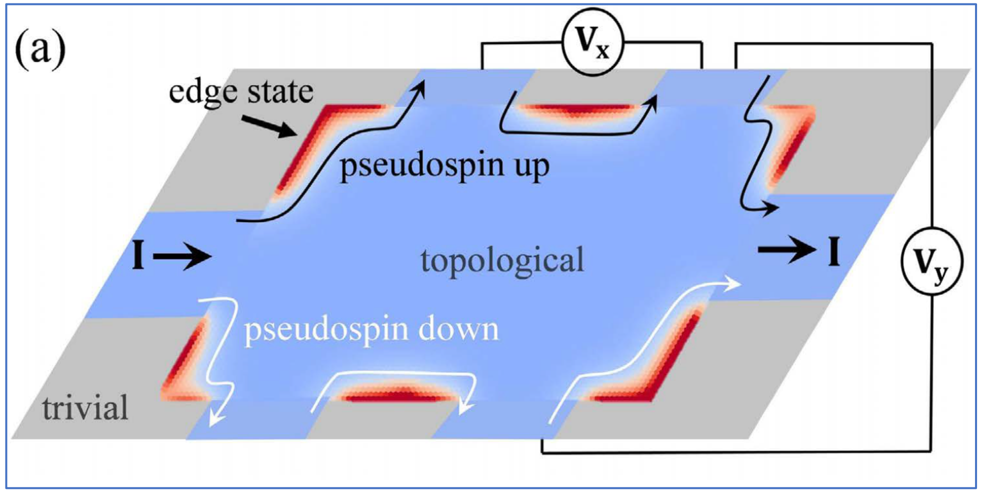
Schematic of the spin-polarized edge channels in a quantum spin Hall insulator.

Phase transition and bulk boundary correspondence







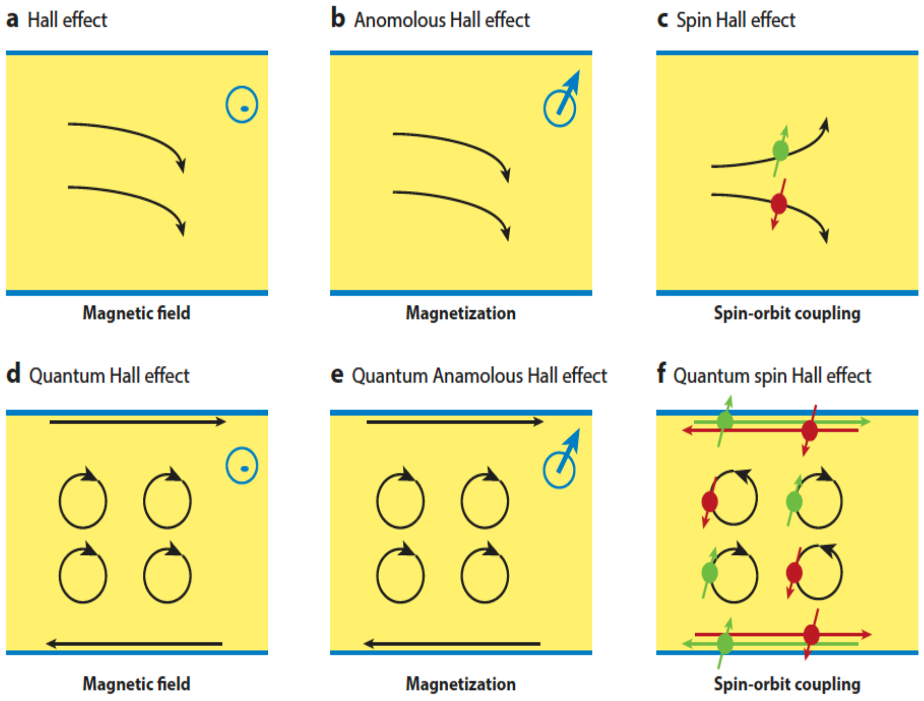


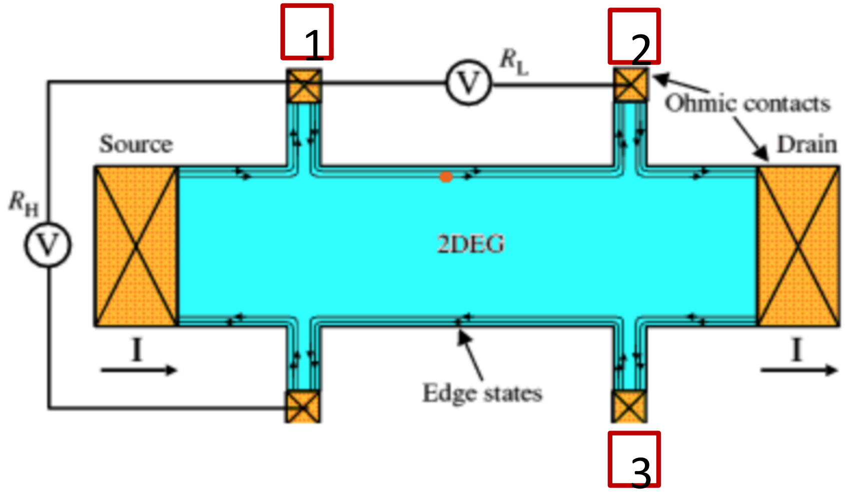
https://topocondmat.org

Sci.Rep, 6, 24347 (2016)

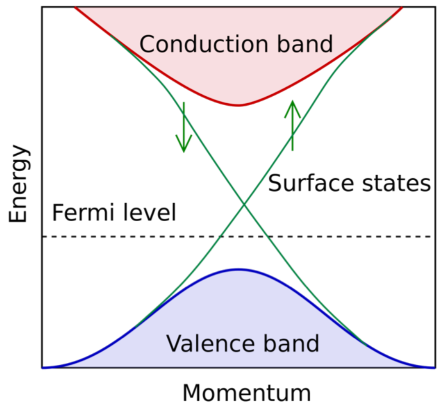
Quantum Effects "visible" at Macroscale!

Family of Quantum Hall Effects :: Starting point of topological stability

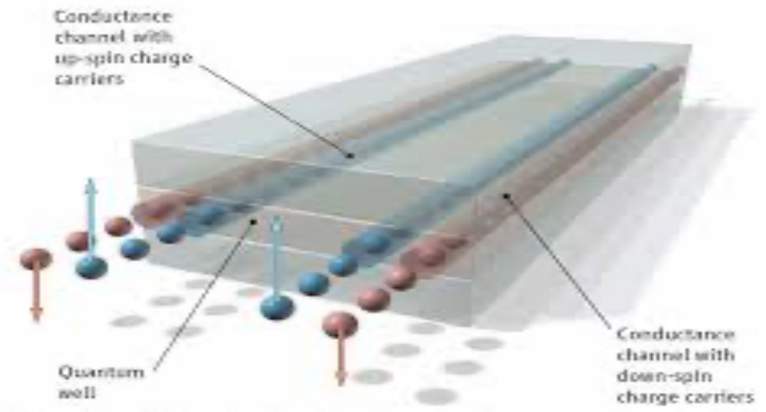




Recurring theme in topological quantum materials

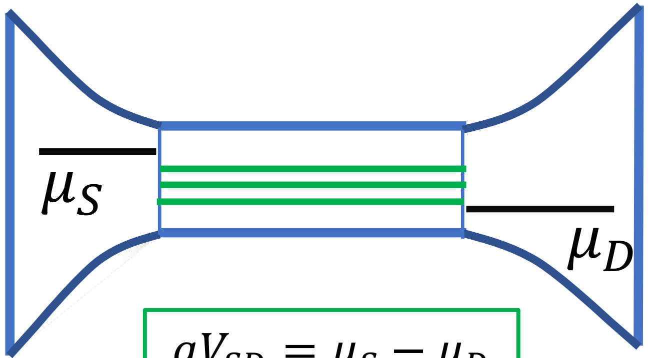


$$\sigma_{xy} = \frac{e^2}{\hbar}\int_{BZ}\frac{d\mathbf{k}}{(2\pi)^2}\Omega_{k_x,k_y} = n\frac{e^2}{h},$$



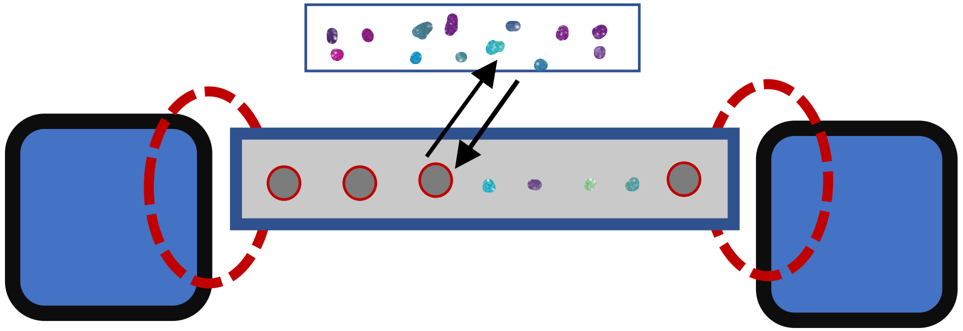
Schematic of the spin-polarized edge channels in a quantum spin Hall insulator.

Formalizing Quantum Lanes



$$qV_{SD} = \mu_S - \mu_D$$

$$I = \frac{e}{h}\int_{-\infty}^{\infty} dE\,T(E)[f(E-\mu_S) - f(E-\mu_D)]$$



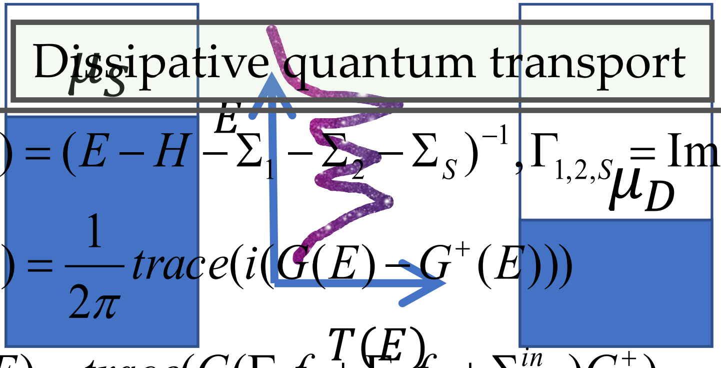
Channel-> Atomistic Hamiltonian

$$\left(i\hbar\frac{\partial}{\partial t} - H\right)\Psi=0$$

$$H\psi = E\psi$$

$$E\{\psi\} = [H+\Sigma]\{\psi\} + \{S\}$$

Dissipative quantum transport



$$G(E) = (E - H - \Sigma_1 - \Sigma_2 - \Sigma_S)^{-1}, \Gamma_{1,2,S} = \mathrm{Im}(\Sigma_{1,2,S})$$

$$D(E) = \frac{1}{2\pi} trace(i(G(E) - G^+(E)))$$

$$G^n(E) = trace(G(\Gamma_1 f_1 + \Gamma_2 f_2 + \Sigma^{in}{}_S)G^+)$$

$$I(E) = trace[\Sigma^{in} A] - trace[\Gamma G^n]$$

Ballistic quantum transport

$$G(E) = (E - H - \Sigma_1 - \Sigma_2)^{-1}, \Gamma_{1,2} = \mathrm{Im}(\Sigma_{1,2})$$

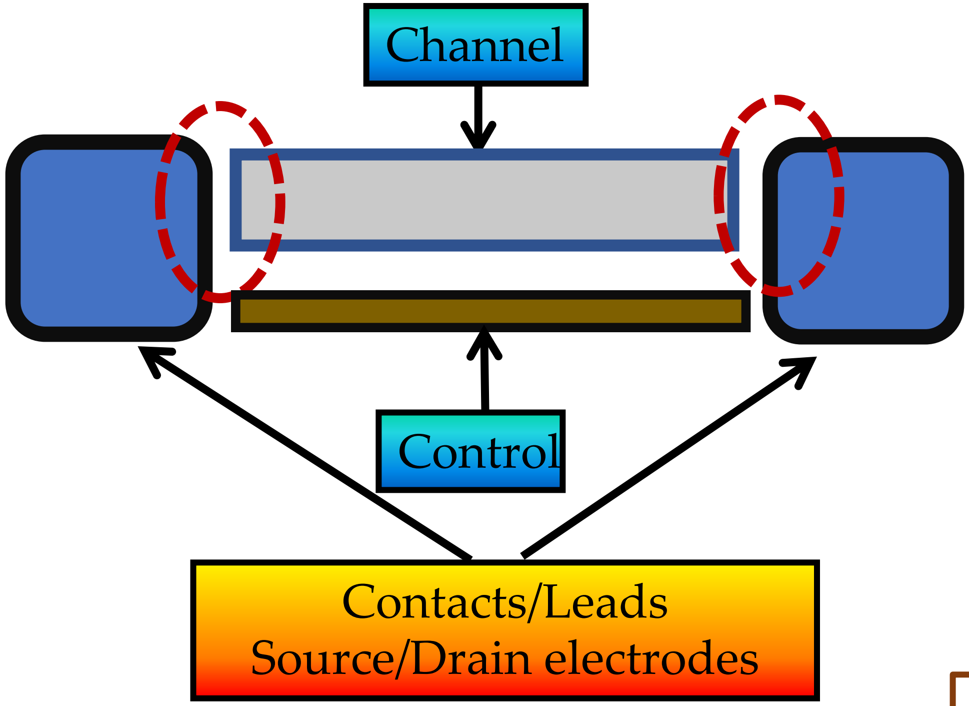
$$D(E) = \frac{1}{2\pi} trace(i(G(E) - G^+(E)))$$

$$\Gamma_{1,2}(E) = i(\Sigma_{1,2} - \Sigma^+{}_{1,2})$$

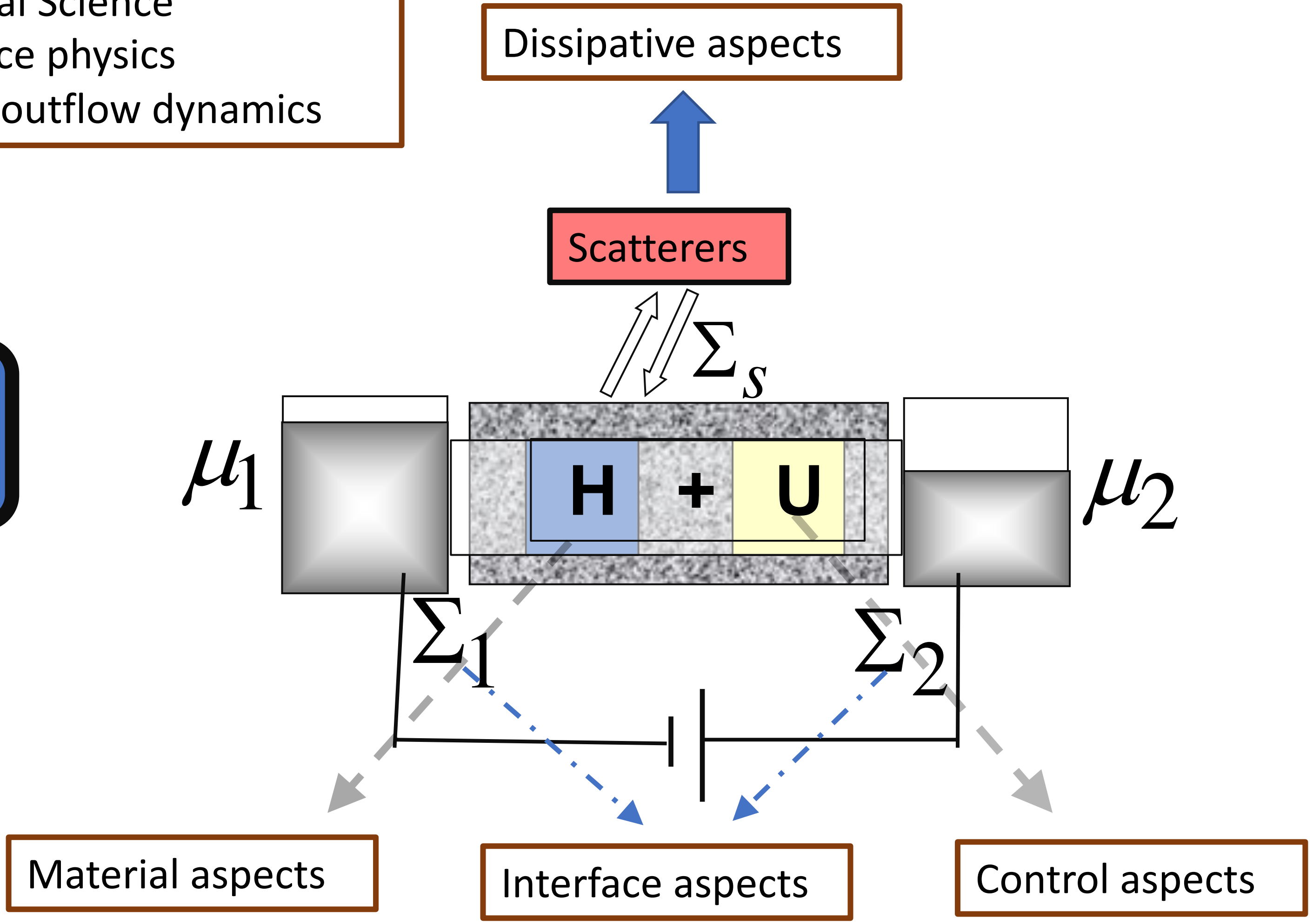
$$G^n(E) = trace(G(\Gamma_1 f_1 + \Gamma_2 f_2)G^+)$$

$$I(E) = \underbrace{trace(\Gamma_1 G \Gamma_2 G^+)}_{T(E)}(f_1(E) - f_2(E))$$

Anatomy of a Nano-Device: Non-equilibrium Green's function formalism



Material Science
Interface physics
Inflow-outflow dynamics



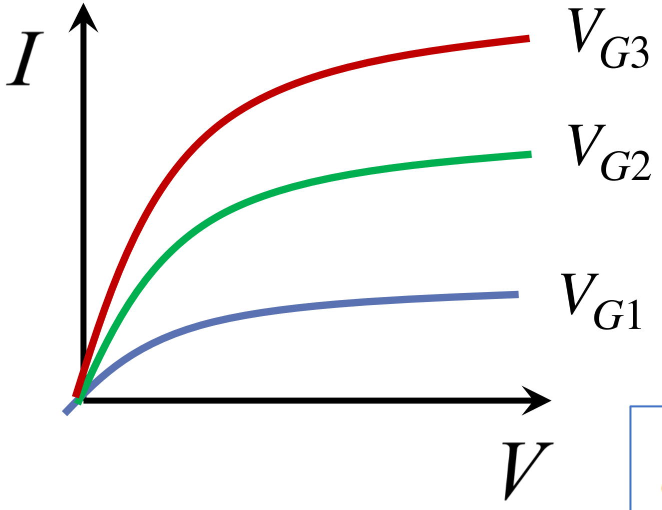
Datta, Quantum Transport, Cambridge Univ Press (2005)

Device Modeling Basics

$$I = \int_{-\infty}^{\infty} dE\,\Xi(E)\big(f_S(E) - f_D(E)\big)$$

Transport Function — Driving

$$qV_{DS} = \mu_S - \mu_D$$



$$I = GV$$

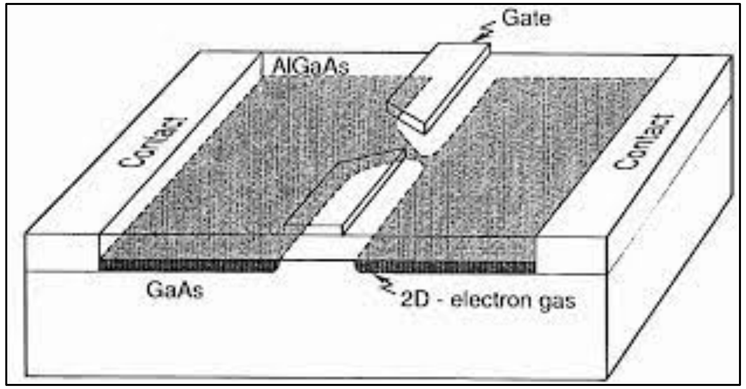
Linear Response

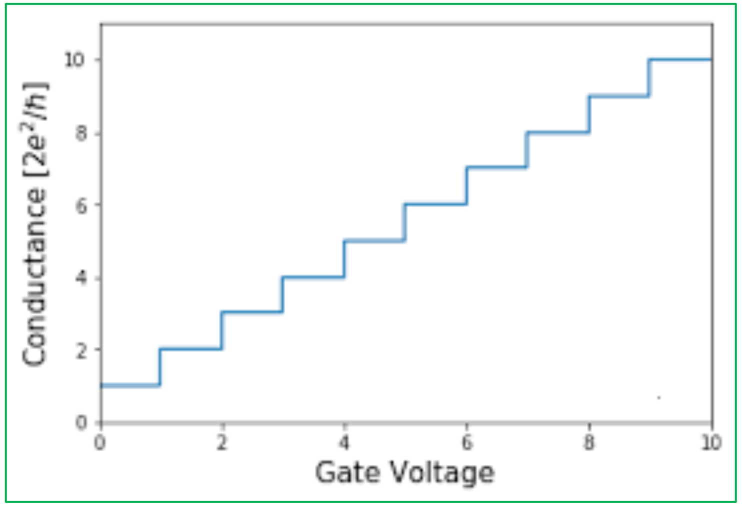
$$G(V_{DS}) = \left.\frac{\partial I}{\partial V}\right|_{V=V_{DS}}$$

$$G(V=0) = \left.\frac{\partial I}{\partial V}\right|_{V=0} = \frac{I}{V}$$

$$G = \frac{2e^2}{h} M$$

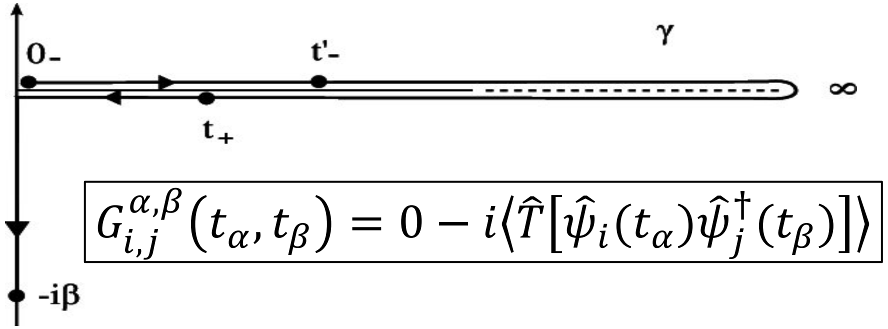
$$\sigma_{xy} = \frac{e^2}{\hbar} \int_{BZ} \frac{d\mathbf{k}}{(2\pi)^2} \Omega_{k_x,k_y} = n\frac{e^2}{h},$$



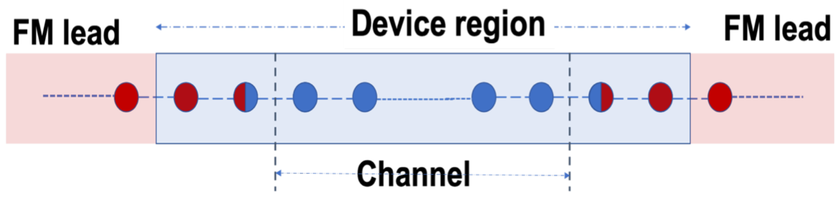


When we have a topological state

Keldysh Non-Equilibrium Green's Function



$$G_{i,j}^{\alpha,\beta}(t_\alpha, t_\beta) = 0 - i\langle \hat{T}[\hat{\psi}_i(t_\alpha)\hat{\psi}_j^\dagger(t_\beta)]\rangle$$



$$\hat{\psi}_i = \begin{pmatrix} \hat{c}_{i\uparrow} \\ \hat{c}_{i\downarrow} \end{pmatrix}$$ → Spinor

$$G_{i,j}^{+,-}(t,t') = i \begin{pmatrix} \langle \hat{c}_{j\uparrow}^\dagger(t')c_{i\uparrow}(t)\rangle & \langle \hat{c}_{j\uparrow}^\dagger(t')c_{i\downarrow}(t)\rangle \\ \langle \hat{c}_{j\downarrow}^\dagger(t')c_{i\uparrow}(t)\rangle & \langle \hat{c}_{j\downarrow}^\dagger(t')c_{i\downarrow}(t)\rangle \end{pmatrix}$$

$$G_{ij}^{+,-}(t,t') = G_{ij}^{+,-}(t-t')$$

$$G_{ij}^{+,-}(t-t') \xrightarrow{\mathcal{F}} G_{ij}^{+,-}(E)$$

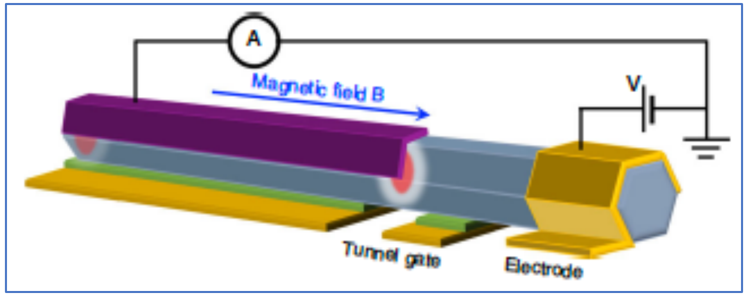
$$G^r(E) = (E\mathbb{I} + i\eta - \mathcal{H} - \Sigma_1 - \Sigma_2)^{-1}$$

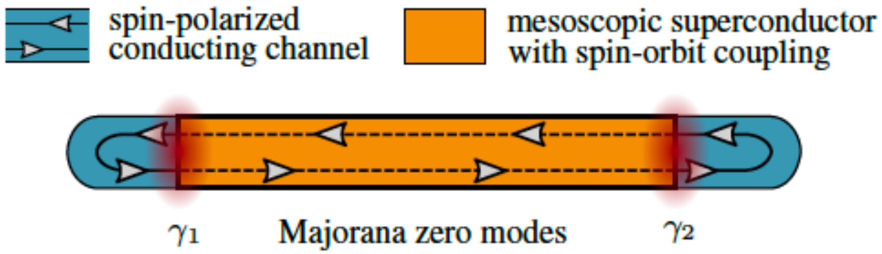
$$I_{op} = \frac{ie}{h}(\mathcal{H}G^n - G^n\mathcal{H})$$

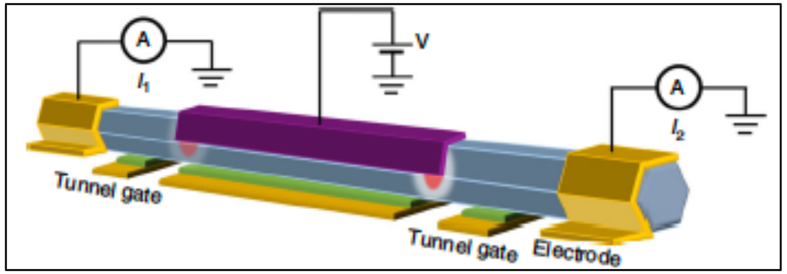
$$G^{+,-} = iG^n$$

Majorana modes and their detection

The "retro" reflection







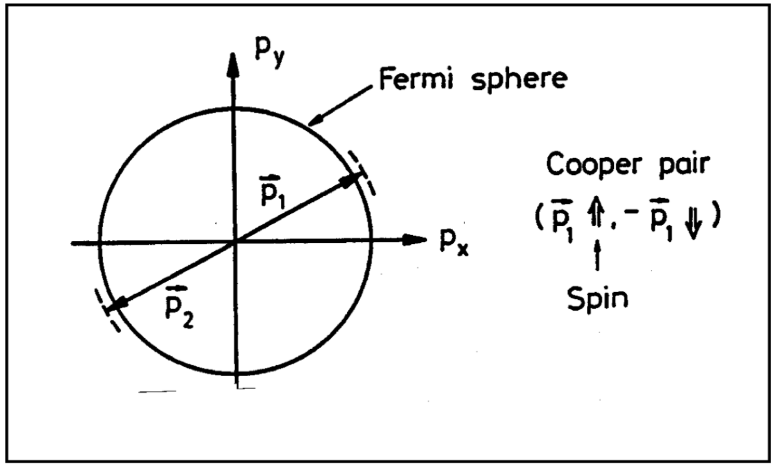
Part II : Transport Spectroscopy

- Andreev bound states
- Detecting Majoranas via quantization
- Issues with conductance spectroscopy

Supercurrent operator

$$I_L^{op}(E) = \frac{1}{2}\left[\frac{e}{h} Tr\left(\tau_z\left[[G^r][\Sigma_L^<] - [\Sigma_L^<][G^a] + [G^<][\Sigma_L^a] - [\Sigma_L^r][G^<]\right]\right)\right]$$

Crash course on superconductor systems



Generic Hamiltonian with e-e interactions

$$H_{BCS} = \sum_{k\sigma} \epsilon_{k\sigma} c^\dagger_{k\sigma} c_{k\sigma} + \sum_{k,k'} \sum_{q,\sigma\sigma'} V(q) c^\dagger_{k+q,\sigma} c^\dagger_{k'-q,\sigma'} c_{k'\sigma'} c_{k\sigma}$$

We then get the BCS Hamiltonian after considering the Cooper pairs

$$H_{BCS} = \sum_{k\sigma} \epsilon_{k\sigma} c^\dagger_{k\sigma} c_{k\sigma} + \sum_{k,k'} V_{k,k'} c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} c_{-k'\downarrow} c_{k'\uparrow}$$

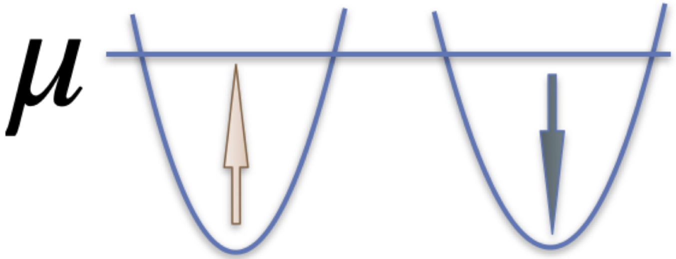
The basic idea of any mean field theory:: $c_{-k\downarrow} c_{k\uparrow} = \langle c_{-k\downarrow} c_{k\uparrow} \rangle + \delta_k$

$$H^{MF}_{BCS} = \sum_{k\sigma} \epsilon_k c^\dagger_{k\sigma} c_{k\sigma} + \sum_{k,k'} V_{kk'} \left[c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} \langle c_{-k'\downarrow} c_{k'\uparrow} \rangle + c_{-k'\downarrow} c_{k'\uparrow} \langle c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} \rangle - \langle c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} \rangle \langle c_{-k'\downarrow} c_{k'\uparrow} \rangle \right]$$

There are other terms: $\langle c^\dagger_{k\uparrow} c_{k\uparrow} \rangle$ are typically the Coulomb terms like Hartree-Fock etc., which will get absorbed into on-site terms

Crash course on superconductor systems

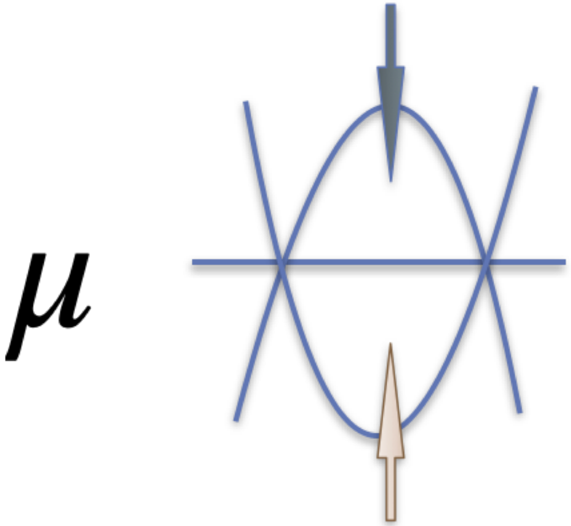
Electrons and holes in superconductors



$$H_{BCS}^{MF} = \sum_{k\sigma} \tilde{\epsilon}_k c_{k\sigma}^\dagger c_{k\sigma} + \sum_{k,k'} V_{kk'} \left[c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger \langle c_{-k'\downarrow} c_{k'\uparrow} \rangle + c_{-k'\downarrow} c_{k'\uparrow} \langle c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger \rangle - \langle c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger \rangle \langle c_{-k'\downarrow} c_{k'\uparrow} \rangle \right]$$

$$\Delta_k = -\sum_{k'} V_{kk'} \langle c_{-k'\downarrow} c_{k'\uparrow} \rangle$$

$$H_{BCS}^{MF} = \sum_{k\sigma} \tilde{\epsilon}_k c_{k\sigma}^\dagger c_{k\sigma} - \sum_k \left(\Delta_k c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger + \Delta_k^* c_{-k\uparrow} c_{k\downarrow} \right)$$



$$H_{BCS}^{MF} = \sum_k \left(\tilde{\epsilon}_k c_{k\uparrow}^\dagger c_{k\uparrow} - \tilde{\epsilon}_k c_{-k\downarrow} c_{-k\downarrow}^\dagger \right) - \sum_k \left(\Delta_k c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger + \Delta_k^* c_{-k\uparrow} c_{k\downarrow} \right)$$

$$H_{BCS}^{MF} = \sum_k \hat{\psi}_k^\dagger H_{BdG} \hat{\psi}_k$$

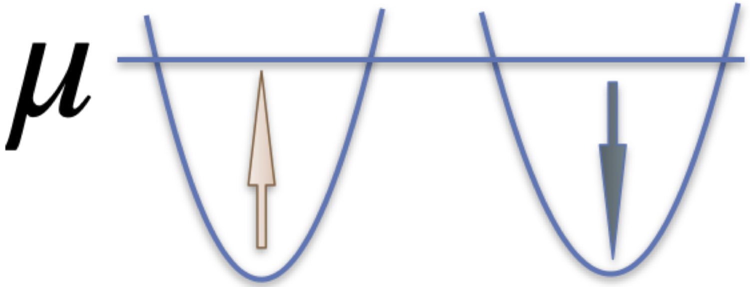
$$\hat{\psi}_k = \begin{pmatrix} c_{k\uparrow} \\ c_{-k\downarrow}^\dagger \end{pmatrix}$$

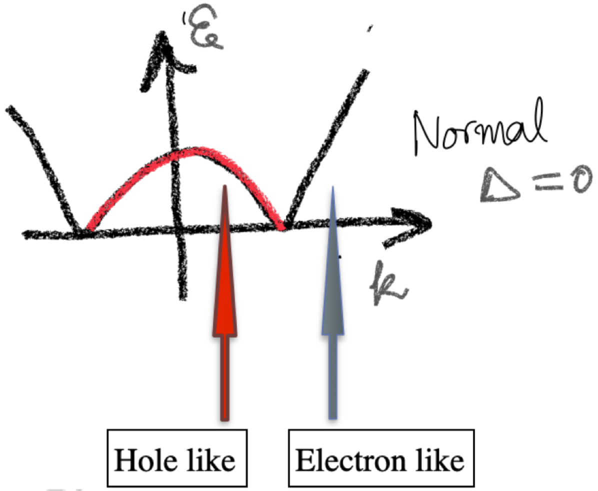
$$H_{BdG} = \begin{pmatrix} \epsilon_k - \mu & \Delta_k \\ \Delta_k^* & -(\epsilon_k - \mu) \end{pmatrix}$$

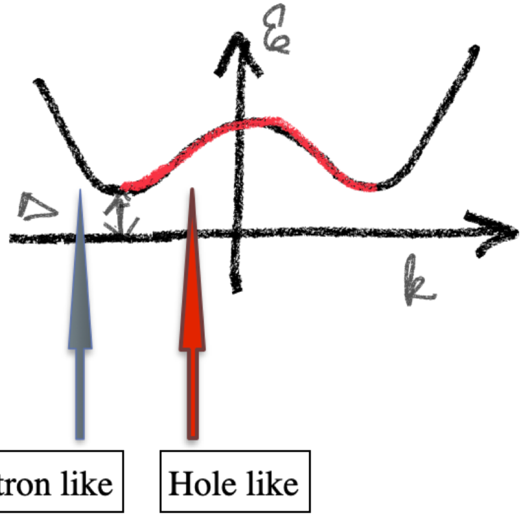
Nambu Spinor

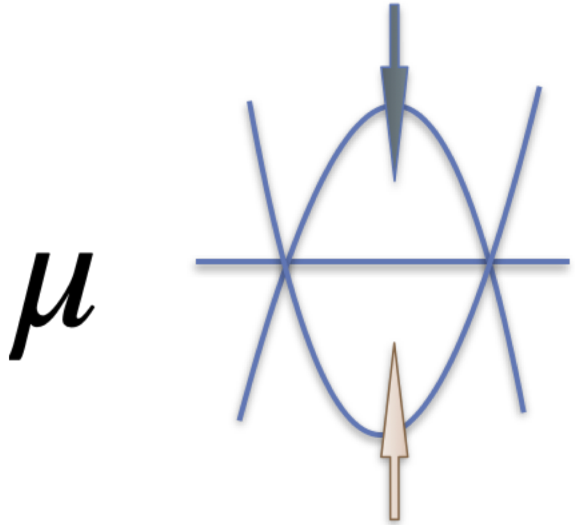
Superconducting hybrid systems

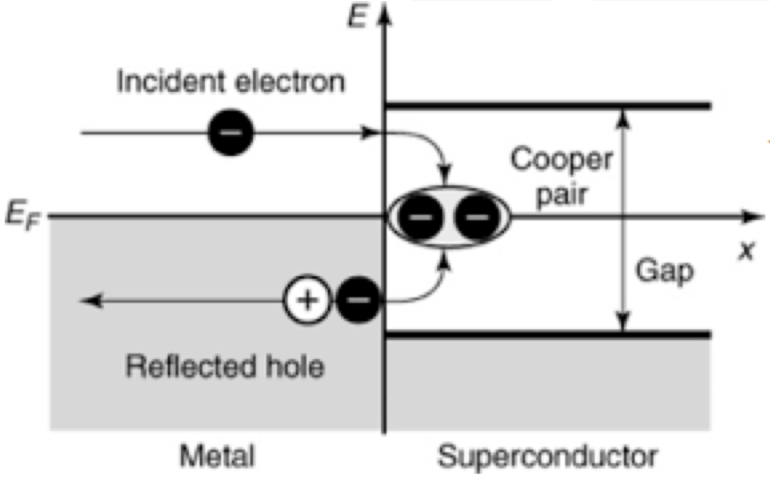
Electrons and holes in superconductors











$$H_{BdG} = \begin{pmatrix} \epsilon_k - \mu & \Delta_k \\ \Delta_k^* & -(\epsilon_k - \mu) \end{pmatrix}$$

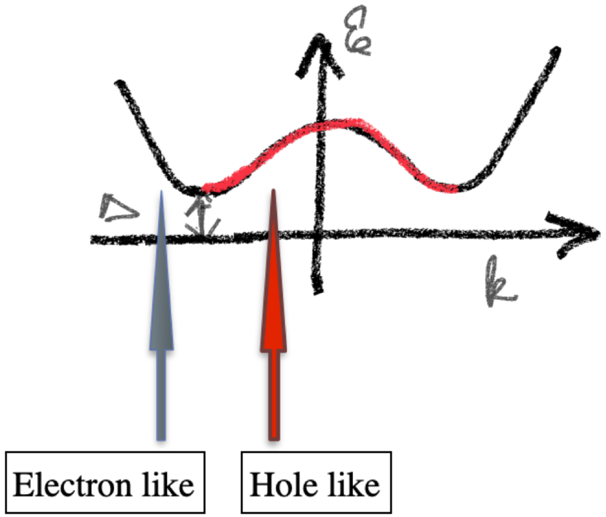
$$\hat{\psi}_k = \begin{pmatrix} c_{k\uparrow} \\ c_{-k\downarrow}^{\dagger} \end{pmatrix}$$

Nambu Spinor

$$I_L^{op}(E) = \frac{1}{2}\left[\frac{e}{h}Tr\left(\tau_z\left[[G^r][\Sigma_L^<] - [\Sigma_L^<][G^a] + [G^<][\Sigma_L^a] - [\Sigma_L^r][G^<]\right]\right)\right]$$

Supercurrent operator

The Majorana Fermion

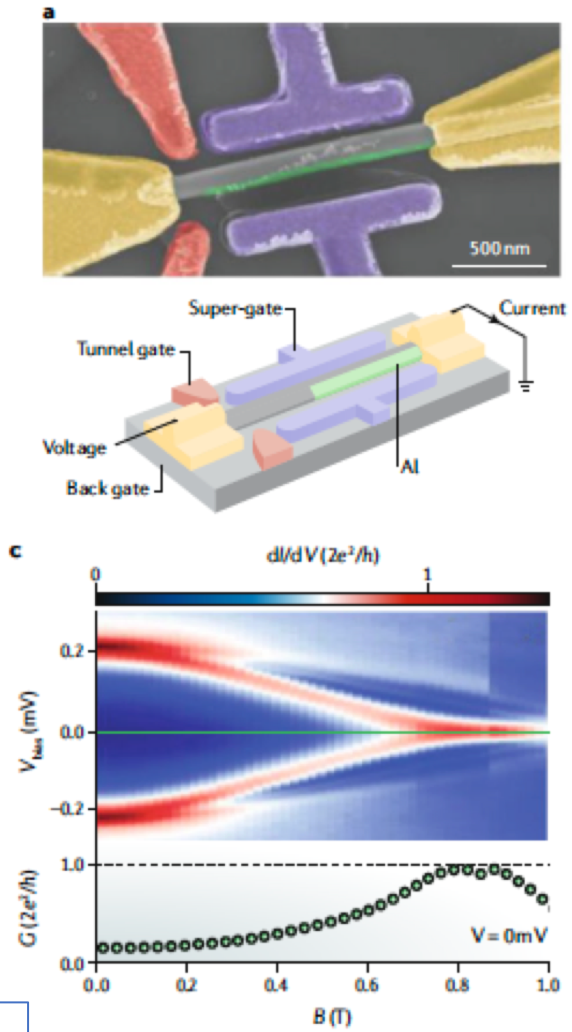


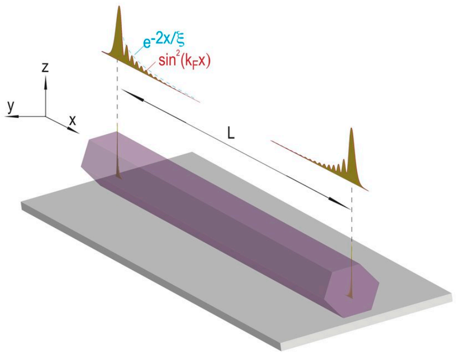
$$\gamma = uc^{\dagger} + vc$$

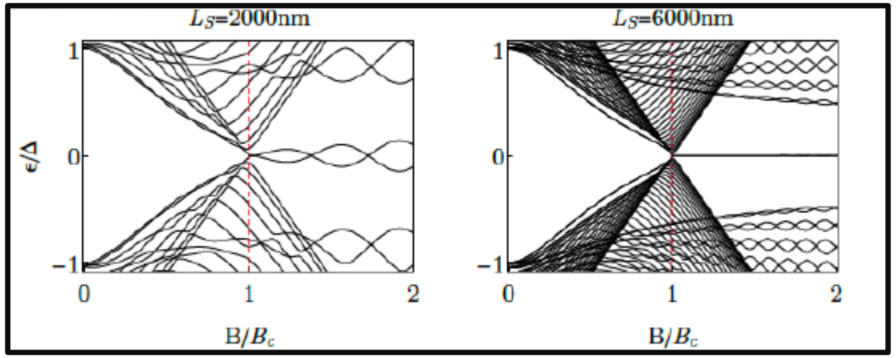
In the middle of the gap we do have |u|=|v|

Such a particle may satisfy: $\gamma^{\dagger} = \gamma$

LPK group Science 2012







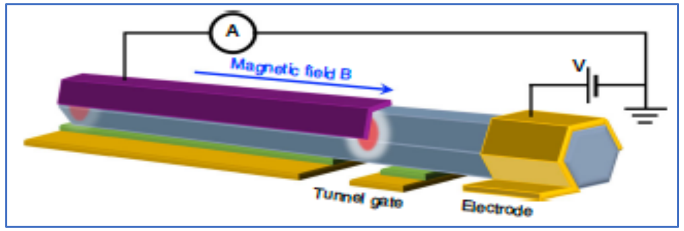
$$G_A = \frac{2e^2}{h}$$

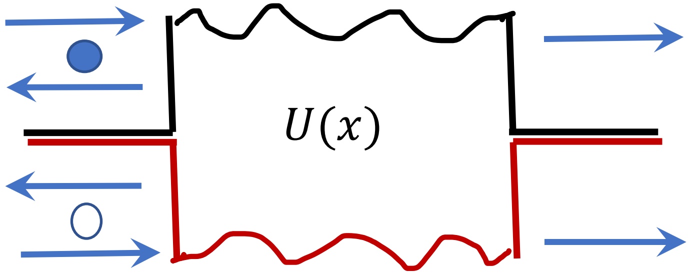
Can be detected via conductance at zero bias!

These end states are non-locally correlated!

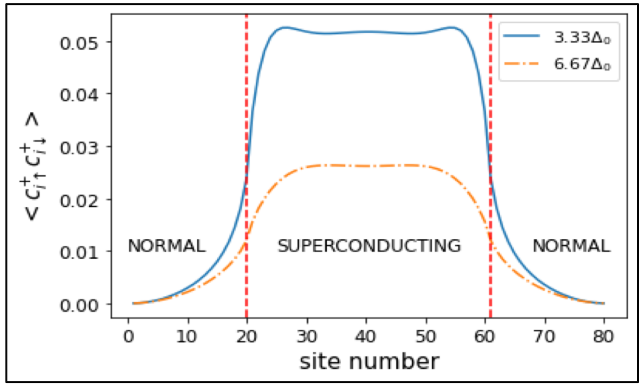
The Andreev "retro" Reflection

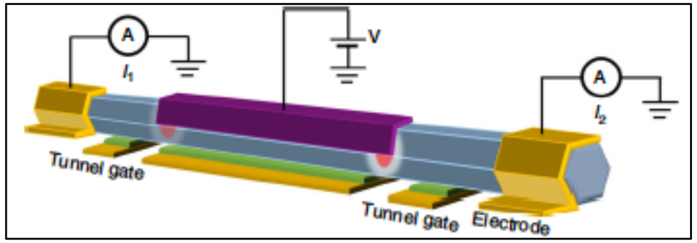
$$H_{BdG} = \begin{pmatrix} (H + U - \mu) & \Delta \\ \Delta^* & -(H + U - \mu) \end{pmatrix}$$

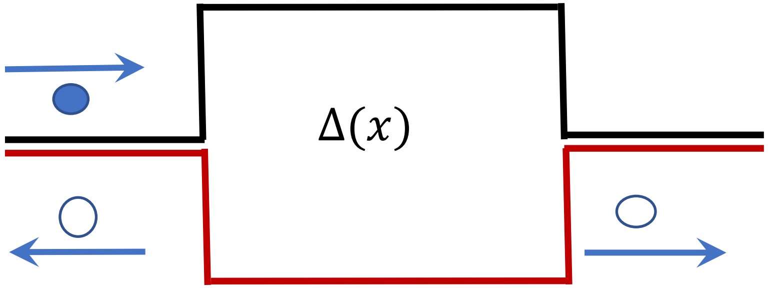


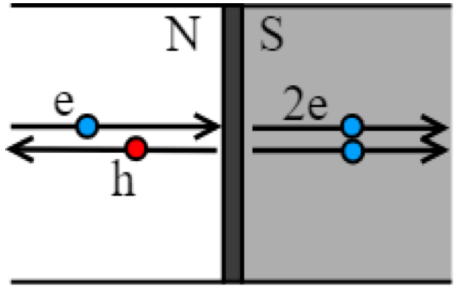


Direct Process

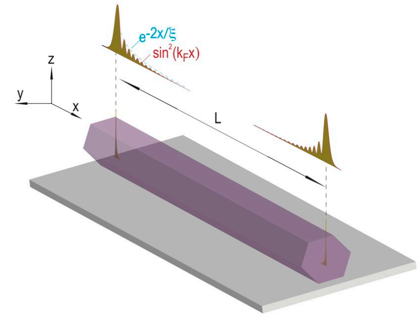


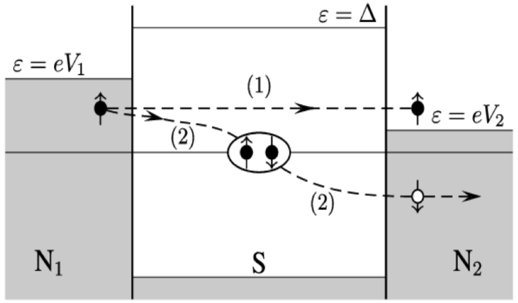






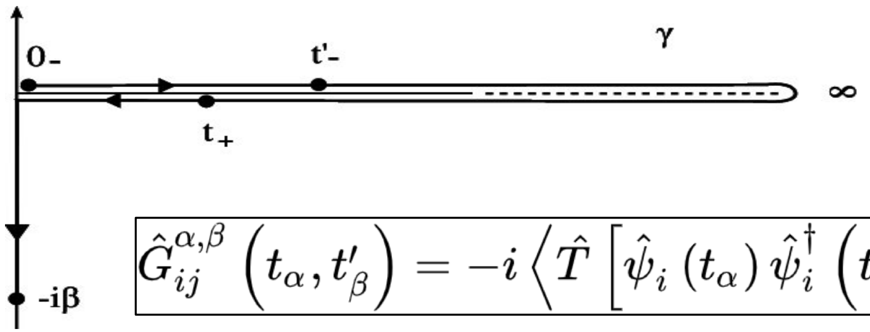
Andreev reflection





Crossed Andreev Reflection

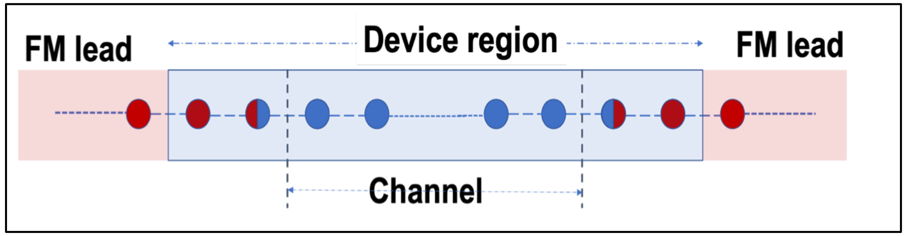
Keldysh Non-Equilibrium Green's Function



$$\hat{G}^{\alpha,\beta}_{ij}\left(t_\alpha, t'_\beta\right) = -i\left\langle \hat{T}\left[\hat{\psi}_i\left(t_\alpha\right)\hat{\psi}^\dagger_i\left(t'_\beta\right)\right]\right\rangle$$

$$G^{+,-}_{ij}(t,t') = G^{+,-}_{ij}(t-t')$$

$$G^{+,-}_{ij}(t-t') \xrightarrow{\mathcal{F}} G^{+,-}_{ij}(E)$$

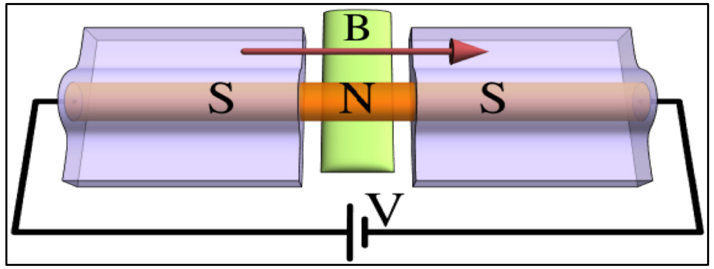


$$G^r(E) = \left(E\mathbb{I} + i\eta - \mathcal{H} - \Sigma_1 - \Sigma_2\right)^{-1}$$

$$\hat{\psi}_i = \begin{pmatrix}\hat{c}_{i\uparrow}\\ \hat{c}_{i\downarrow}\end{pmatrix}$$ → Spinor

$$\hat{\psi}_i = \begin{pmatrix}\hat{c}_{i\uparrow}\\ \hat{c}^\dagger_{i\downarrow}\end{pmatrix}$$ → Nambu Spinor

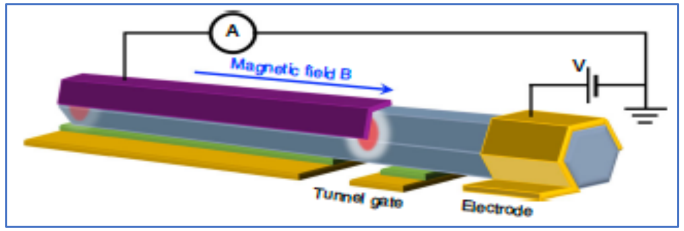
$$I_{op} = \frac{ie}{h}\left(\mathcal{H}G^n - G^n\mathcal{H}\right)$$

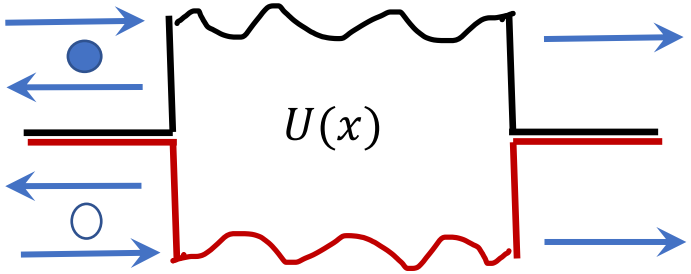


$$I^{op}_L(E) = \frac{1}{2}\left[\frac{e}{h}Tr\left(\tau_z\left[[G^r][\Sigma^<_L] - [\Sigma^<_L][G^a] + [G^<][\Sigma^a_L] - [\Sigma^r_L][][G^<]\right]\right)\right]$$

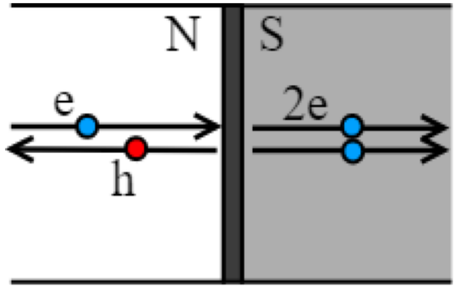
The Andreev "retro" Reflection

$$H_{BdG} = \begin{pmatrix} (H+U-\mu) & \Delta \\ \Delta^* & -(H+U-\mu) \end{pmatrix}$$

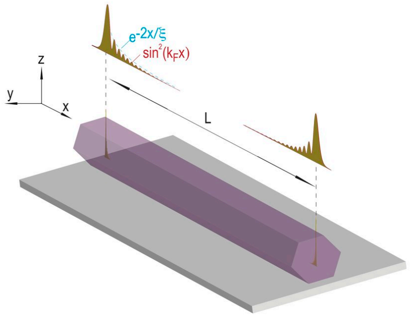




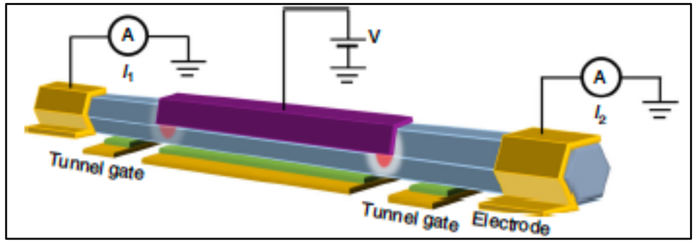
Direct Process

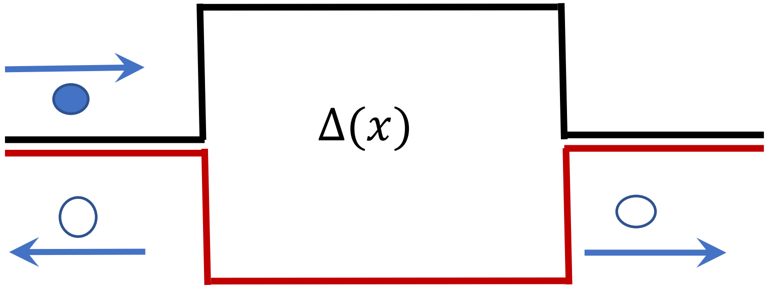


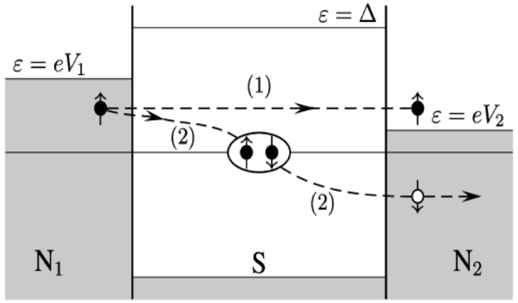
Andreev reflection



$$I_L^e = \int dE \frac{e}{h} \left(T_D^e(E) \left[f_L^{ee}(E-eV_L) - f_R^{ee}(E-eV_R)\right]\right)$$
$$+ \int dE \frac{e}{h} \left(T_A^e(E) \left[f_L^{ee}(E-eV_L) - f_L^{hh}(E+eV_L)\right]\right)$$
$$+ \int dE \frac{e}{h} \left(T_{CA}^e(E) \left[f_L^{ee}(E-eV_L) - f_R^{hh}(E+eV_R)\right]\right)$$







Crossed Andreev Reflection

Device Modeling Basics

$$I = \int_{-\infty}^{\infty} dE\,\Xi(E)\left(f_S(E) - f_D(E)\right)$$

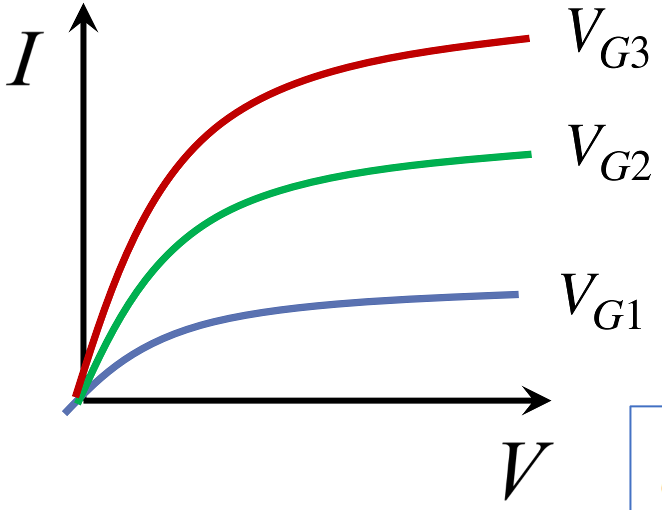
Transport Function — Driving

$$I = GV$$

Linear Response

$$G(V_{DS}) = \left.\frac{\partial I}{\partial V}\right|_{V=V_{DS}}$$

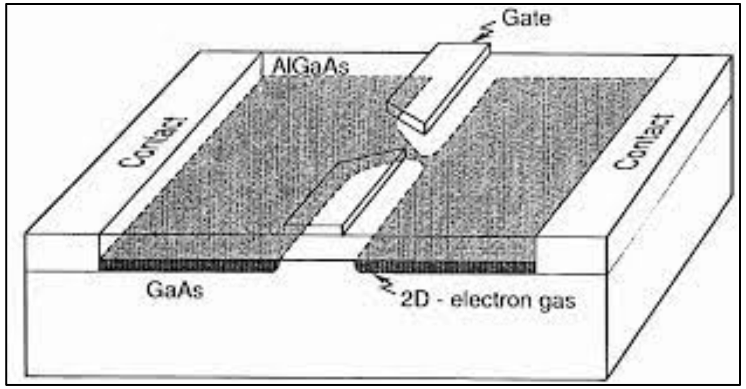
$$qV_{DS} = \mu_S - \mu_D$$

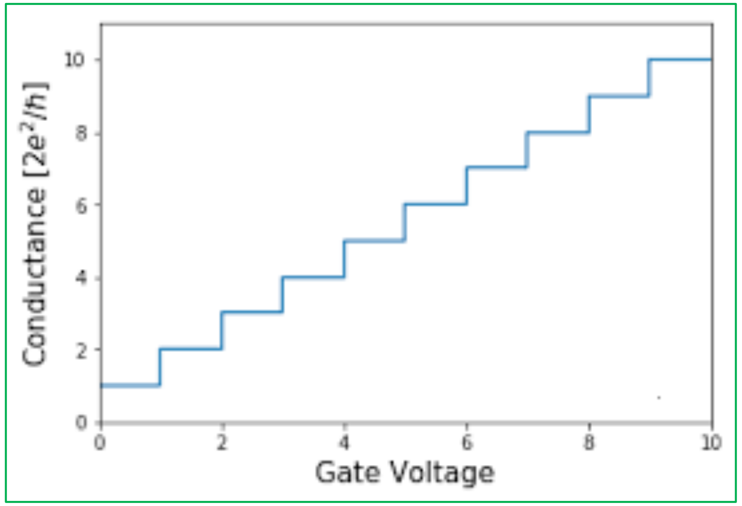


$$G(V=0) = \left.\frac{\partial I}{\partial V}\right|_{V=0} = \frac{I}{V}$$

$$G = \frac{2e^2}{h}M$$

$$\sigma_{xy} = \frac{e^2}{\hbar}\int_{BZ}\frac{d\mathbf{k}}{(2\pi)^2}\Omega_{k_x,k_y} = n\frac{e^2}{h},$$





When we have a topological state

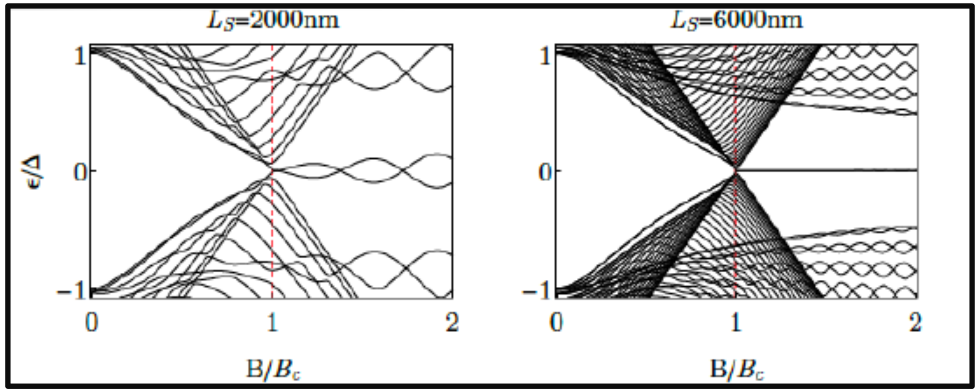
Conductance "Spectroscopy"

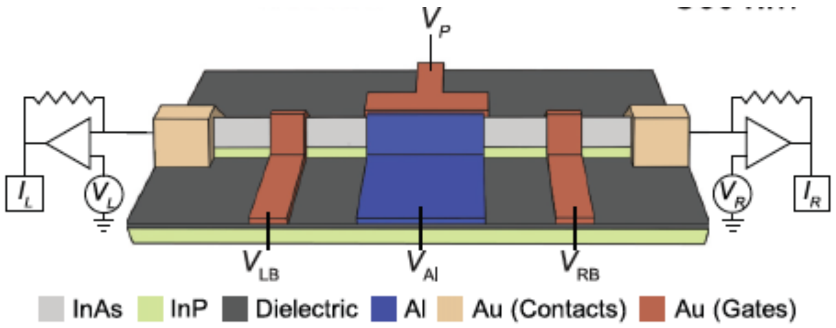
How to "probe" the spectra?

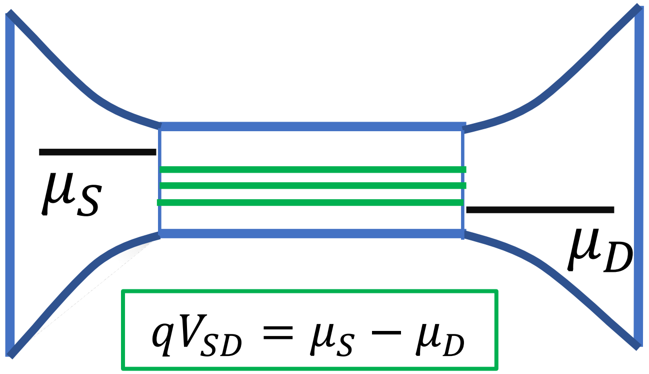
$$G(V=0) = \left.\frac{\partial I}{\partial V}\right|_{V=0} = \frac{I}{V}$$

$$G(V_{DS}) = \left.\frac{\partial I}{\partial V}\right|_{V=V_{DS}}$$

$$I = \int_{-\infty}^{\infty} dE\,\Xi(E)\big(f_S(E) - f_D(E)\big)$$







$$qV_{SD} = \mu_S - \mu_D$$

$$I = GV$$

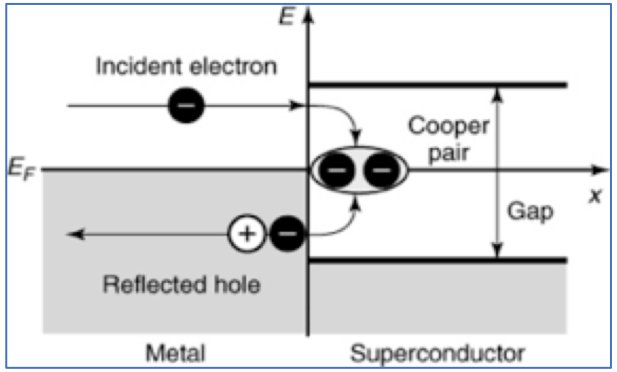
Linear Response

$$qV_{DS} = \mu_S - \mu_D$$

Role of "Energy Axis"

$$[G] = \begin{pmatrix} G_{LL} & G_{LR} \\ G_{RL} & G_{RR} \end{pmatrix} = \begin{pmatrix} \left.\frac{\partial I_L}{\partial V_L}\right|_{V_R=0} & \left.\frac{\partial I_L}{\partial V_R}\right|_{V_L=0} \\ \left.\frac{\partial I_R}{\partial V_L}\right|_{V_R=0} & \left.\frac{\partial I_R}{\partial V_R}\right|_{V_L=0} \end{pmatrix}$$

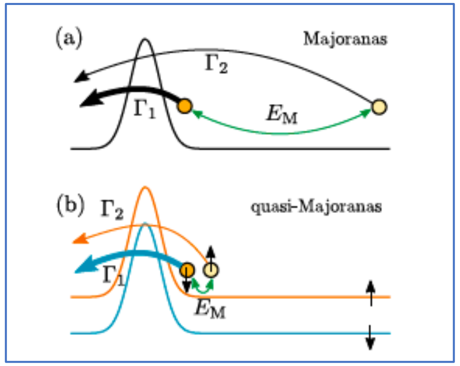
Tunneling Conductance Spectroscopy



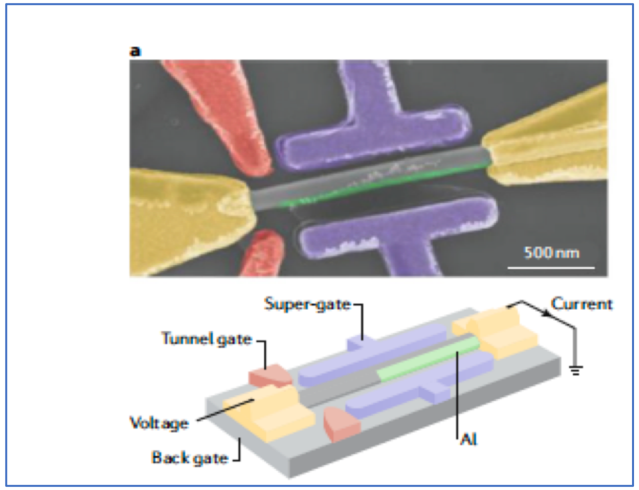
$$\gamma = uc^{\dagger} + vc$$

$$\gamma^{\dagger} = \gamma$$

For Majoranas -> exact mid gap Andreev reflection -> quantized conductance

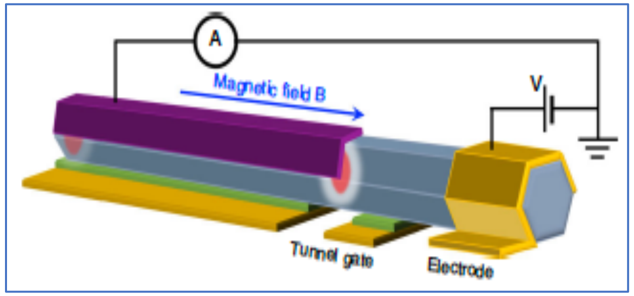


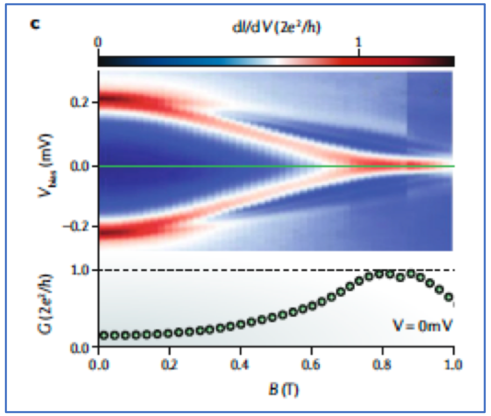
Vuik et.al., Sci. Post, 7, 061 (2019)

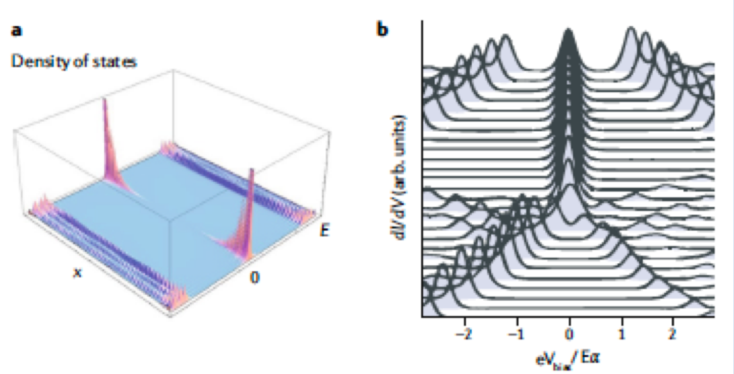


False positive? Due to disorder and localized Andreev modes :(

[1] Mourik et.al., Science, 336, 1003, (2012).
[2] Zhang et. al., Nature, 74, 556, (2018)
→ ArXiv: 2101.11456 (2021)
[3] Prada et.al., Nat. Phys. Rev, 2, 575, (2020)



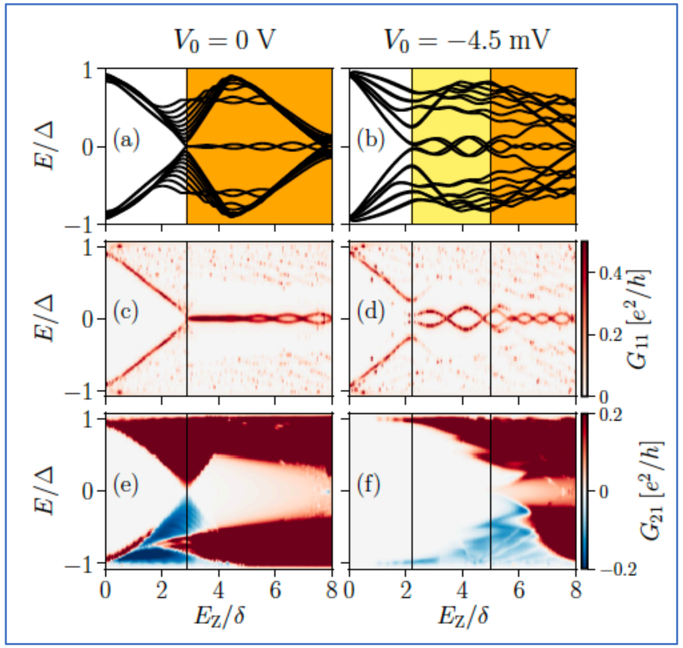




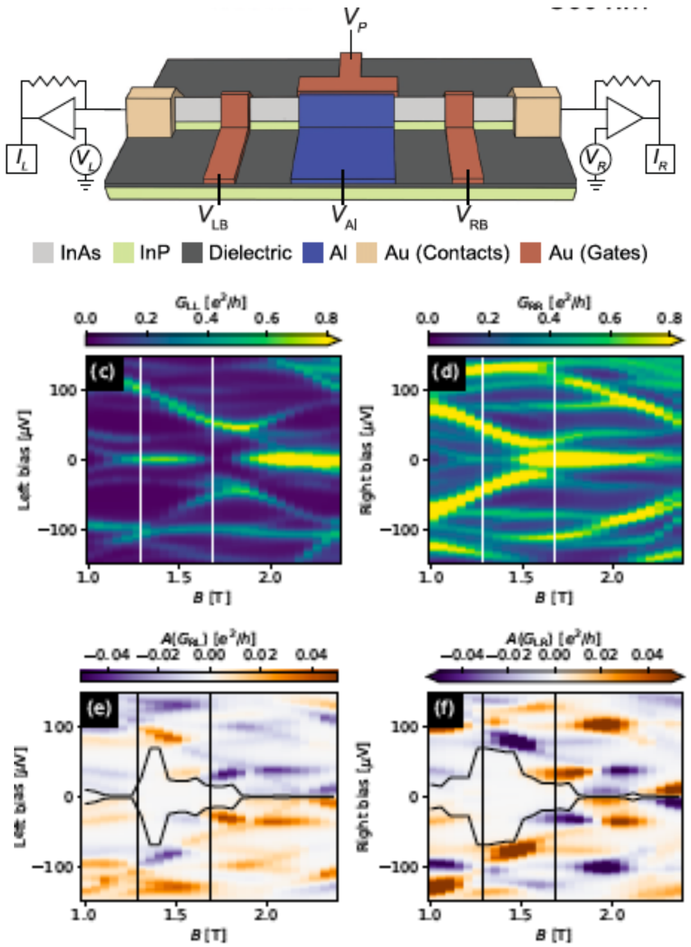
Understanding non-local correlations: The Gap Protocol!

These end states are non-locally correlated!

$$[G] = \begin{pmatrix} G_{LL} & G_{LR} \\ G_{RL} & G_{RR} \end{pmatrix} = \begin{pmatrix} \left.\frac{\partial I_L}{\partial V_L}\right|_{V_R=0} & \left.\frac{\partial I_L}{\partial V_R}\right|_{V_L=0} \\ \left.\frac{\partial I_R}{\partial V_L}\right|_{V_R=0} & \left.\frac{\partial I_R}{\partial V_R}\right|_{V_L=0} \end{pmatrix}$$

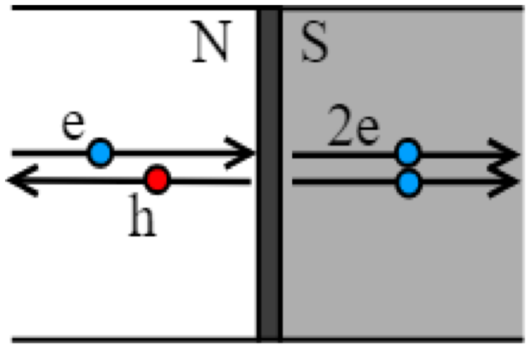


Rosdahl et.al., PRB, 97, 045421 (2018)

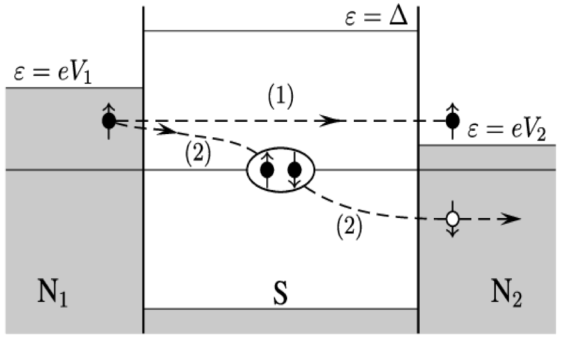


Microsoft Quantum, ArXiv:2207.02472 (2022)

Andreev



Andreev reflection

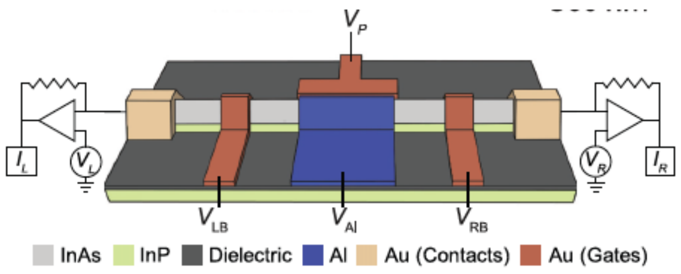


Crossed Andreev

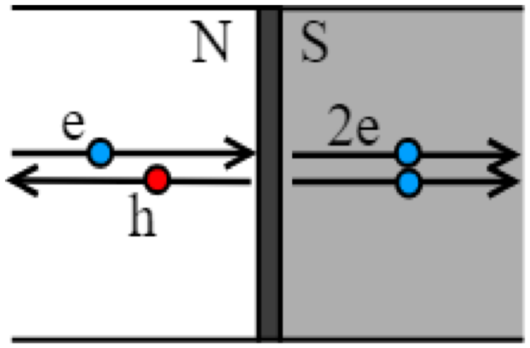
Understanding non-local correlations: The non-local conductance

These end states are non-locally correlated!

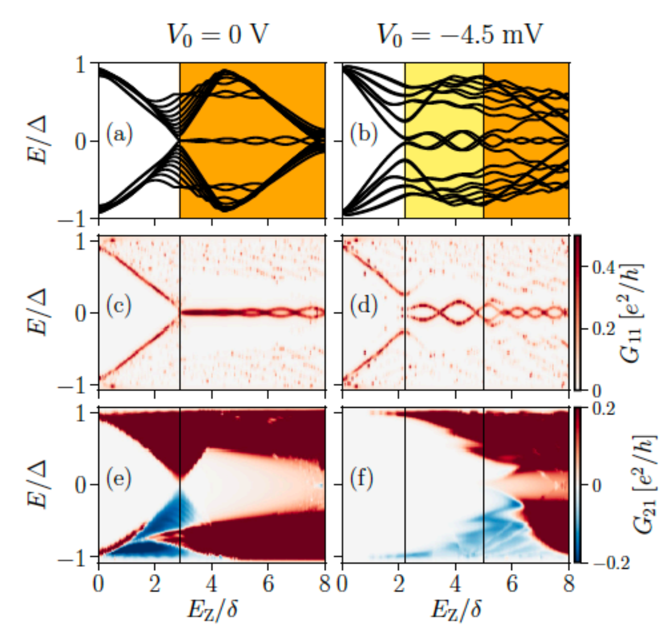
$$[G] = \begin{pmatrix} G_{LL} & G_{LR} \\ G_{RL} & G_{RR} \end{pmatrix} = \begin{pmatrix} \left.\frac{\partial I_L}{\partial V_L}\right|_{V_R=0} & \left.\frac{\partial I_L}{\partial V_R}\right|_{V_L=0} \\ \left.\frac{\partial I_R}{\partial V_L}\right|_{V_R=0} & \left.\frac{\partial I_R}{\partial V_R}\right|_{V_L=0} \end{pmatrix}$$

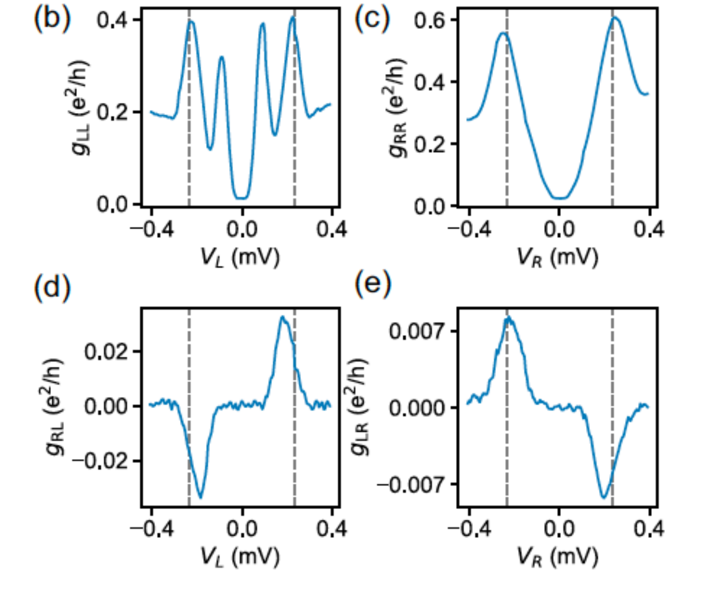


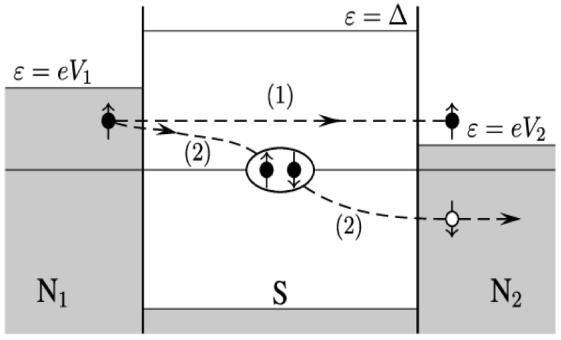
Andreev



Andreev reflection







Crossed Andreev

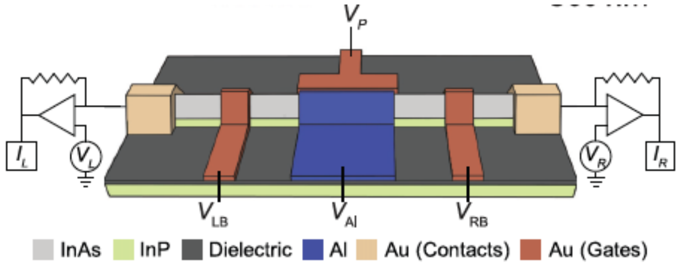
Rosdahl et.al., PRB, 97, 045421 (2018)

Puglia et.al., PRB, 103, 235201 (2021)
Microsoft Quantum, ArXiv:2207.02472 (2022)

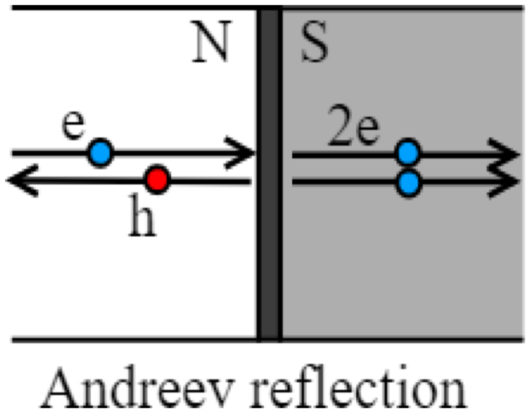
Understanding non-local correlations:
The non-local conductance

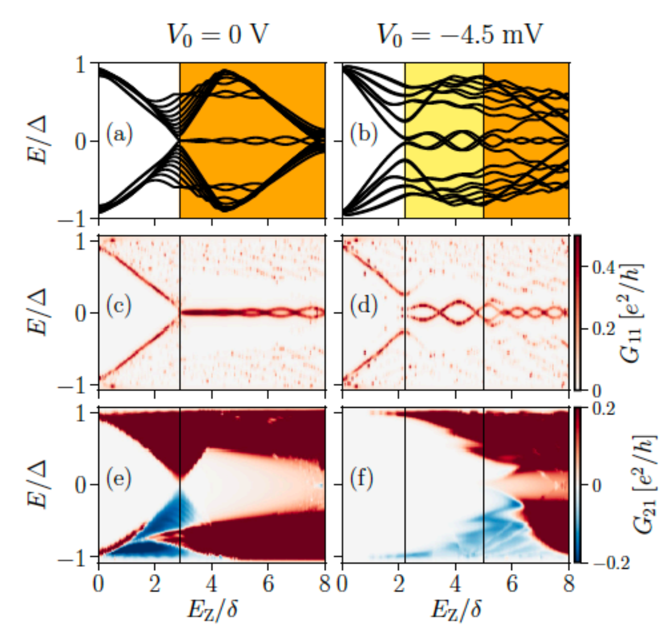
$$[G] = \begin{pmatrix} G_{LL} & G_{LR} \\ G_{RL} & G_{RR} \end{pmatrix} = \begin{pmatrix} \left.\frac{\partial I_L}{\partial V_L}\right|_{V_R=0} & \left.\frac{\partial I_L}{\partial V_R}\right|_{V_L=0} \\ \left.\frac{\partial I_R}{\partial V_L}\right|_{V_R=0} & \left.\frac{\partial I_R}{\partial V_R}\right|_{V_L=0} \end{pmatrix}$$

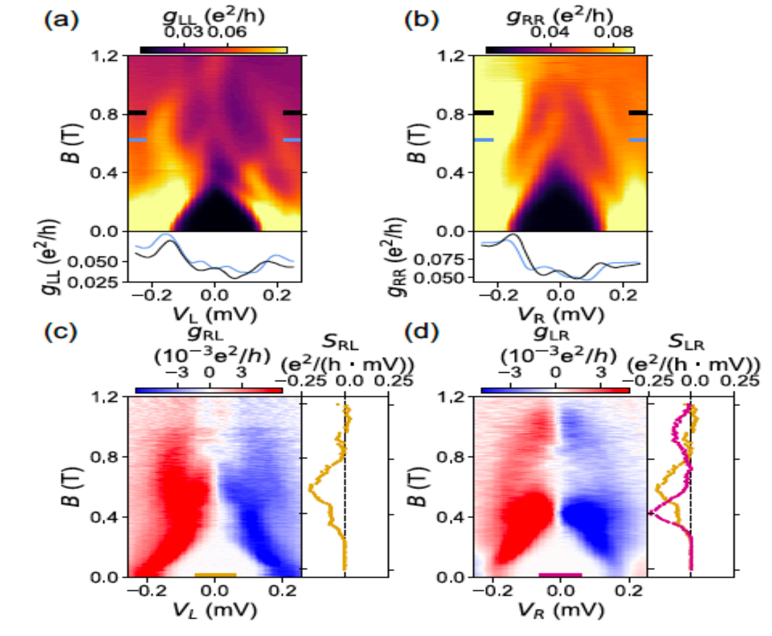
These end states are non-locally correlated!

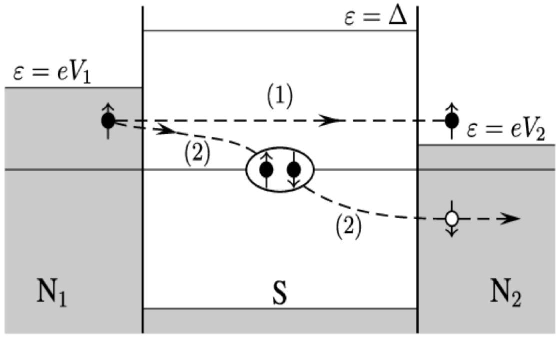


Andreev







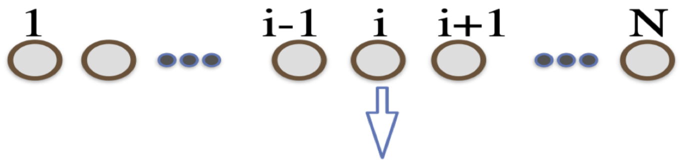


Crossed Andreev

Rosdahl et.al., PRB, 97, 045421 (2018)

Puglia et.al., PRB, 103, 235201 (2021)
Microsoft Quantum, ArXiv:2207.02472 (2022)

The Kitaev Model



$$\gamma_i^A = (c_i + c_i^\dagger)$$

$$\gamma_i^B = i(c_i^\dagger - c_i)$$

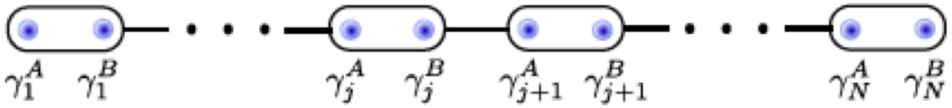
$$H = -\mu \sum_{i=1}^{N} (c_i^\dagger c_i - \frac{1}{2}) + \sum_{i=1}^{N-1} \left[-t \left(c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i \right) + \Delta c_i c_{i+1} + \Delta^* c_{i+1}^\dagger c_i^\dagger \right]$$

$$c_i = \frac{1}{2}\left(\gamma_i^A + i\gamma_i^B\right)$$

$$c_i^\dagger = \frac{1}{2}\left(\gamma_i^A - i\gamma_i^B\right)$$

$$H = -i\frac{\mu}{2} \sum_{i=1}^{N} \gamma_i^A \gamma_i^B + \frac{i}{2} \sum_{i=1}^{N-1} \left[(\Delta + t)\gamma_i^B \gamma_{i+1}^A + (\Delta - t)\gamma_i^A \gamma_{i+1}^B \right]$$

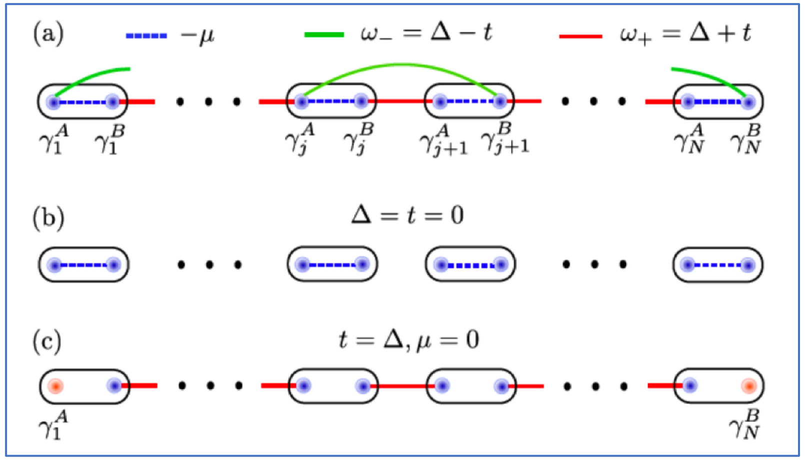
JPCM, 32, 445302, (2020)



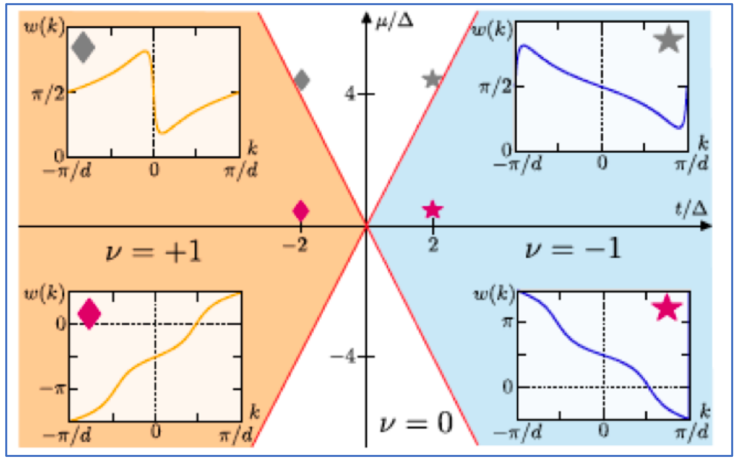
Generic Chain

Trivial Chain

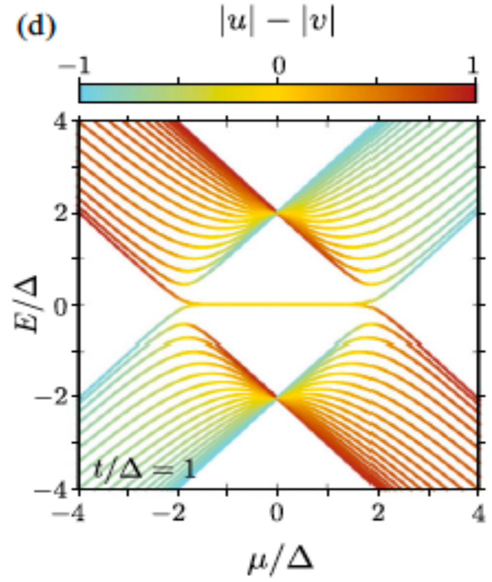
Topological Chain

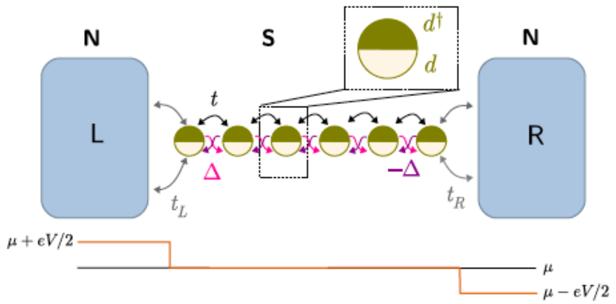


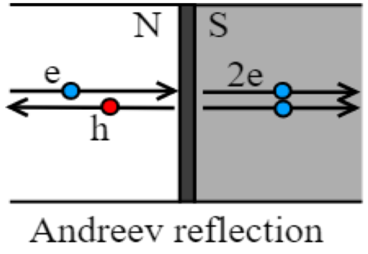
Phase diagram of the Kitaev model

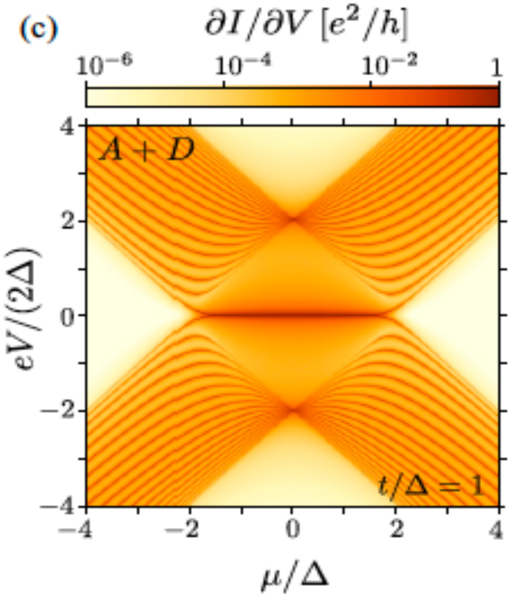


Theory-Local conductance spectroscopy

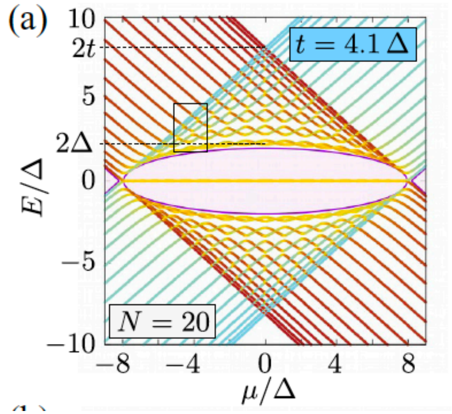


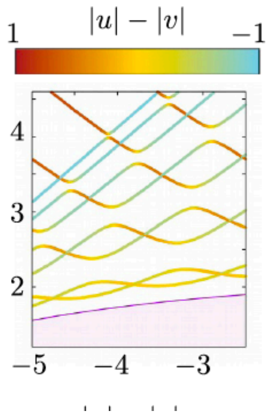






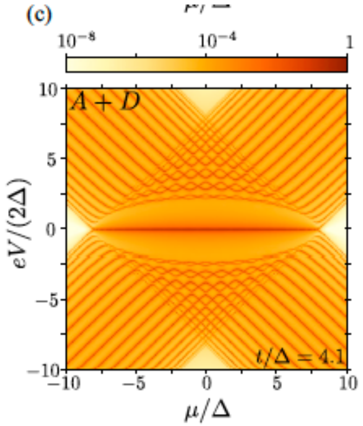
$$I_L^{op}(E)=\frac{1}{2}\left[\frac{e}{h}Tr\left(\tau_z\left[[G^r][\Sigma_L^<]-[\Sigma_L^<][G^a]+[G^<][\Sigma_L^a]-[\Sigma_L^r][G^<]\right]\right)\right]$$

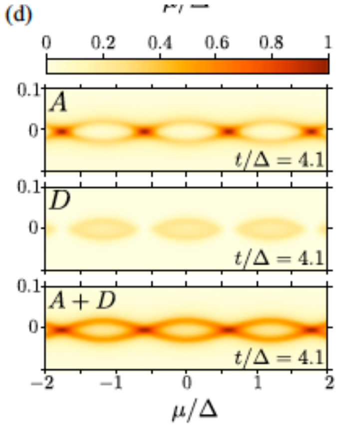




$$\begin{aligned}I_L^e=&\int dE\frac{e}{h}\left(T_D^e(E)\left[f_L^{ee}(E-eV_L)-f_R^{ee}(E-eV_R)\right]\right)\\&+\int dE\frac{e}{h}\left(T_A^e(E)\left[f_L^{ee}(E-eV_L)-f_L^{hh}(E+eV_L)\right]\right)\\&+\int dE\frac{e}{h}\left(T_{CA}^e(E)\left[f_L^{ee}(E-eV_L)-f_R^{hh}(E+eV_R)\right]\right)\end{aligned}$$

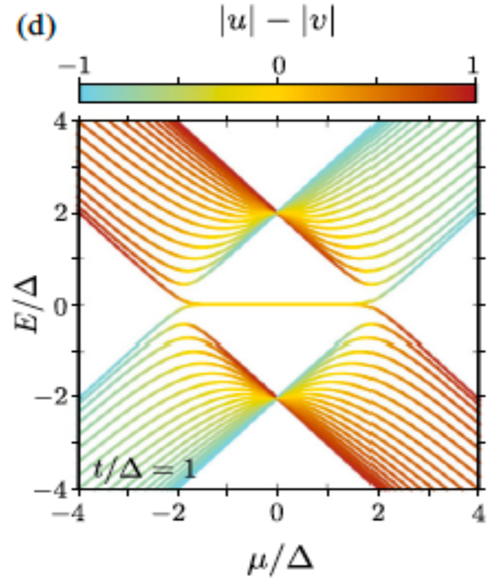
$$G_{LL}=\frac{e^2}{h}T_A^e(E=eV_L)+\frac{e^2}{h}T_A^e(E=-eV_L)$$

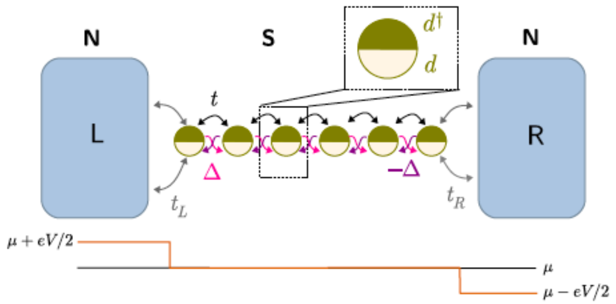


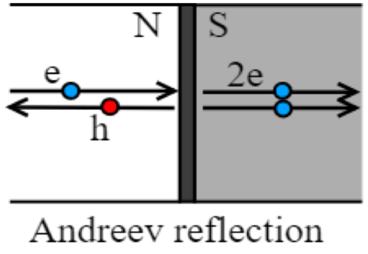


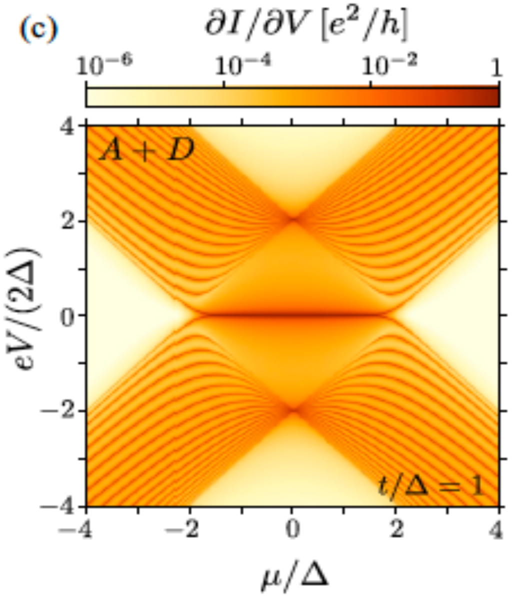
Phys Rev B, 103, 165432, (2021)

Theory-Local conductance spectroscopy

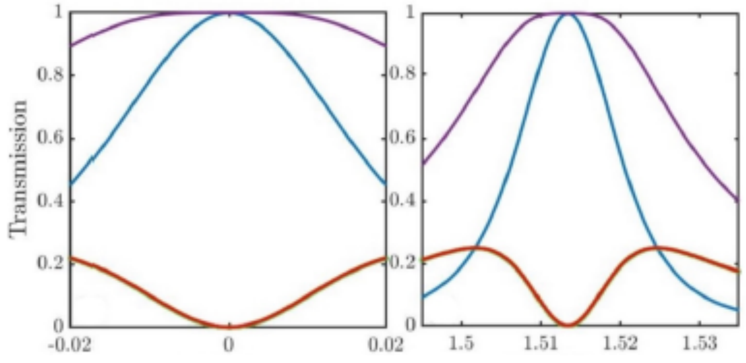






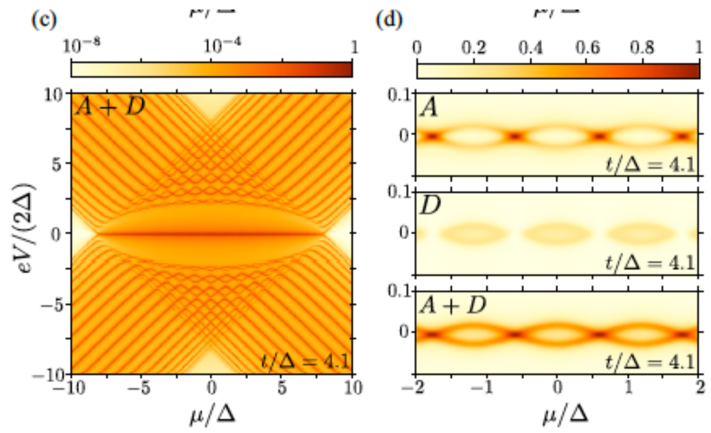


$$I_L^{op}(E) = \frac{1}{2}\left[\frac{e}{h}Tr\left(\tau_z\left[[G^r][\Sigma_L^<] - [\Sigma_L^<][G^a] + [G^<][\Sigma_L^a] - [\Sigma_L^r][G^<]\right]\right)\right]$$



$$\begin{aligned} I_L^e &= \int dE \frac{e}{h}\left(T_D^e(E)\left[f_L^{ee}(E - eV_L) - f_R^{ee}(E - eV_R)\right]\right) \\ &+ \int dE \frac{e}{h}\left(T_A^e(E)\left[f_L^{ee}(E - eV_L) - f_L^{hh}(E + eV_L)\right]\right) \\ &+ \int dE \frac{e}{h}\left(T_{CA}^e(E)\left[f_L^{ee}(E - eV_L) - f_R^{hh}(E + eV_R)\right]\right) \end{aligned}$$

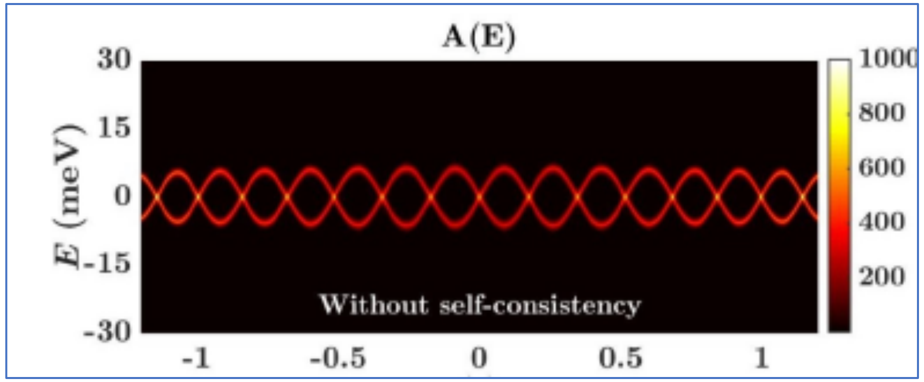
$$G_{LL} = \frac{e^2}{h}T_A^e(E = eV_L) + \frac{e^2}{h}T_A^e(E = -eV_L)$$

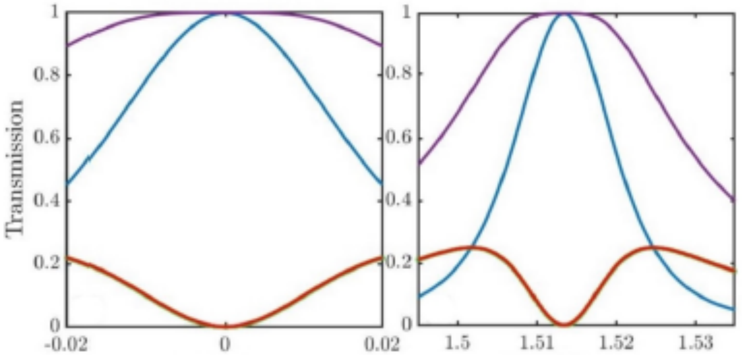


JPCM, 33, 365301, (2021)

Phys Rev B, 103, 165432, (2021)

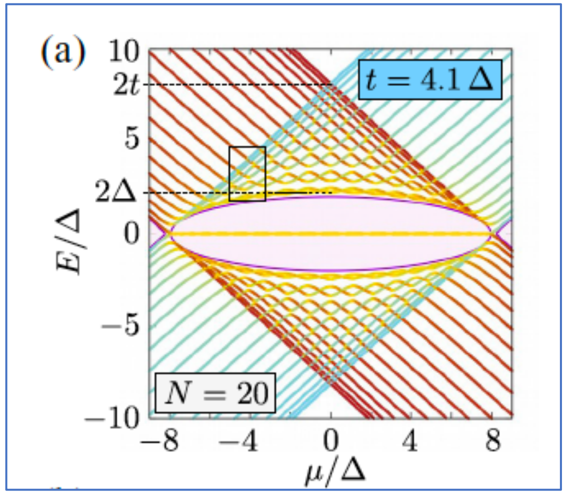
Theory - Local conductance

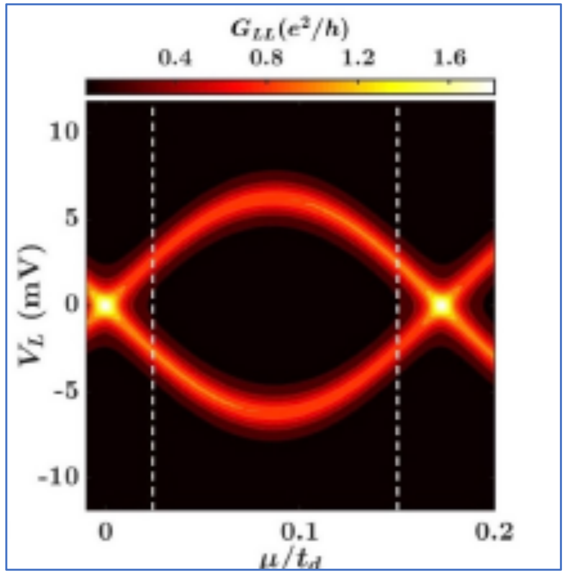




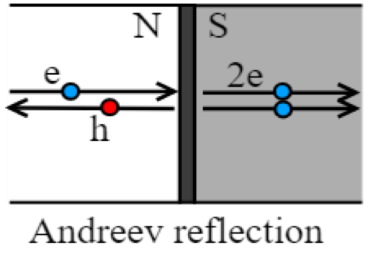
$$I_L^{op}(E) = \frac{1}{2}\left[\frac{e}{h}Tr\left(\tau_z\left[[G^r][\Sigma_L^<]-[\Sigma_L^<][G^a]+[G^<][\Sigma_L^a]-[\Sigma_L^r][][G^<]\right]\right)\right]$$

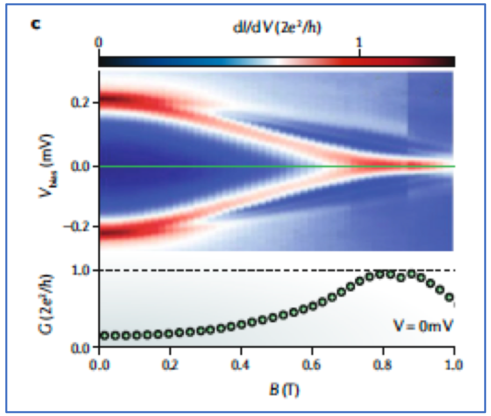
$$G_{LL} = \frac{e^2}{h}T_A^e(E=eV_L)+\frac{e^2}{h}T_A^e(E=-eV_L)$$

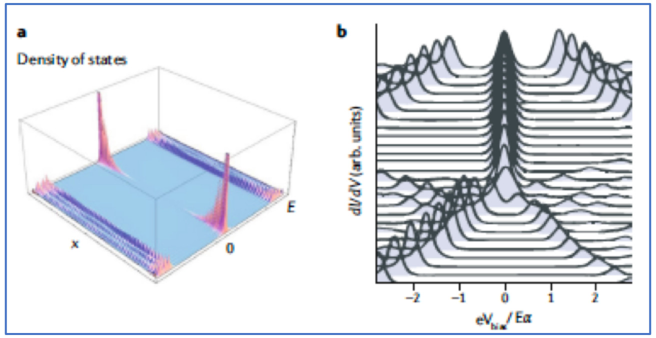




Phys Rev B, 103, 165432, (2021)

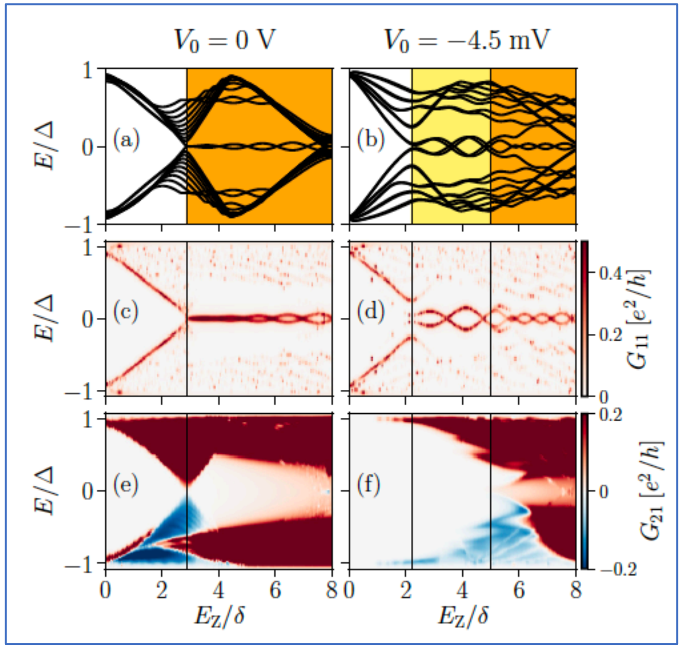




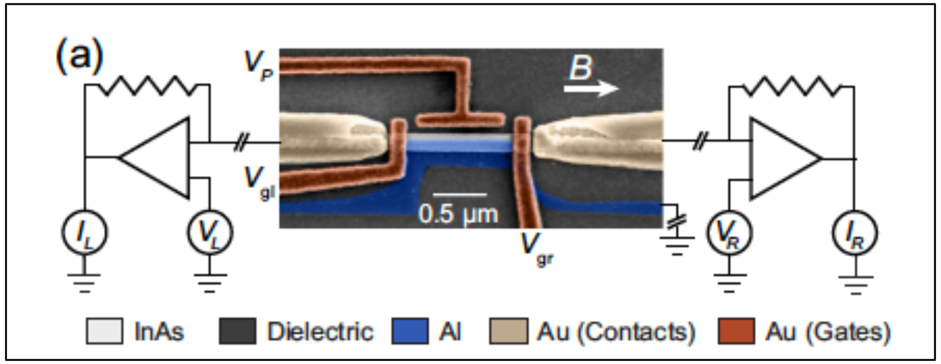


Understanding non-local correlations:
The non-local conductance

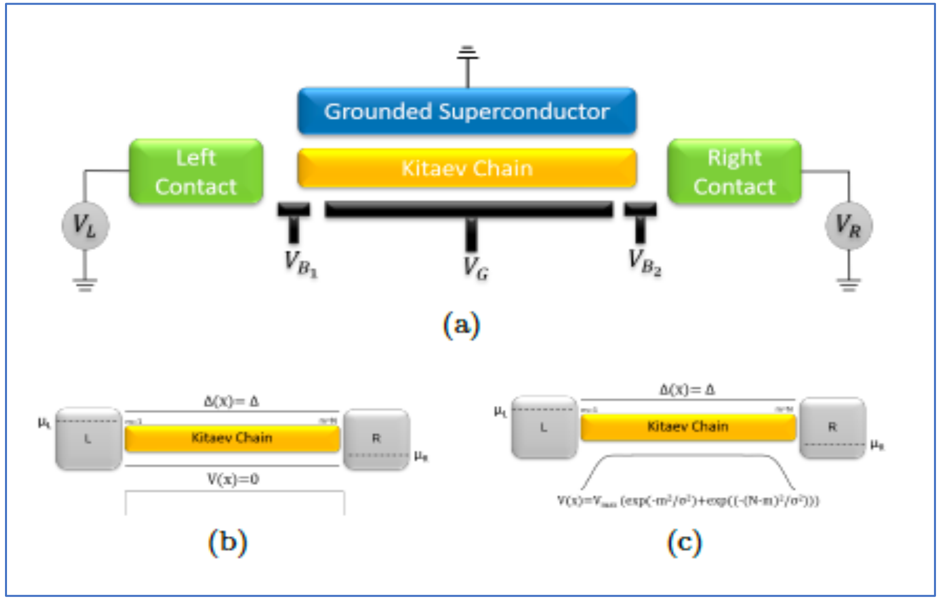
$$[G] = \begin{pmatrix} G_{LL} & G_{LR} \\ G_{RL} & G_{RR} \end{pmatrix} = \begin{pmatrix} \left.\frac{\partial I_L}{\partial V_L}\right|_{V_R=0} & \left.\frac{\partial I_L}{\partial V_R}\right|_{V_L=0} \\ \left.\frac{\partial I_R}{\partial V_L}\right|_{V_R=0} & \left.\frac{\partial I_R}{\partial V_R}\right|_{V_L=0} \end{pmatrix}$$



Rosdahl et.al., PRB, 97, 045421 (2018)

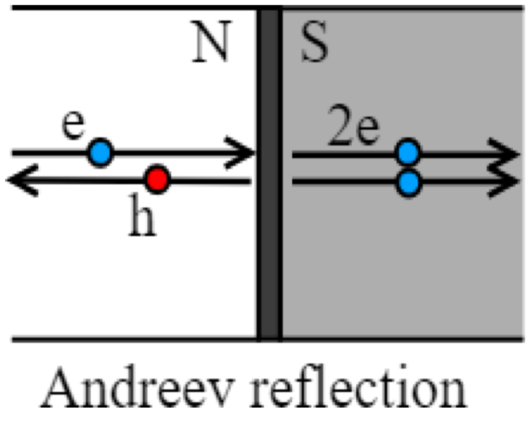


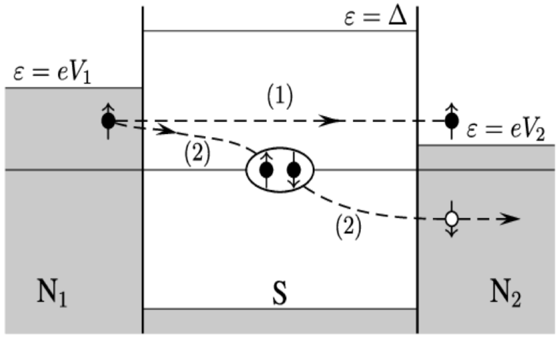
These end states are non-locally correlated!



JPCM, 33, 365301, (2021)
PRB Letter, 105,161403, (2022)

Andreev





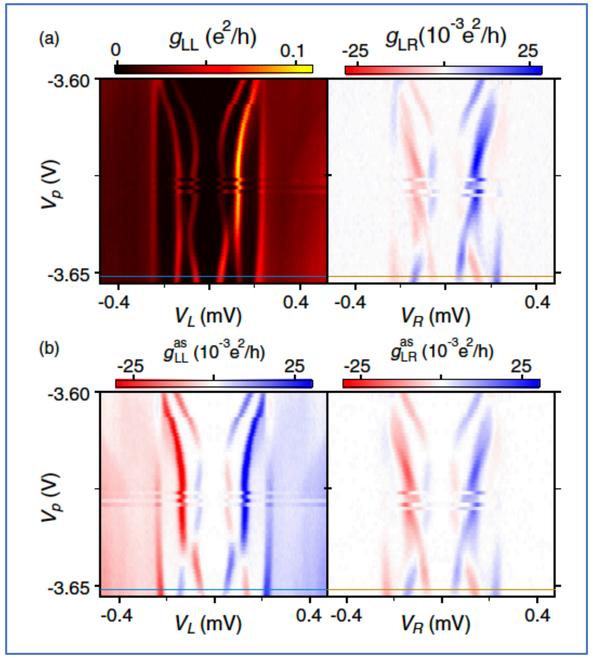
Crossed Andreev

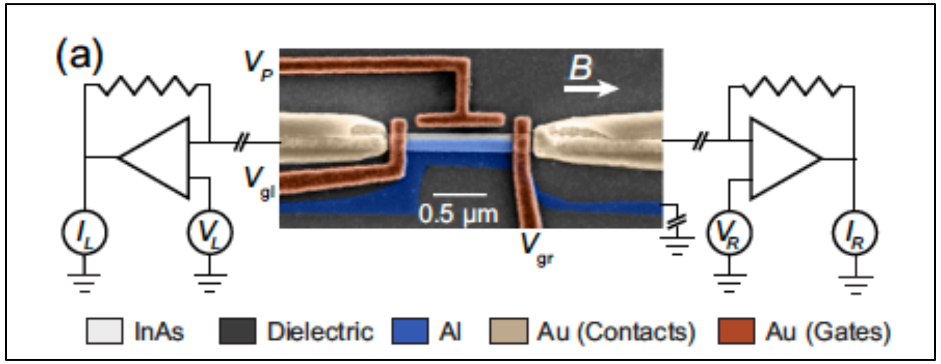
Understanding non-local correlations: The non-local conductance

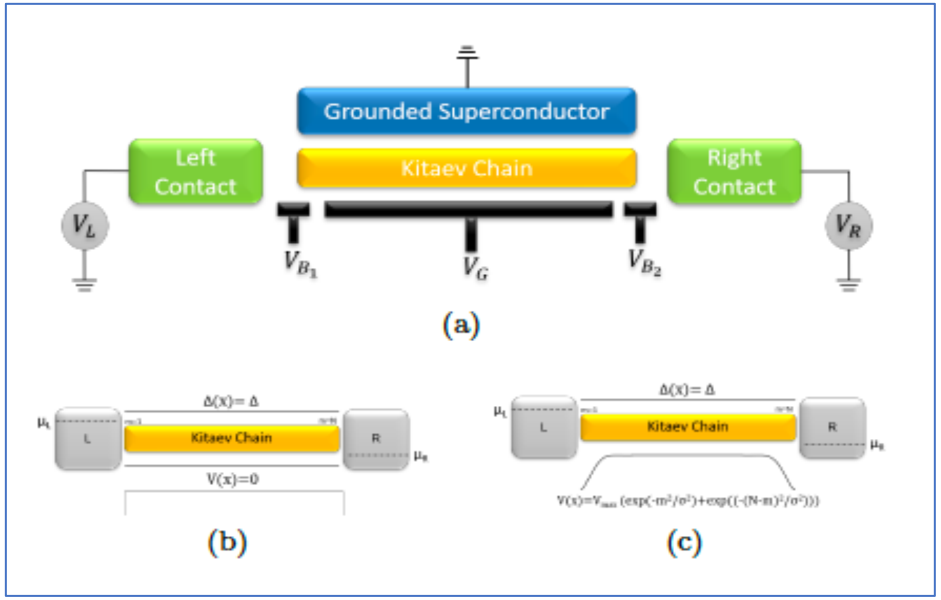
$$G_{LL}(V) + G_{LR}(V) = G_{LL}(-V) + G_{LR}(-V)$$

$$G_{\alpha\beta}^{\mathrm{sym/asym}}(V) \equiv \frac{G_{\alpha\beta}(V) \pm G_{\alpha\beta}(-V)}{2}$$

$$G_{LL}^{\mathrm{asym}}(V) = -G_{LR}^{\mathrm{asym}}(V)$$

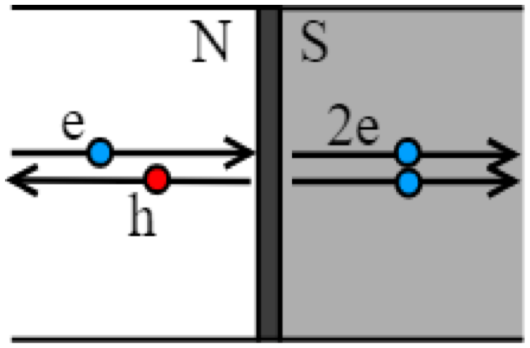




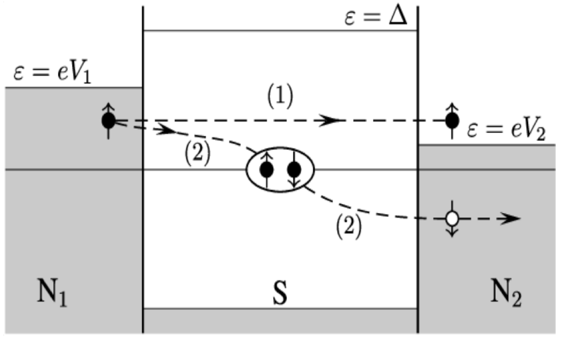


Danon et.al., PRL, 124, 036801 (2020)
Menard et.al., PRL, 124,036802, (2020)

Andreev

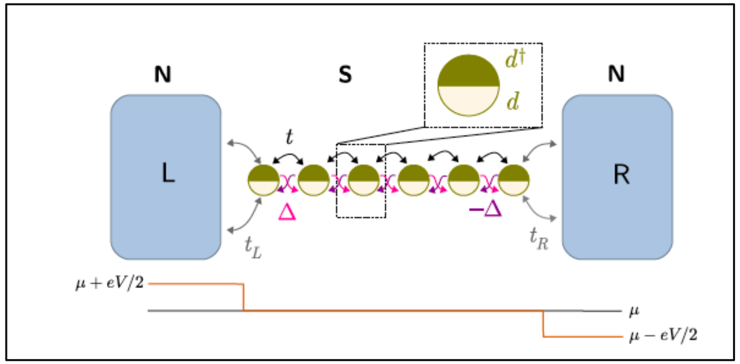


Andreev reflection

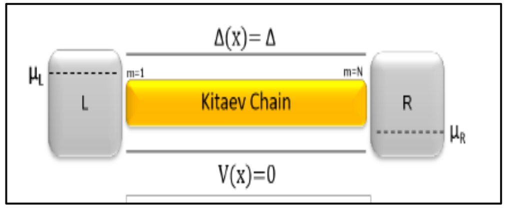


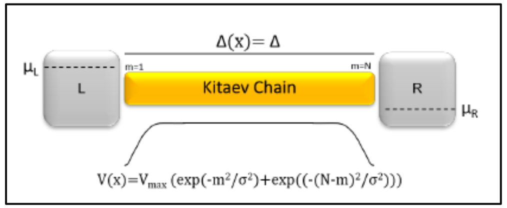
Crossed Andreev

Theory: Non-local conductance

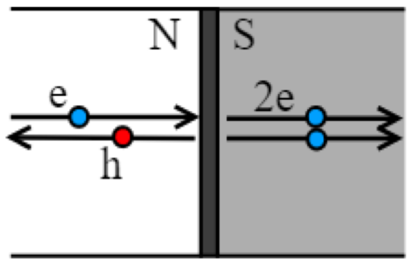


$$\hat{H} = -\sum_{i=1}^{N}(\mu + V(i))c_i^\dagger c_i + \sum_{i=1}^{N-1}\left(\Delta c_i^\dagger c_{i+1}^\dagger - t c_{i+1}^\dagger c_i + \text{h.c.}\right)$$

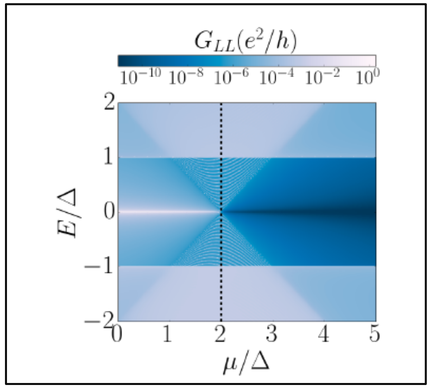




Andreev

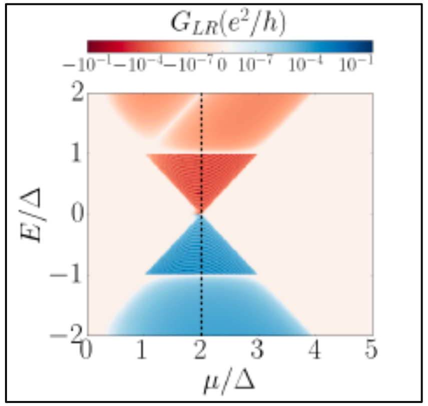


Andreev reflection



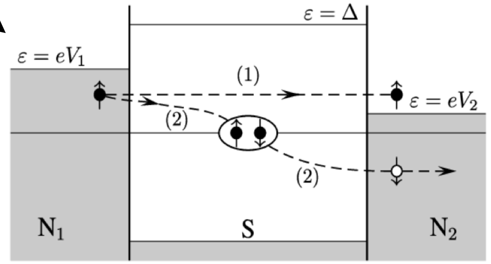
$$I_L^e = \int dE \frac{e}{h}\left(T_D^e(E)\left[f_L^{ee}(E - eV_L) - f_R^{ee}(E - eV_R)\right]\right)$$
$$+ \int dE \frac{e}{h}\left(T_A^e(E)\left[f_L^{ee}(E - eV_L) - f_L^{hh}(E + eV_L)\right]\right)$$
$$+ \int dE \frac{e}{h}\left(T_{CA}^e(E)\left[f_L^{ee}(E - eV_L) - f_R^{hh}(E + eV_R)\right]\right)$$

Crossed Andreev



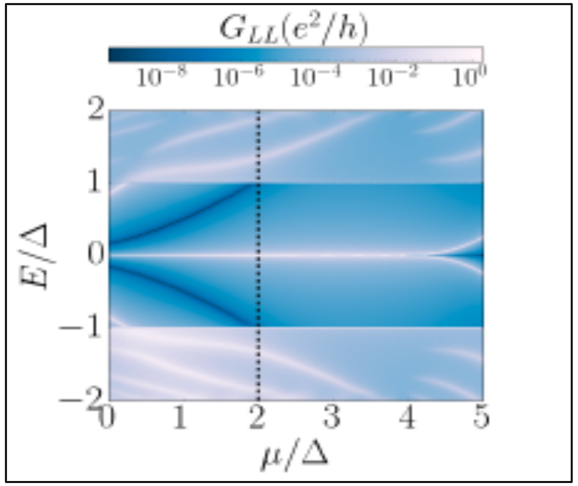
$$G_{LL} = \frac{e^2}{h}T_A^e(E = eV_L) + \frac{e^2}{h}T_A^e(E = -eV_L) + \frac{e^2}{h}T_{CA}^e(E = -eV_L) + \frac{e^2}{h}T_D^e(E = eV_L),$$

$$G_{LR} = \frac{e^2}{h}T_D^e(E = eV_R) - \frac{e^2}{h}T_{CA}^e(E = -eV_R)$$

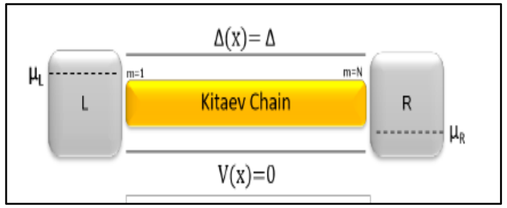


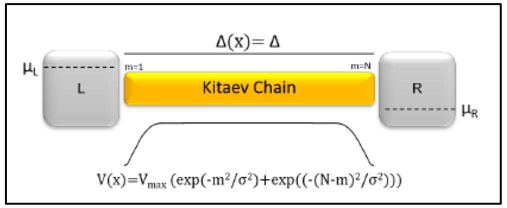
Phys Rev B, 103, 165432, (2021)

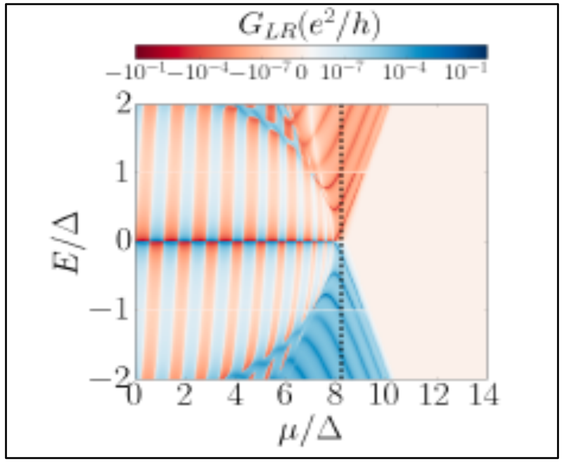
Theory: Non-local conductance

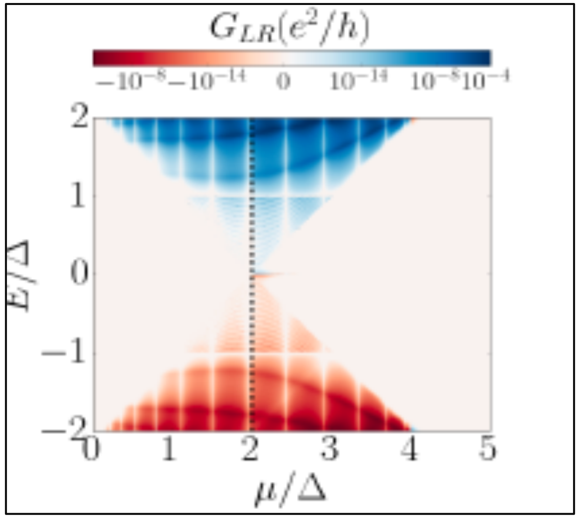


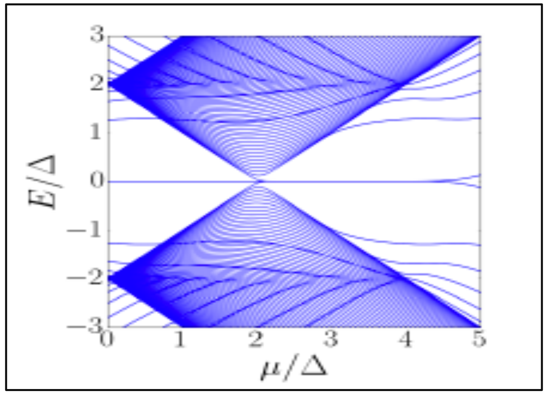
$$\hat{H} = -\sum_{i=1}^{N}(\mu+V(i))c_i^\dagger c_i + \sum_{i=1}^{N-1}\left(\Delta c_i^\dagger c_{i+1}^\dagger - t c_{i+1}^\dagger c_i + \text{h.c.}\right)$$

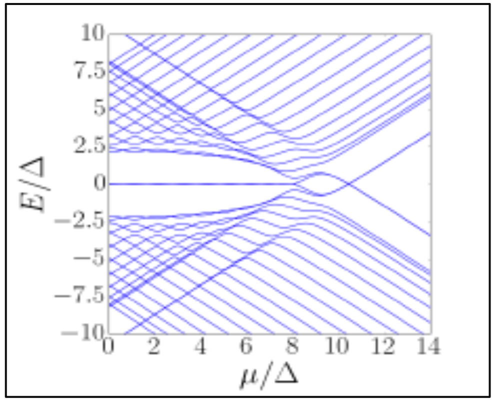


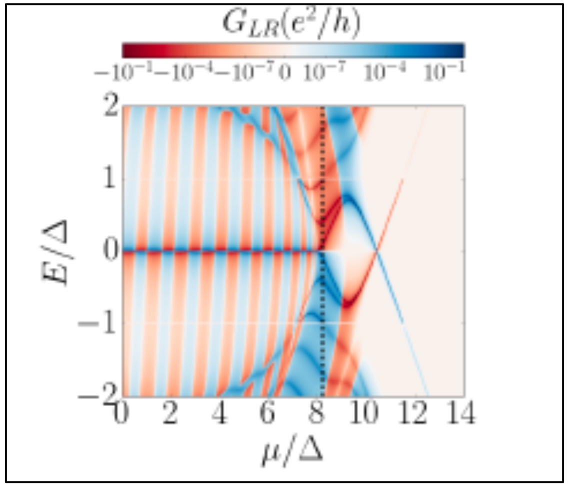








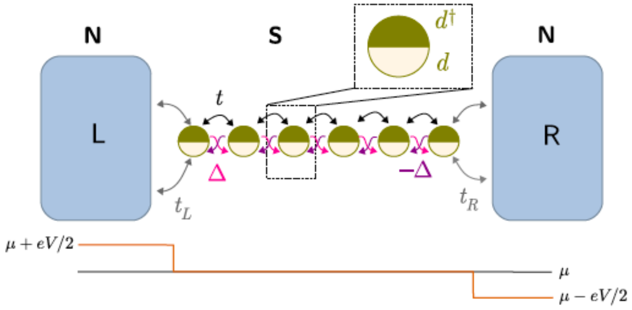




$$G_{LL} = \frac{e^2}{h}T_A^e(E=eV_L) + \frac{e^2}{h}T_A^e(E=-eV_L) + \frac{e^2}{h}T_{CA}^e(E=-eV_L) + \frac{e^2}{h}T_D^e(E=eV_L),$$

$$G_{LR} = \frac{e^2}{h}T_D^e(E=eV_R) - \frac{e^2}{h}T_{CA}^e(E=-eV_R)$$

Recap …



$$[G] = \begin{pmatrix} G_{LL} & G_{LR} \\ G_{RL} & G_{RR} \end{pmatrix} = \begin{pmatrix} \left.\frac{\partial I_L}{\partial V_L}\right|_{V_R=0} & \left.\frac{\partial I_L}{\partial V_R}\right|_{V_L=0} \\ \left.\frac{\partial I_R}{\partial V_L}\right|_{V_R=0} & \left.\frac{\partial I_R}{\partial V_R}\right|_{V_L=0} \end{pmatrix}$$

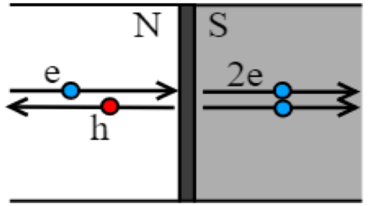
$$G_{LL} = \frac{e^2}{h} T_A^e(E = eV_L) + \frac{e^2}{h} T_A^e(E = -eV_L) + \frac{e^2}{h} T_{CA}^e(E = -eV_L) + \frac{e^2}{h} T_D^e(E = eV_L),$$

$$G_{LR} = \frac{e^2}{h} T_D^e(E = eV_R) - \frac{e^2}{h} T_{CA}^e(E = -eV_R)$$

$$I_L^e = \int dE \frac{e}{h} \left(T_D^e(E) \left[f_L^{ee}(E - eV_L) - f_R^{ee}(E - eV_R)\right]\right) + \int dE \frac{e}{h} \left(T_A^e(E) \left[f_L^{ee}(E - eV_L) - f_L^{hh}(E + eV_L)\right]\right) + \int dE \frac{e}{h} \left(T_{CA}^e(E) \left[f_L^{ee}(E - eV_L) - f_R^{hh}(E + eV_R)\right]\right)$$

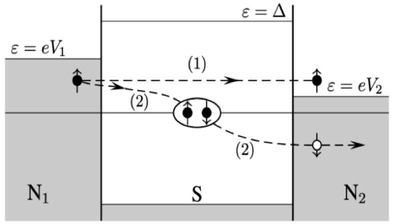
- Many issues regarding false positives
- Local conductance prone to such issues
- Non-local conductance may lead to conclusive sighting
- No conclusive experiment yet.
- Possible clues from entropic arguments

Andreev

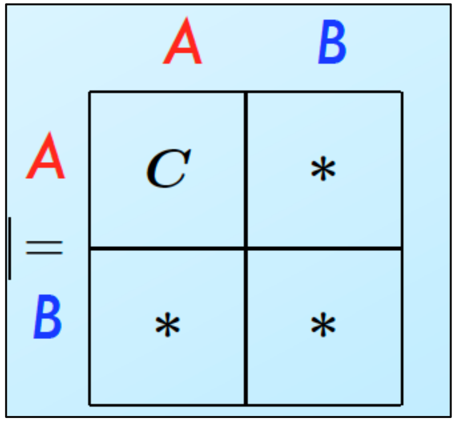


Andreev reflection

Crossed Andreev



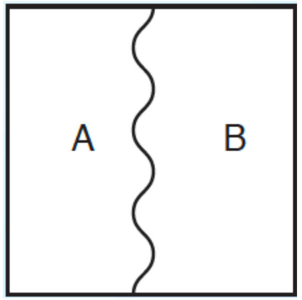
The von-Neumann entropy



$$\hat{H} = -\sum_{i=1}^{N}(\mu + V(i))c_i^\dagger c_i + \sum_{i=1}^{N-1}\left(\Delta c_i^\dagger c_{i+1}^\dagger - t c_{i+1}^\dagger c_i + \text{h.c.}\right)$$

$$\text{System} = A \oplus B$$

$$\text{State} = \sum \Psi_A \otimes \Psi_B$$



$$\langle c_j^\dagger c_k \rangle = (P)_{kj}$$

$$|\Psi_{AB}\rangle = \frac{1}{\sqrt{D}} \sum_j^D |\Psi_A^i\rangle \otimes |\Psi_B^i\rangle$$

$$\rho_{AB} = \frac{1}{D}\sum_{jk}^{D} |\Psi_A^j\rangle\langle\Psi_A^k| \otimes |\Psi_B^j\rangle\langle\Psi_B^k|$$

$$C_{nm} = \begin{pmatrix} \langle c^\dagger c \rangle & \langle c^\dagger c^\dagger \rangle \\ \langle cc \rangle & \langle c c^\dagger \rangle \end{pmatrix}_{nm}$$

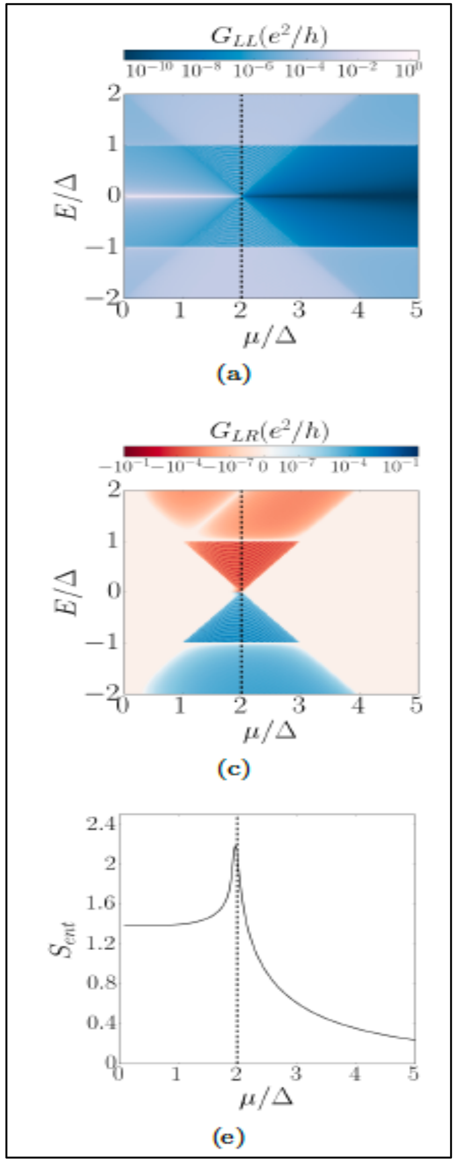
$$\rho_{AB} = |\Psi_{AB}\rangle\langle\Psi_{AB}|$$

$$\rho_A = \mathrm{Tr}_B\, \rho_{AB}$$

$$= \frac{1}{D}\sum_j^D |\Psi_A^j\rangle\langle\Psi_A^j|$$

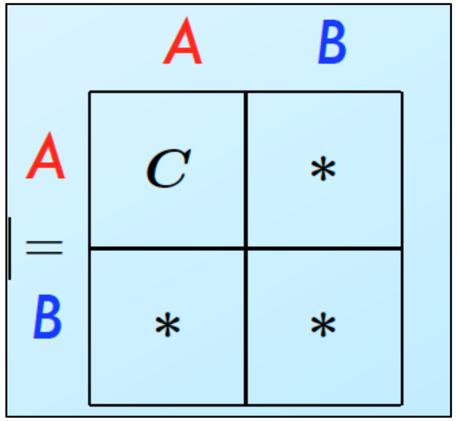
Pure State $D = 1$ → *Mixed State*

$$S_{ent} = -\sum_{k=1}^{M}\left[\xi_k \log \xi_k + (1-\xi_k)\log(1-\xi_k)\right]$$



Ryu and Hatsugai., PRB, 73, 245115 (2006)
Hegde et.al., ArXiv: 2108.1460, (2021)

The von-Neumann entropy



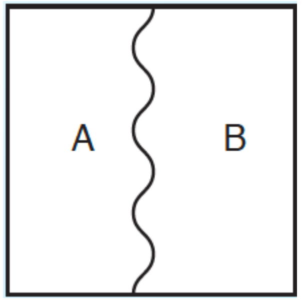
$$\langle c_j^\dagger c_k \rangle = (\boldsymbol{P})_{kj}$$

$$C_{nm} = \begin{pmatrix} \langle c^\dagger c \rangle & \langle c^\dagger c^\dagger \rangle \\ \langle cc \rangle & \langle c c^\dagger \rangle \end{pmatrix}_{nm}$$

$$S_{ent} = -\sum_{k=1}^{M} \left[\xi_k \log \xi_k + (1-\xi_k) \log (1-\xi_k)\right]$$

$$\hat{H} = -\sum_{i=1}^{N} (\mu + V(i)) c_i^\dagger c_i + \sum_{i=1}^{N-1} \left(\Delta c_i^\dagger c_{i+1}^\dagger - t c_{i+1}^\dagger c_i + \text{h.c.} \right)$$

$$\text{System} = A \oplus B$$
$$\text{State} = \sum \Psi_A \otimes \Psi_B$$



$$|\Psi_{AB}\rangle = \frac{1}{\sqrt{D}} \sum_j^D |\Psi_A^i\rangle \otimes |\Psi_B^i\rangle$$

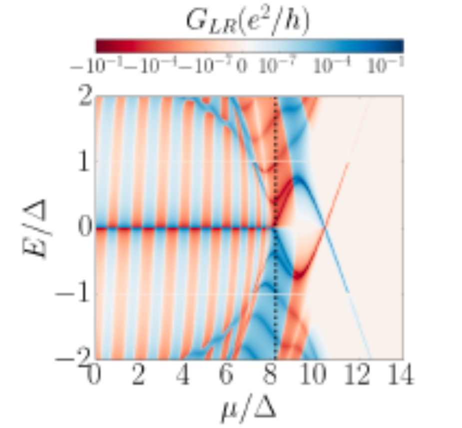
$$\rho_{AB} = \frac{1}{D} \sum_{jk}^{D} |\Psi_A^j\rangle\langle\Psi_A^k| \otimes |\Psi_B^j\rangle\langle\Psi_B^k|$$

$$\rho_{AB} = |\Psi_{AB}\rangle\langle\Psi_{AB}|$$
$$\rho_A = \text{Tr}_B\, \rho_{AB}$$
$$= \frac{1}{D} \sum_{j}^{D} |\Psi_A^j\rangle\langle\Psi_A^j|$$

Pure State

$D = 1$

Mixed State

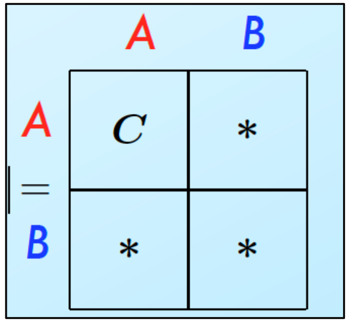


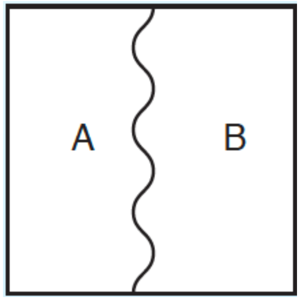
(b)

(d)

Ryu and Hatsugai., PRB, 73, 245115 (2006)
Hegde et.al., ArXiv: 2108.1460, (2021)

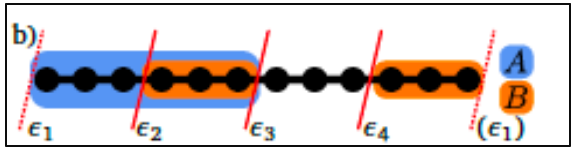
The Topological entanglement entropy





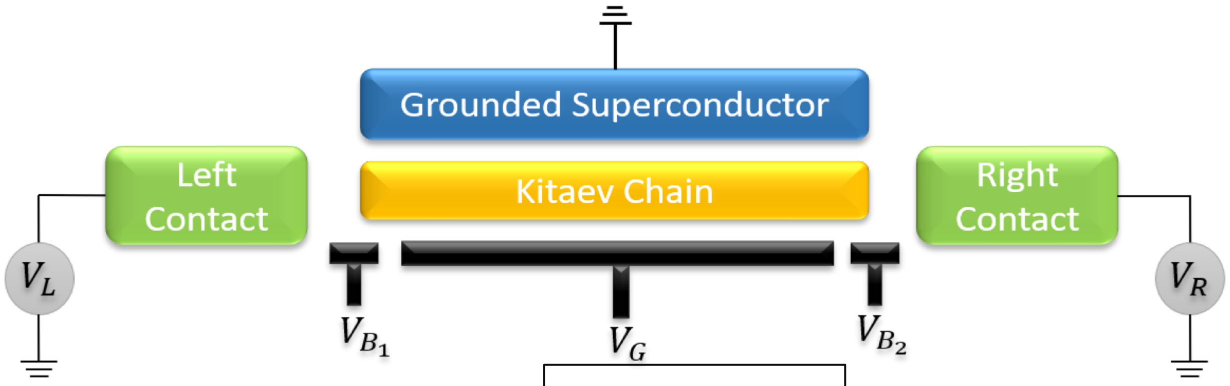
$$\langle c_j^\dagger c_k \rangle = (P)_{kj}$$

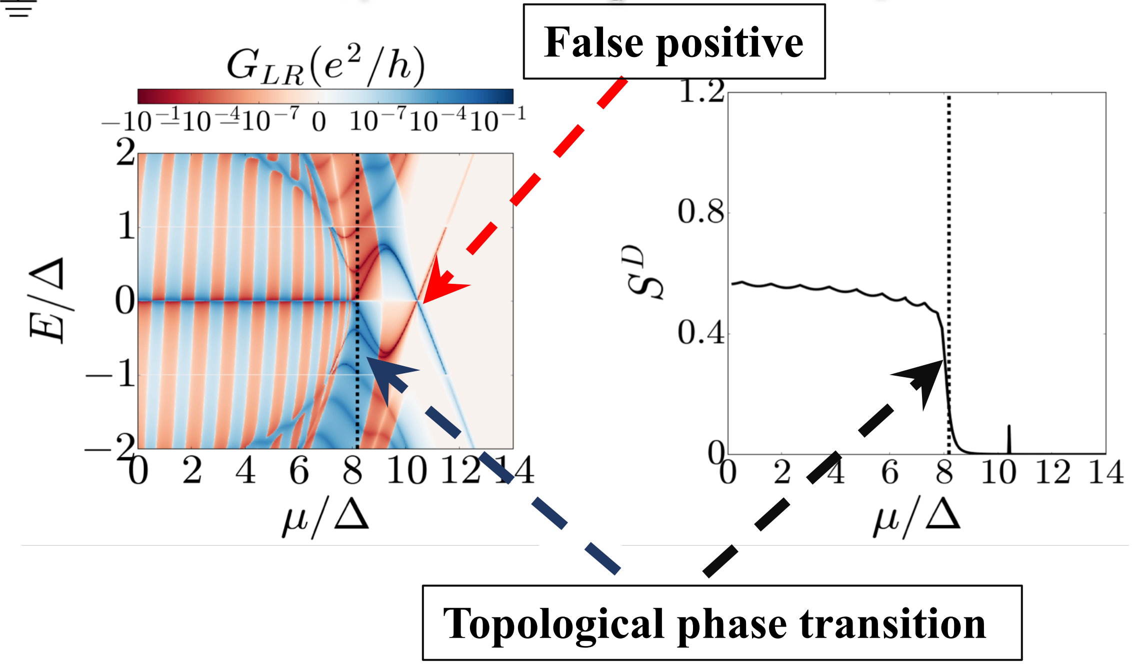
$$S_{ent} = -\sum_{k=1}^{M} [\xi_k \log \xi_k + (1-\xi_k) \log (1-\xi_k)]$$



$$S^D = S_A + S_B - S_{A\cup B} - S_{A\cap B}$$

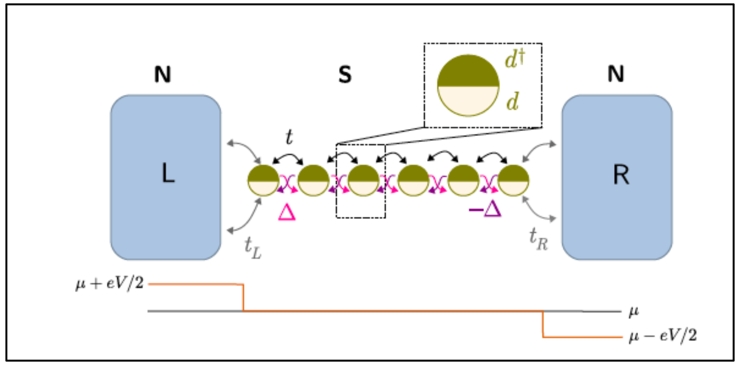
A.Kejriwal and **B. Muralidharan,** Phys Rev B (Letter), 105, 161403 (2022). [Editors' suggestion]





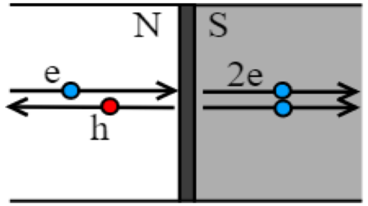
Ryu and Hatsugai., PRB, 73, 245115 (2006)
Sci. Post Phys. Core, 3, 12 (2020).

Recap …



$$[\mathrm{G}] = \begin{pmatrix} G_{LL} & G_{LR} \\ G_{RL} & G_{RR} \end{pmatrix} = \begin{pmatrix} \left.\frac{\partial I_L}{\partial V_L}\right|_{V_R=0} & \left.\frac{\partial I_L}{\partial V_R}\right|_{V_L=0} \\ \left.\frac{\partial I_R}{\partial V_L}\right|_{V_R=0} & \left.\frac{\partial I_R}{\partial V_R}\right|_{V_L=0} \end{pmatrix}$$

Andreev



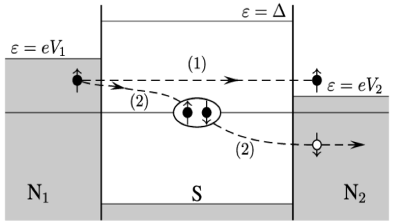
Andreev reflection

$$G_{LL} = \frac{e^2}{h}T_A^e(E=eV_L) + \frac{e^2}{h}T_A^e(E=-eV_L) + \frac{e^2}{h}T_{CA}^e(E=-eV_L) + \frac{e^2}{h}T_D^e(E=eV_L),$$

$$G_{LR} = \frac{e^2}{h}T_D^e(E=eV_R) - \frac{e^2}{h}T_{CA}^e(E=-eV_R)$$

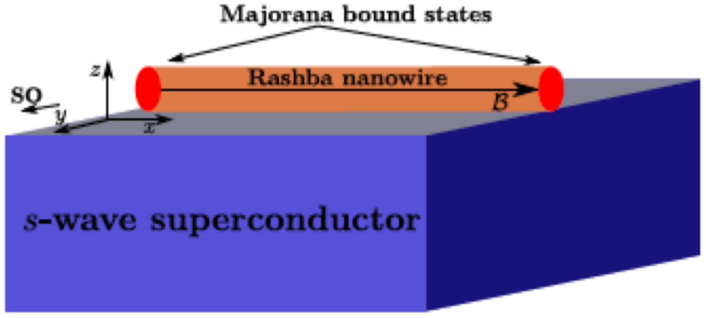
$$\begin{aligned} I_L^e = &\int dE \frac{e}{h}\left(T_D^e(E)\left[f_L^{ee}(E-eV_L) - f_R^{ee}(E-eV_R)\right]\right) \\ &+ \int dE \frac{e}{h}\left(T_A^e(E)\left[f_L^{ee}(E-eV_L) - f_L^{hh}(E+eV_L)\right]\right) \\ &+ \int dE \frac{e}{h}\left(T_{CA}^e(E)\left[f_L^{ee}(E-eV_L) - f_R^{hh}(E+eV_R)\right]\right) \end{aligned}$$

Crossed Andreev



- Many issues regarding false positives
- Local conductance prone to such issues
- Non-local conductance may lead to conclusive sighting
- No conclusive experiment yet.
- Possible clues from entropic arguments

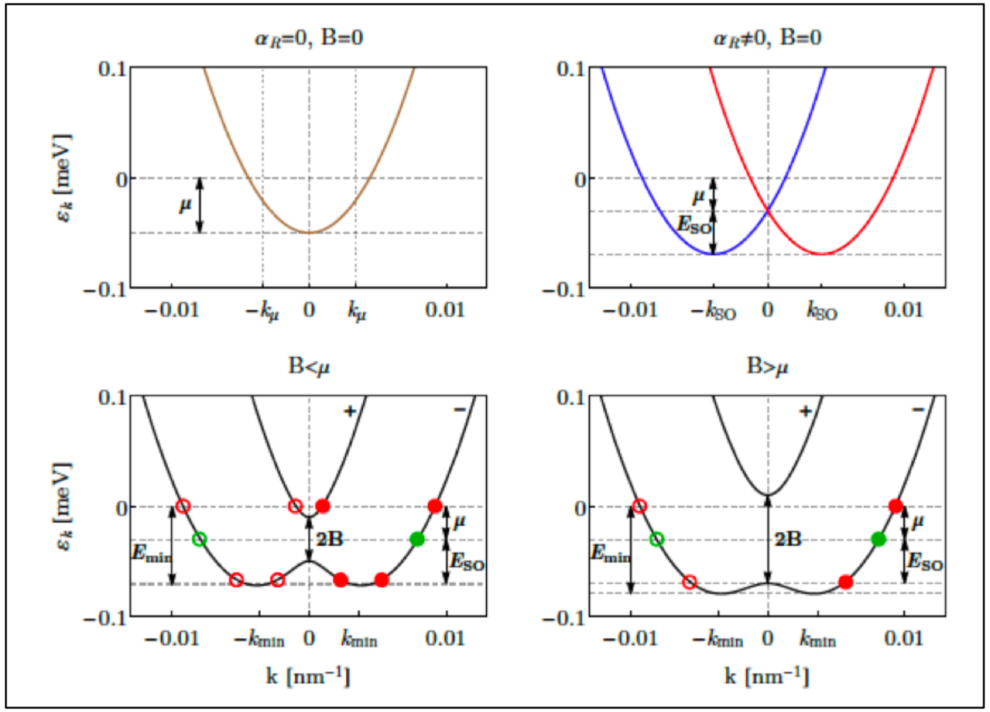
Rashba Nanowire in the BdG form

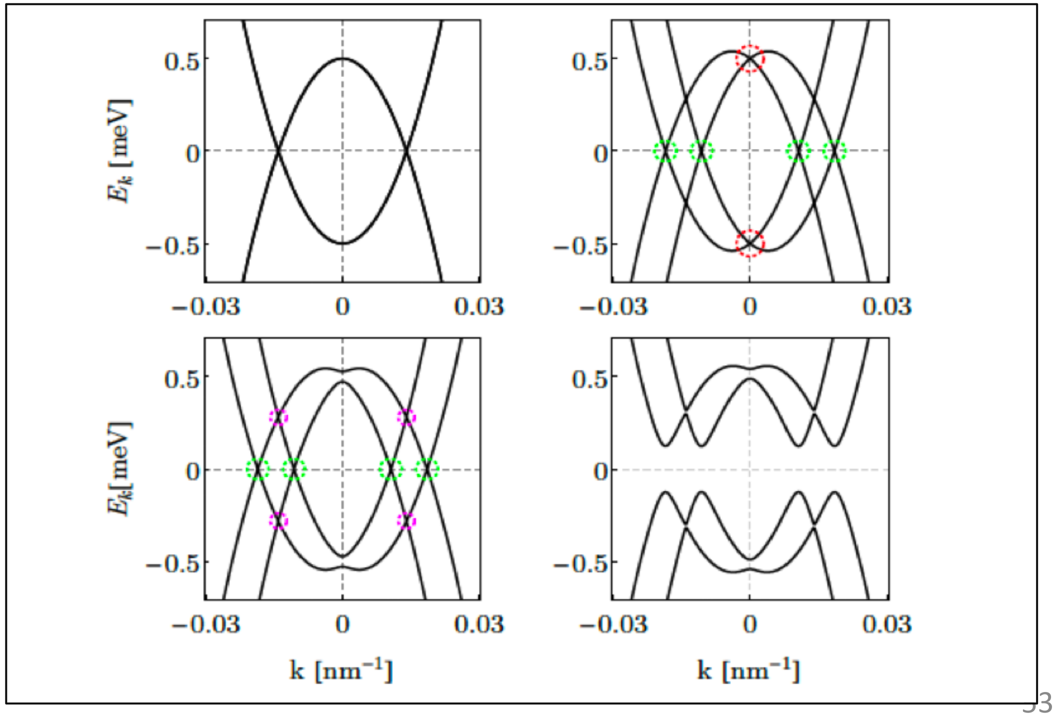


$$H_0 = \frac{p_x^2}{2m} - \mu - \frac{\alpha_R}{\hbar}\sigma_y p_x + \frac{g\mu_B}{2}\sigma_x B$$

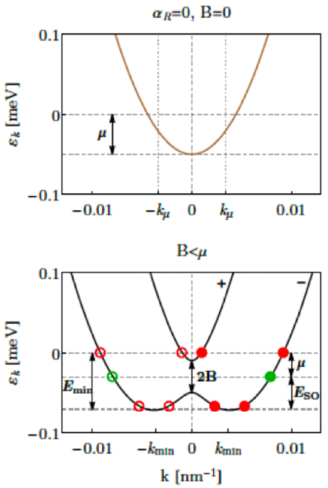
$$H_{BdG} = \begin{pmatrix} H_0 & -i\Delta\sigma_y \\ i\Delta\sigma_y & -H_0^* \end{pmatrix}$$

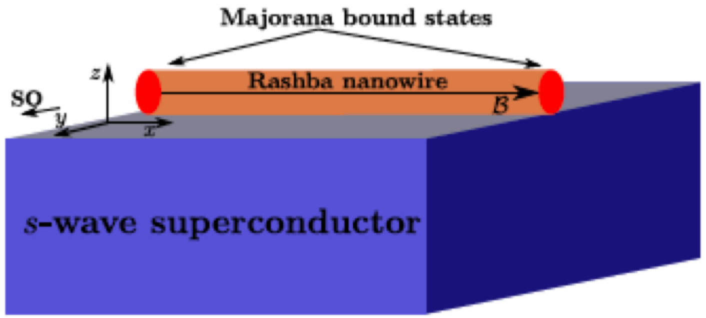
$$\mathcal{H} = \frac{1}{2}\int dx\, \Psi_k^\dagger H_{BdG}\, \Psi_k\,, \qquad \Psi_k = \left(\psi_\uparrow(k), \psi_\downarrow(k), \psi_\uparrow^\dagger(-k), \psi_\downarrow^\dagger(-k)\right)^\dagger$$

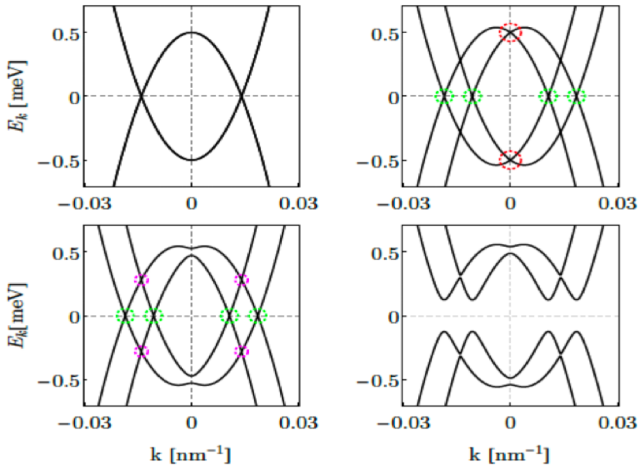


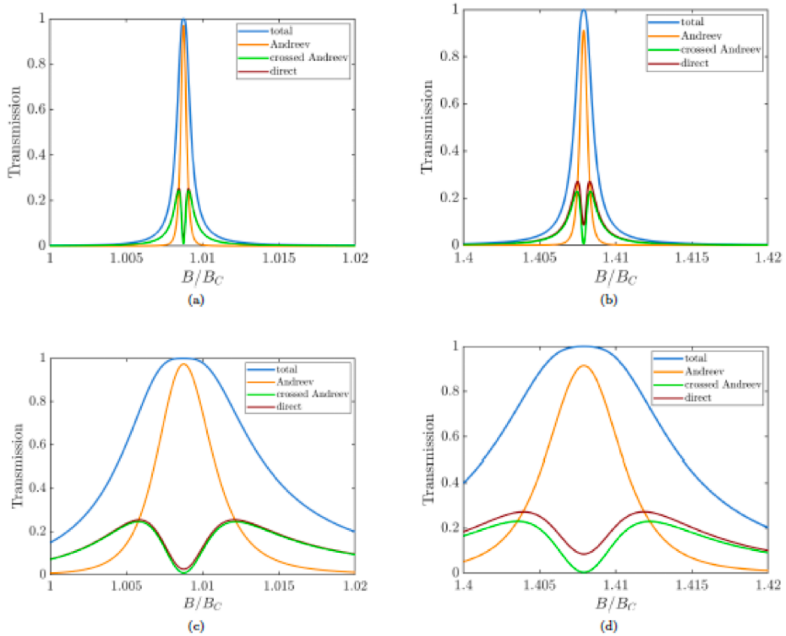


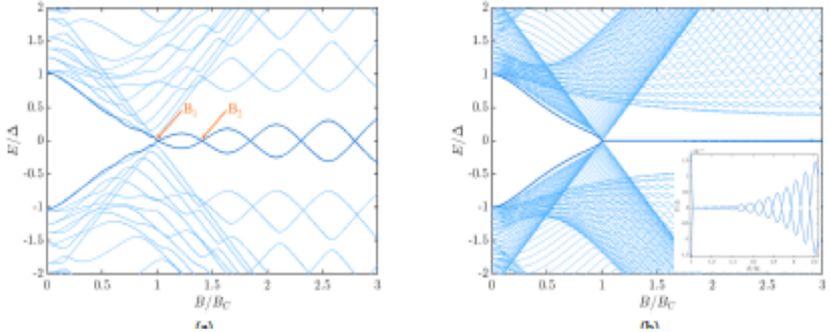
Rashba Nanowire NEGF

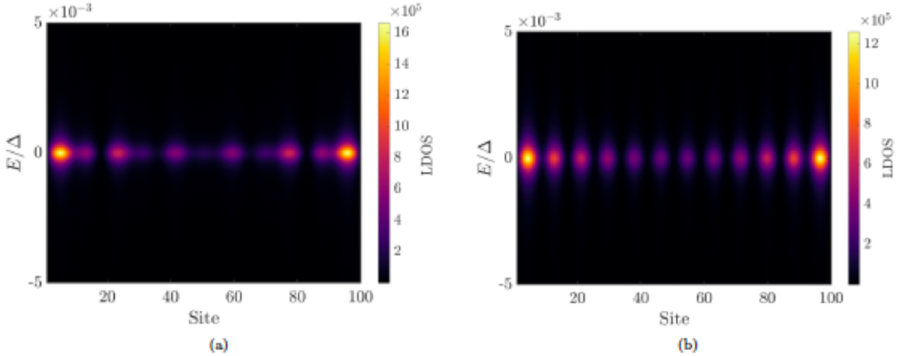






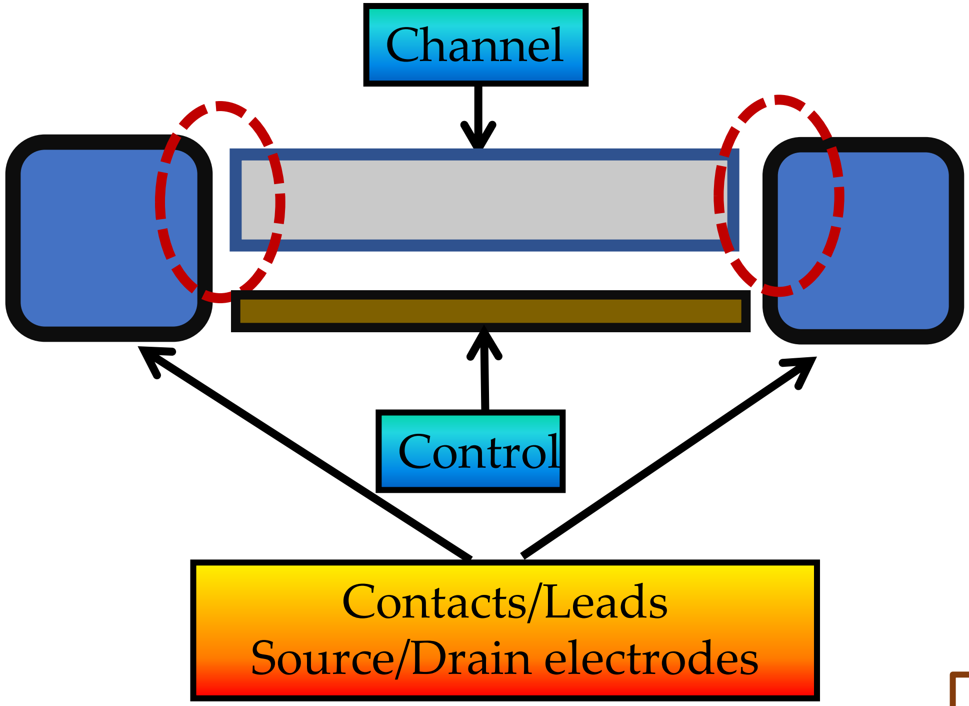




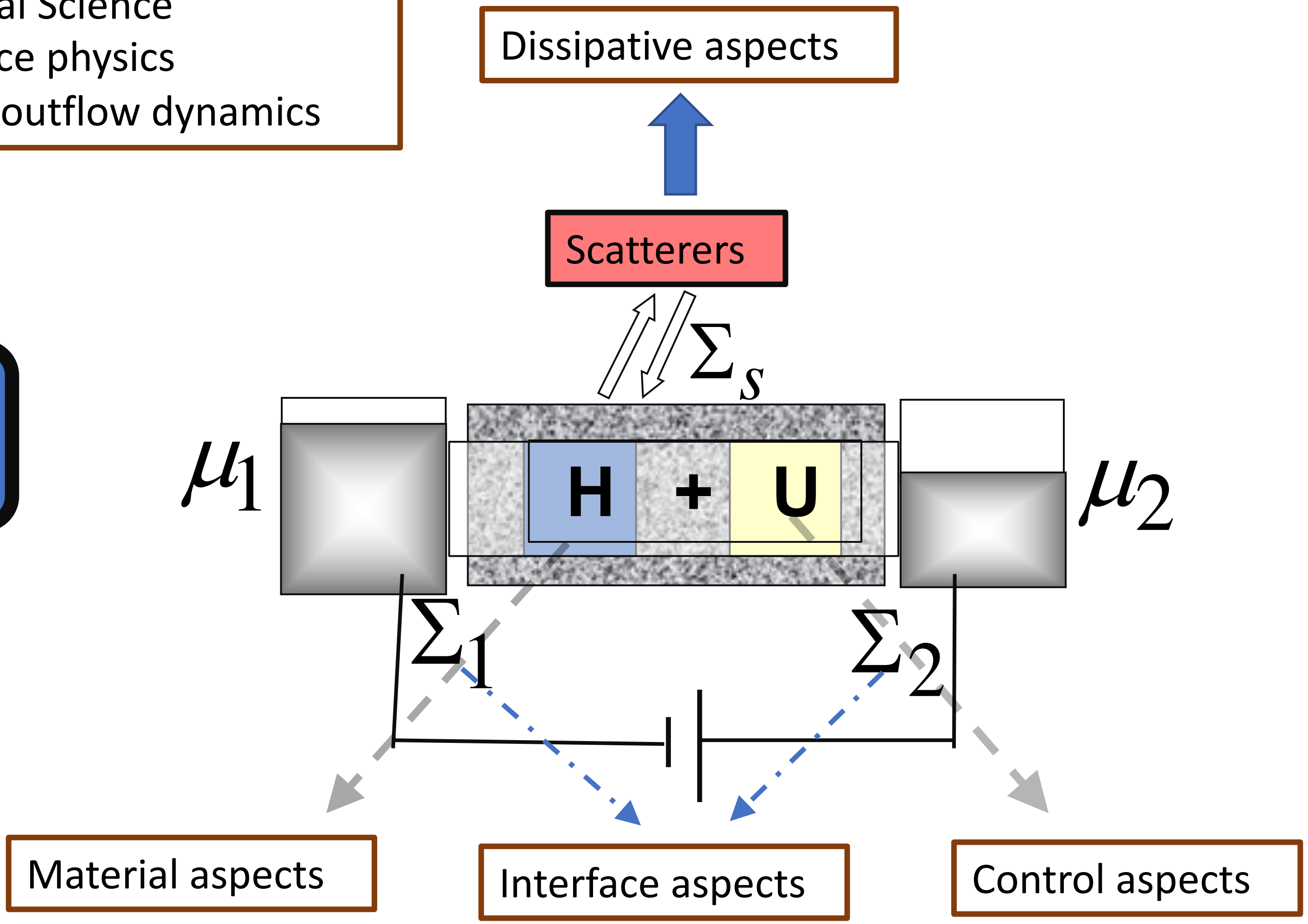


JPCM, 33, 365301, (2021)

Anatomy of a Nano-Device: Non-equilibrium Green's function formalism

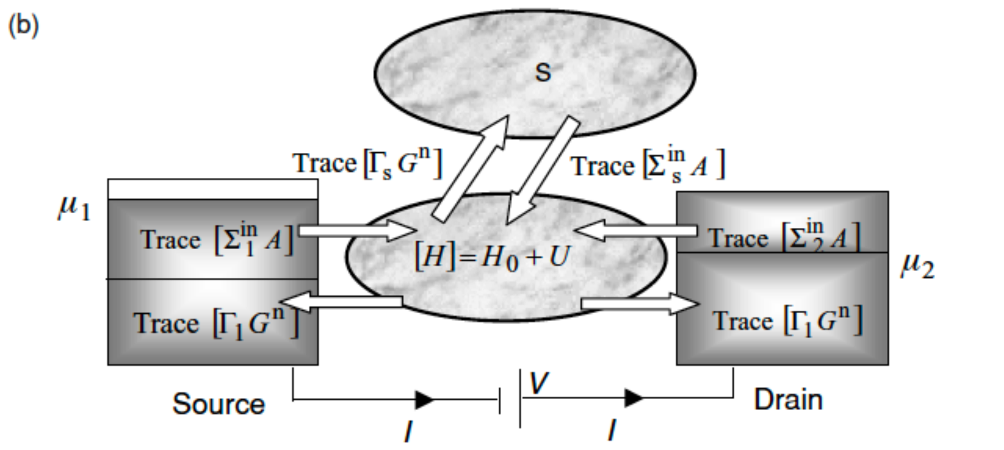


Material Science
Interface physics
Inflow-outflow dynamics



Datta, Quantum Transport, Cambridge Univ Press (2005)

Scattering self-energy



$$G = [EI - H_0 - U - \Sigma]^{-1}$$
$$A = i[G - G^+] \qquad \Gamma = i[\Sigma - \Sigma^+]$$

$$\Sigma^{in} = \Sigma_1^{in} + \Sigma_2^{in} + \Sigma_s^{in}$$
$$\Sigma = \Sigma_1 + \Sigma_2 + \Sigma_s$$

$$G^n = G\,\Sigma^{in}\,G^+$$

$$\tilde{I}_i = \text{Trace}[\Sigma_i^{in} A] - \text{Trace}[\Gamma_i G^n]$$

$$\rho = \int dE\; G^n(E)/2\pi$$

$$I_i = (q/\hbar) \int_{-\infty}^{+\infty} dE\, \tilde{I}_i\,(E)/2\pi$$

For dephasing interactions

$$\Sigma_s^{in}(E) = D_0 G^n(E)$$
$$\Gamma_s(E) = D_0 A(E)$$
$$\Sigma_s(E) = D_0 G(E)$$

For any inelastic interaction in general:

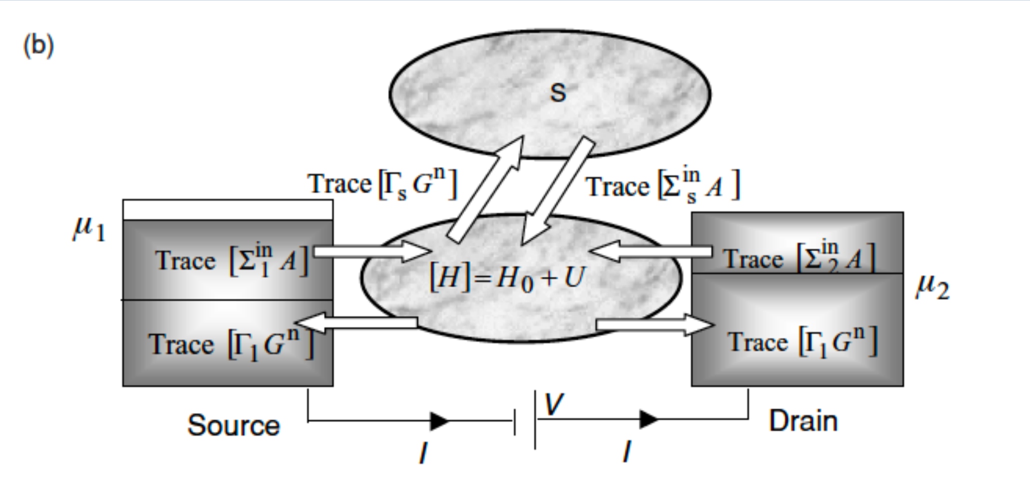
$$\Sigma_s^{in}(E) = \int_0^{\infty} \frac{d(\hbar\omega)}{2\pi} \begin{pmatrix} D^{em}(\hbar\omega) \cdot G^n(E + \hbar\omega) \\ + D^{ab}(\hbar\omega) \cdot G^n(E - \hbar\omega) \end{pmatrix}$$

$$\Gamma_s(E) = \int_0^{\infty} \frac{d(\hbar\omega)}{2\pi} \begin{pmatrix} D^{em}(\hbar\omega) \cdot [G^p(E - \hbar\omega) + G^n(E + \hbar\omega)] \\ + D^{ab}(\hbar\omega) \cdot [G^n(E - \hbar\omega) + G^p(E + \hbar\omega)] \end{pmatrix}$$

Single-frequency inelastic processes

$$\Sigma_s^{in}(E) = D_0^{em} G^n(E + \hbar\omega_0) + D_0^{ab} G^n(E - \hbar\omega_0)$$
$$\Gamma_s(E) = D_0^{em}[G^p(E - \hbar\omega_0) + G^n(E + \hbar\omega_0)]$$
$$+ D_0^{ab}[G^n(E - \hbar\omega_0) + G^p(E + \hbar\omega_0)]$$

Dephasing Self-Energies



$$[\Sigma_s^r]_{ij} = D_{ijkl}[\mathbf{G}^r]_{kl},$$
$$[\Sigma_s^<]_{ij} = D_{ijkl}[\mathbf{G}^<]_{kl},$$

$$\bar{D}(i,j) = \langle U_s(i) U_s^*(j) \rangle,$$
$$\bar{D}(i,j) = D_m \delta_{ij}.$$

Phase+momentum

$$D_{ijkl} = D_M \delta_{ij}\delta_{ik}\delta_{jl},$$

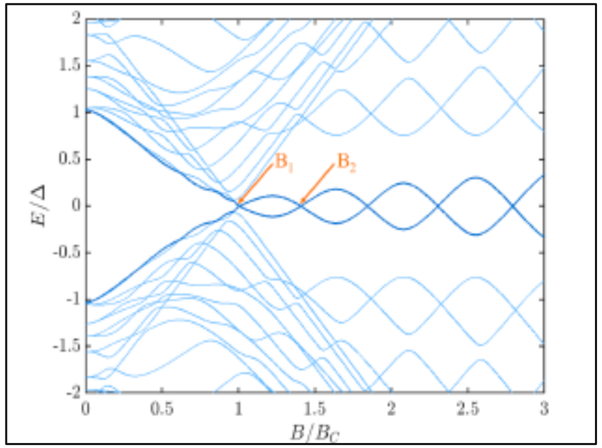
Pure phase

$$D_{ijkl} = D_P \delta_{ik}\delta_{jl}$$

Spin dephasing

$$[\Sigma_s^r]_{ij} = D_S \left(\sigma_x \mathbf{G}_{i,j}^r \sigma_x + \sigma_y \mathbf{G}_{i,j}^r \sigma_y + \sigma_z \mathbf{G}_{i,j}^r \sigma_z\right)$$
$$[\Sigma_s^<]_{ij} = D_S \left(\sigma_x \mathbf{G}_{i,j}^< \sigma_x + \sigma_y \mathbf{G}_{i,j}^< \sigma_y + \sigma_z \mathbf{G}_{i,j}^< \sigma_z\right).$$

A. Yanik et.al., Phys Rev B, 76,045203, (2007)

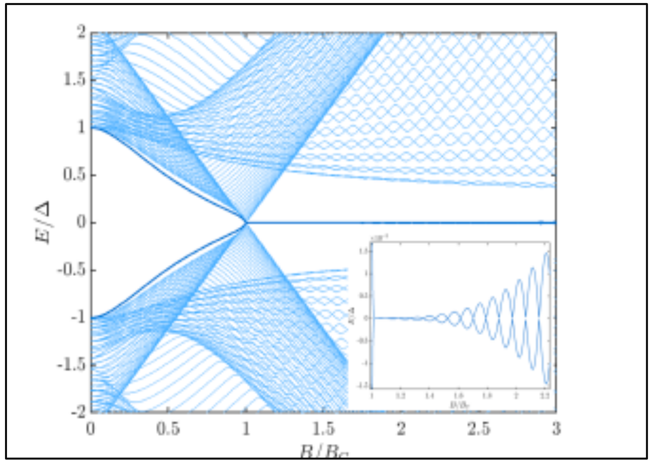


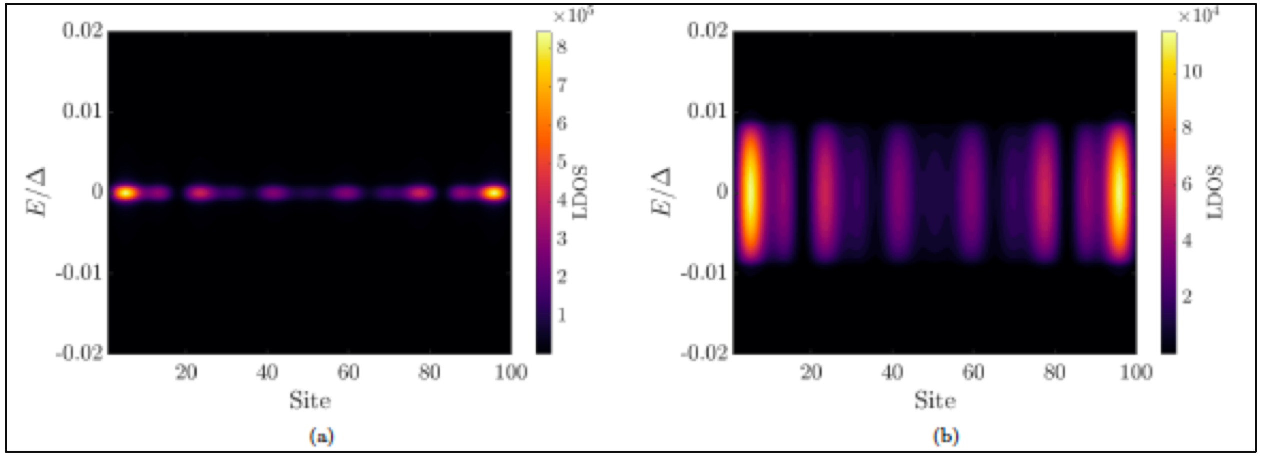
Dephasing

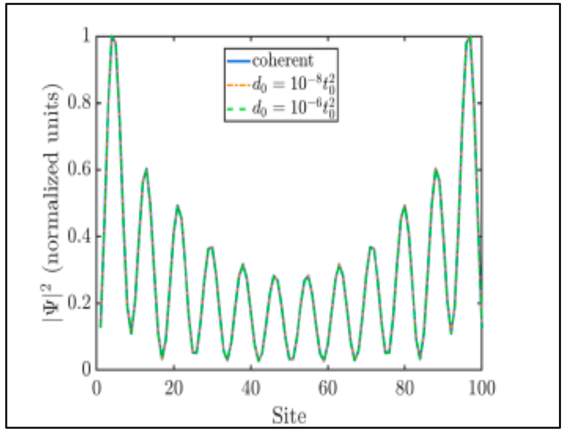
$$\bar{D}(i,j) \propto \langle U_s(i) U_s^*(j) \rangle$$

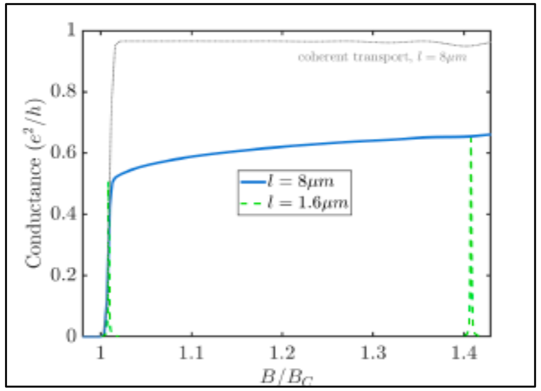
$$\bar{D}(i,j) = d_m \delta_{ij} \quad \text{(“Momentum relaxing”)}$$

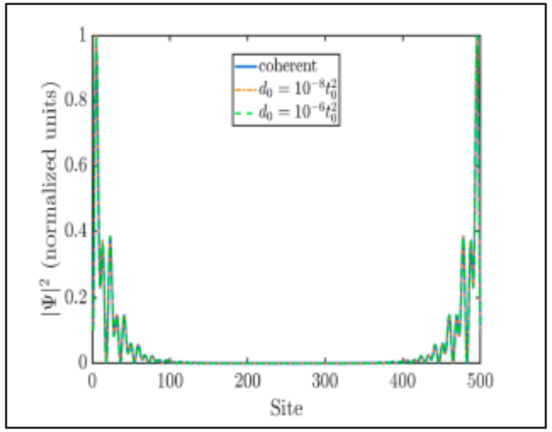
$$G(E) = (EI - H - \Sigma_L - \Sigma_R - \Sigma_s)^{-1}$$





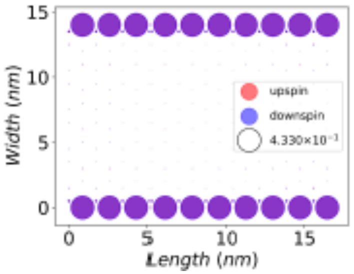


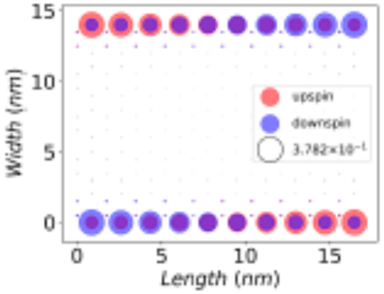


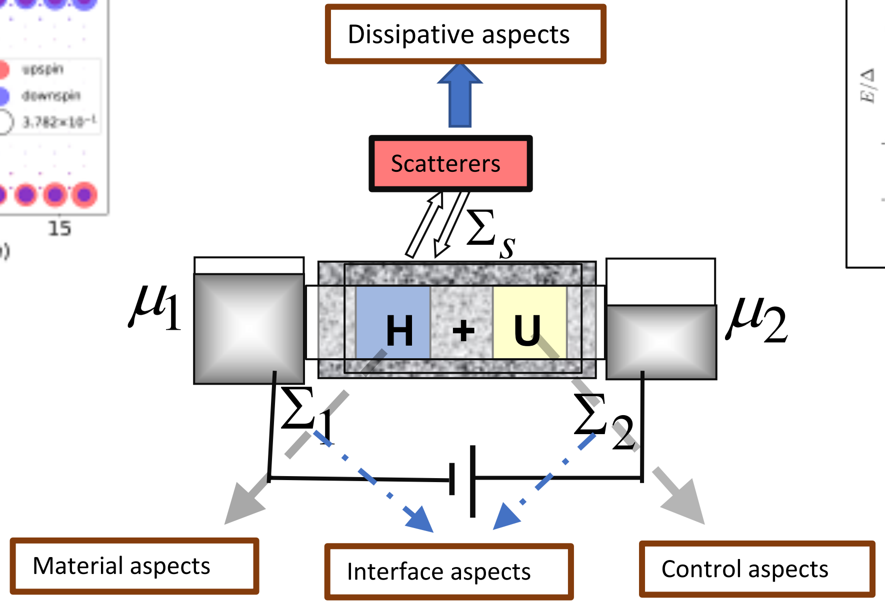


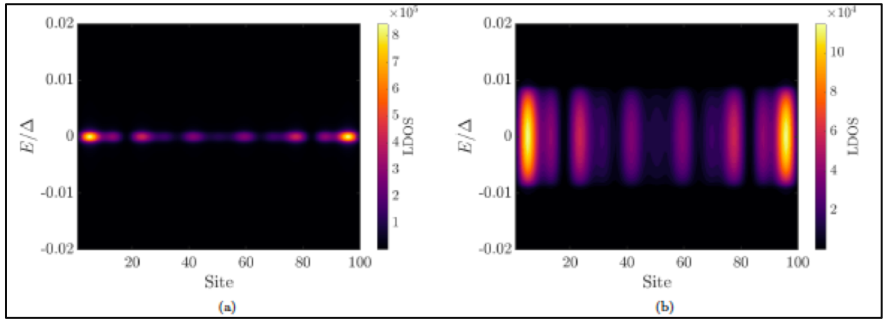
JPCM, 33, 365301, (2021)

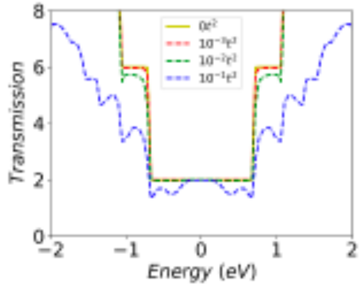
Dephasing Interactions

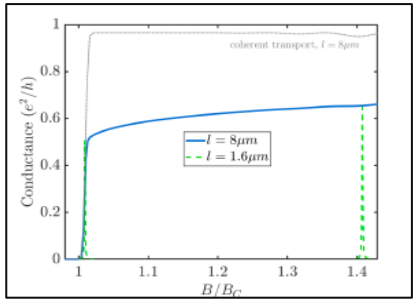












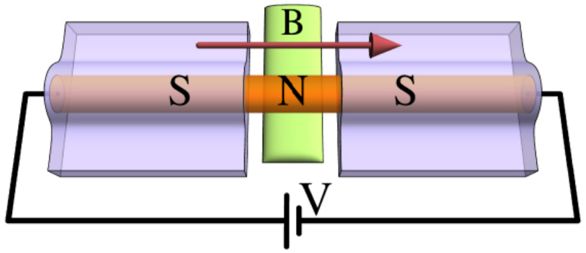
$$\bar{D}(i,j) \propto \langle U_s(i) U_s^*(j) \rangle$$

$$\Sigma_s(i,j) = \bar{D}(i,j) G(i,j),$$

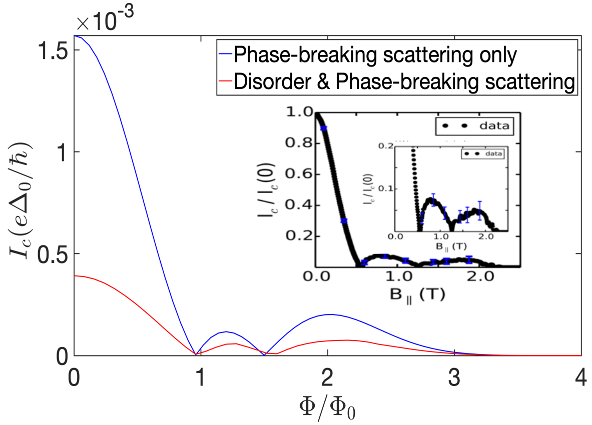
$$\Sigma_s^{in,out}(i,j) = \bar{D}(i,j) G^{n,p}(i,j).$$

$$\bar{D}(i,j) = d_m \delta_{ij} \quad \text{(“Momentum relaxing”)},$$

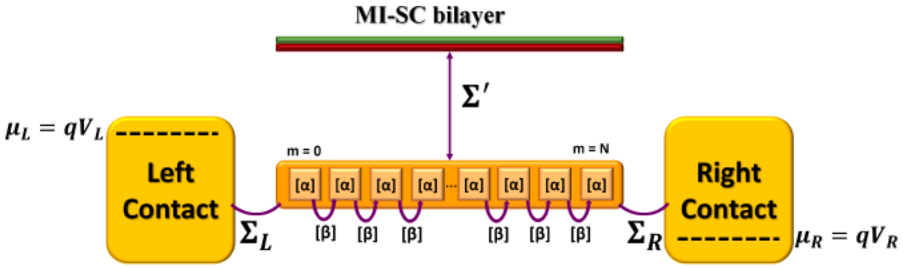
$$\bar{D}(i,j) = d_p \quad \text{for all } i,j \quad \text{(“Momentum conserving”)}$$

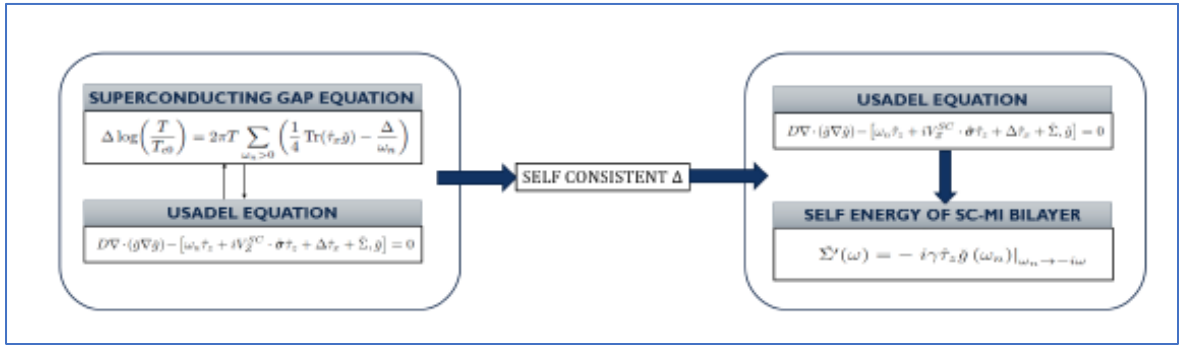


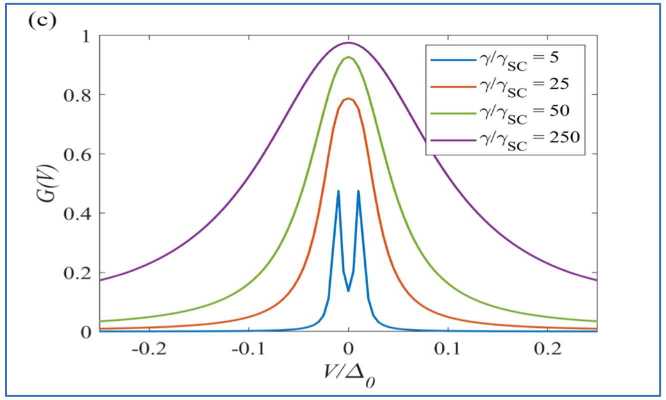
Phys Rev B, 100, 155431, (2019)
Phys Rev B, 98, 125417, (2018)
JPCM (2021), JPD (2021)

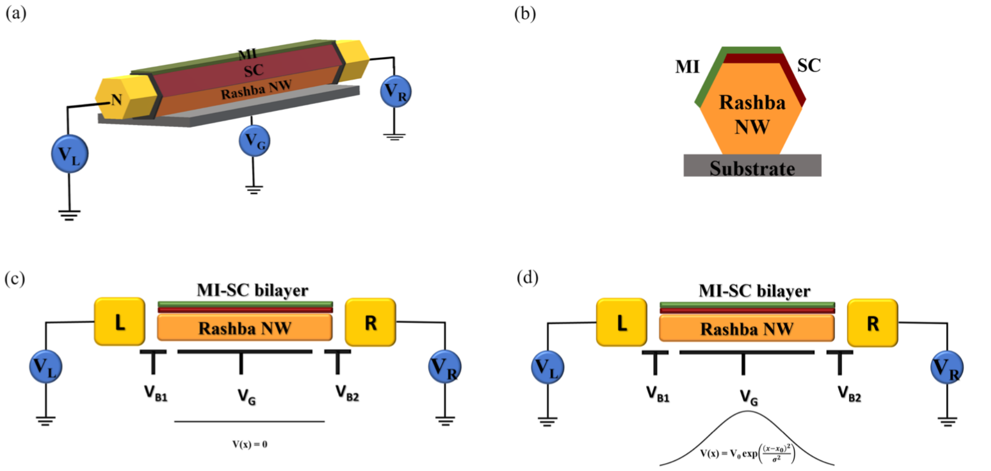


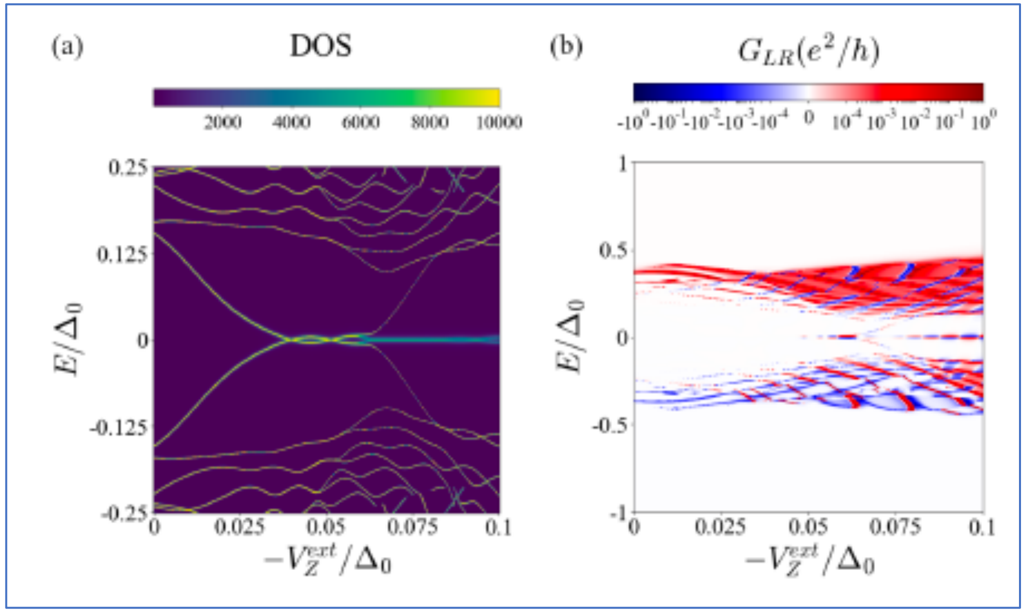
Magnetic insulator Hybrid NW







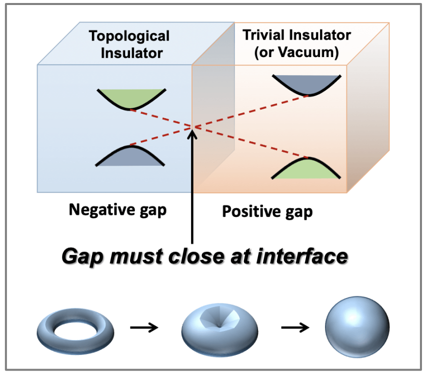




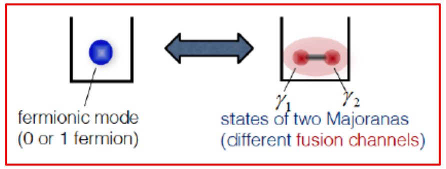
Poster Session!

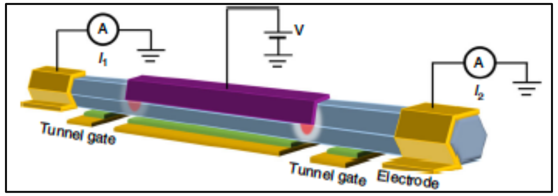
ArXiv:2203.08413, (2022)

What was this tutorial all about??

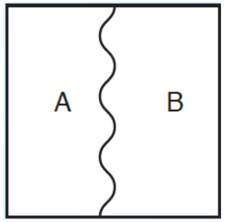


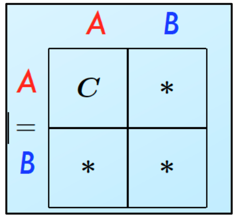
Topology and gap closure
Connection to Conductance

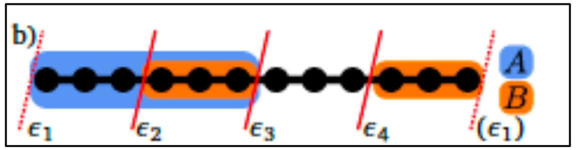




Transport spectroscopy
Detection of MZMs
Quantum transport
Gap closing and false postiives

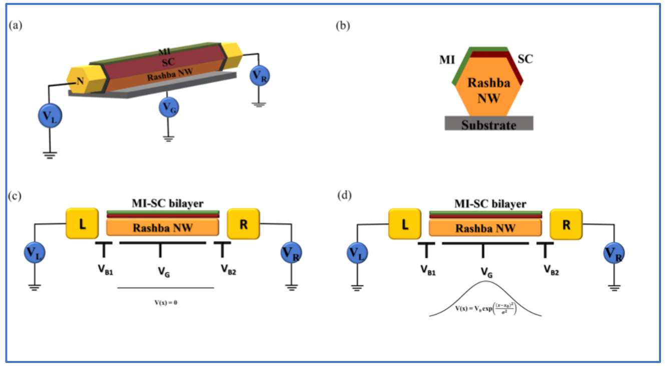


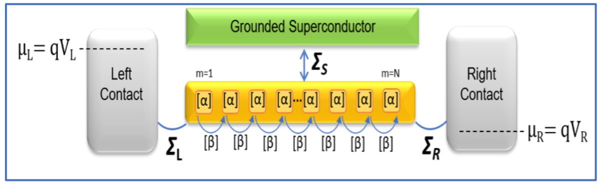




Quantum Materials
Entanglement Entropy

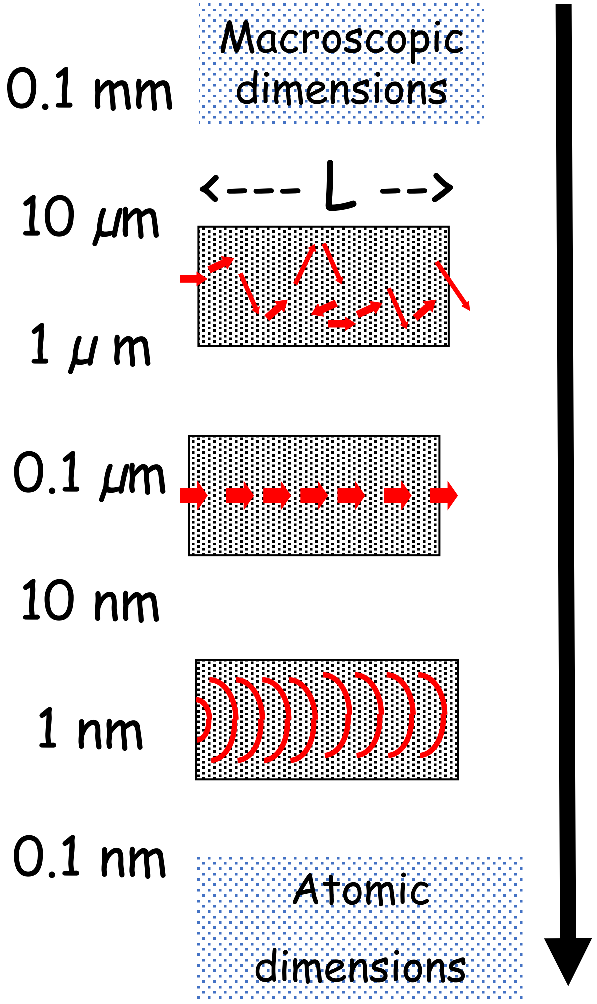
Advanced Device Modeling
Rashba nanowire systems
Dephasing
Magnetic Insulator hybrids

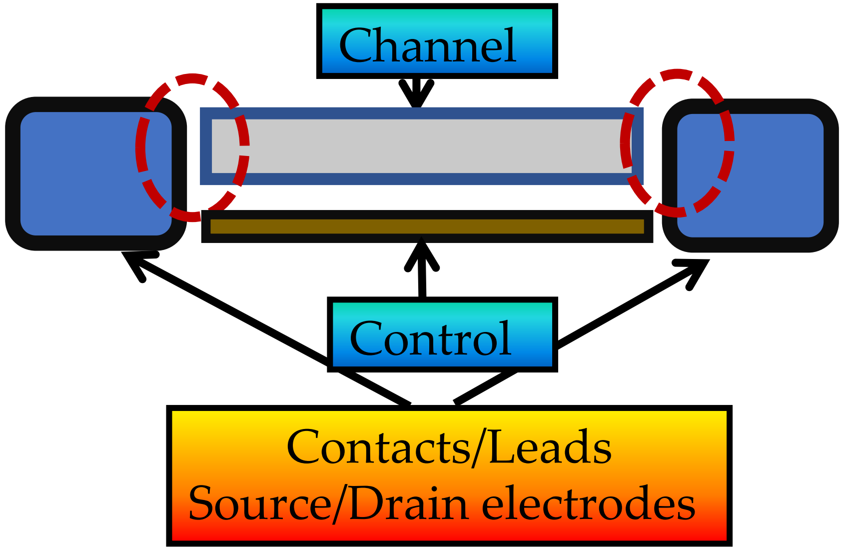


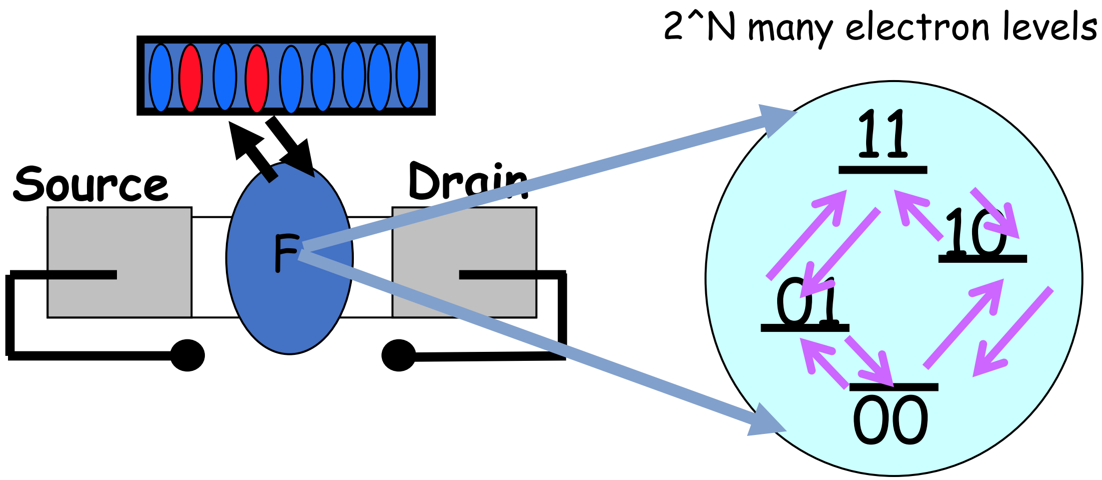


JPCM, 33, 365301, (2021)
PRB, 103, 165432, (2021)
PRB (L), 105,161403, (2022)
ArXiv: 2203.08413 (2022)

World of Quantum transport-> Many more explorations!

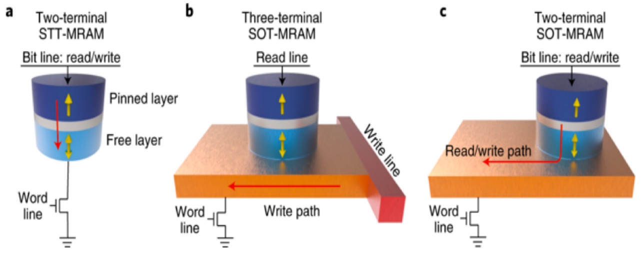






“Beyond Moore” Device Research Highlights

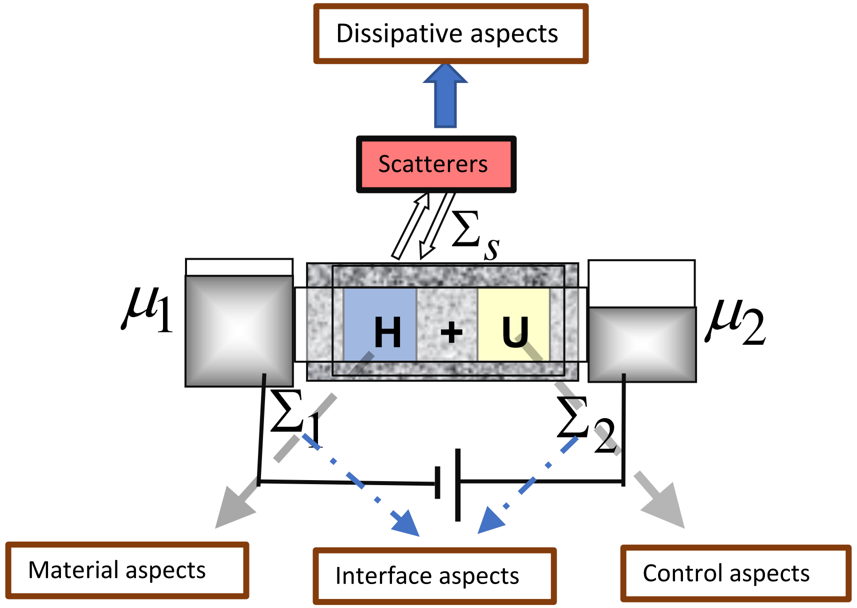
Spintronics: MRAM



Recent Publications
IEEE Trans.Elec Dev., 63, 4527-4563, (2016).
Phys. Rev. Applied, 8, 064014, (2017).
Appl. Phys. Lett., 112, 192404, (2018).
Phys. Rev. Applied, 12, 024038, (2019).
IEEE Trans. Nano., 19, 469, (2020).

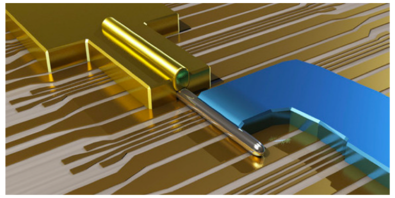
- Spin filtering devices
- STT-MRAM
- Toward Neuromorphic

- 2D topological spintronics
- Materials -> Devices -> Functionalities



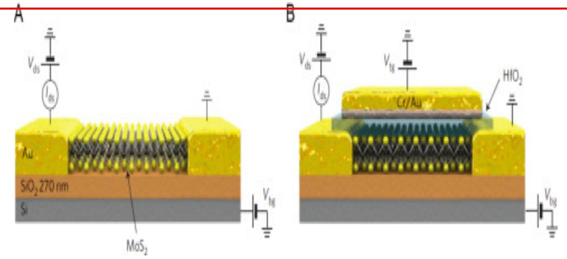
Unified Quantum Device Simulation Platform

Topological hybrid quantum systems



Recent Publications
Phys. Rev. B, 98, 125417, (2018).
Phys. Rev B, 100, 155431, (2019)
Phys. Rev. Research 2, 043430, (2020).
Phys. Rev B,103,165432,(2021).
Phys Rev B, 105, L161403, (2022)

2D Quantum Materials and Devices



Recent Publications
Phys. Rev. Applied, 10, 014022, (2018).
Phys. Rev. B, 99, 075415, (2019).
Phys. Rev. B (Rapid Comm), 100, 081403, (2019).
Phys. Rev. Materials, 3, 124005, (2019).
Phys. Rev. Research, 2, 043041, (2020)
npj 2D materials., 6, 19, (2022)

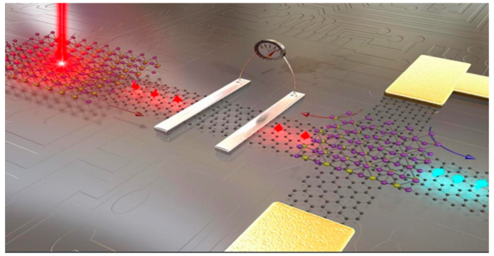
- Quantum Hall hybrid systems
- Straintronics
- Topotronics

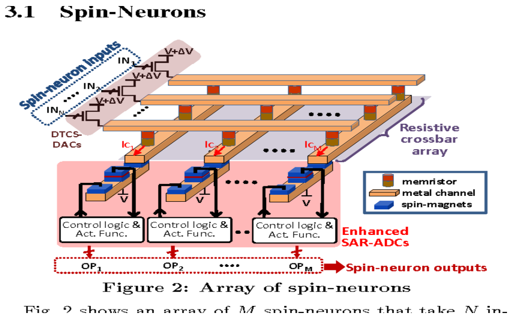
- 1-D Majorana devices
- Topological vs trivial
- Entropic signatures
- Magnetic insulator hybrids

Many more frontiers!

Spintronics

2D Spintronics
Neuromorphic spintronics
Modular spintronics-> Device to Circuits





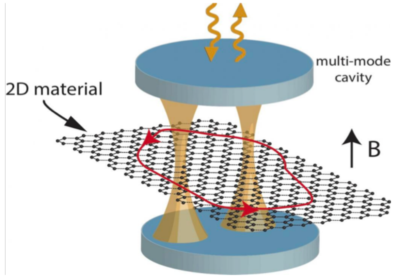
Expertise:
1) 2D Quantum materials and Devices
2) 2D Spintronics
3) Frontier areas like hybrid quantum devices for quantum technologies

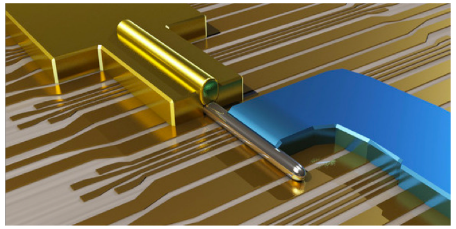
Research

Frontier areas ->Devices Structures -> Functionalities

Topotronics and Hybrid Quantum Devices

Majorana Braiding Architectures
Topological Quantum Computing





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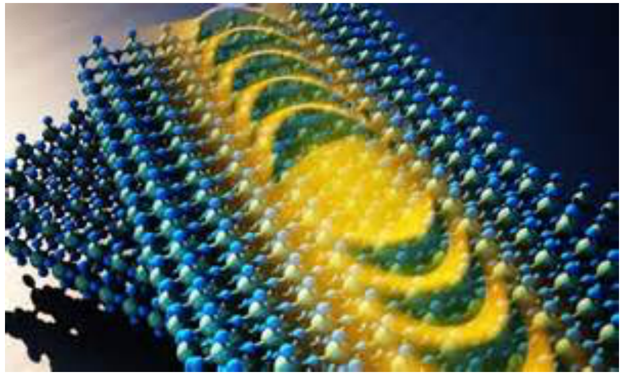
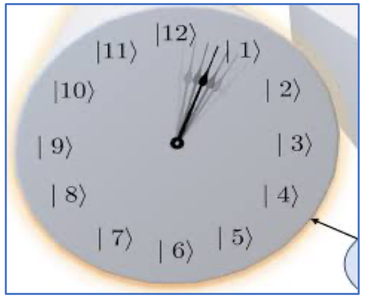
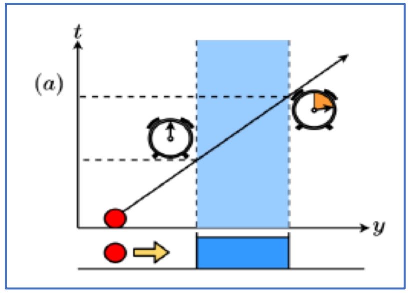
2D Electronics

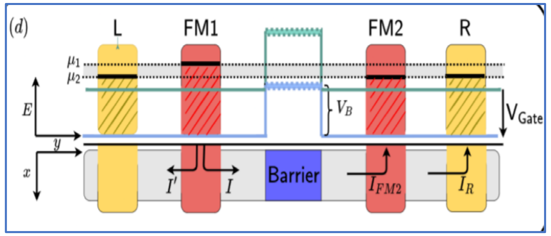
Valleytronics
Topotronics
Straintronics
Twistronics

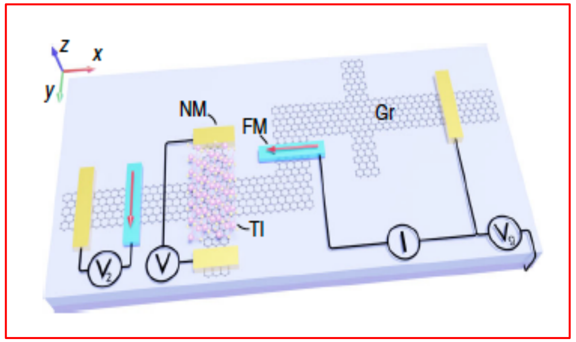


Novel Possibilities- Weak value amplification







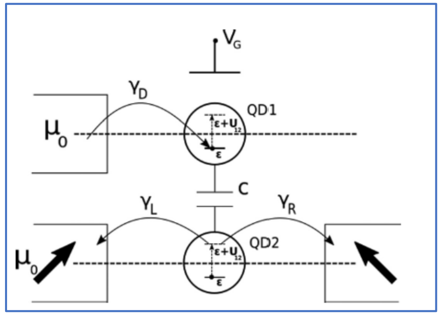


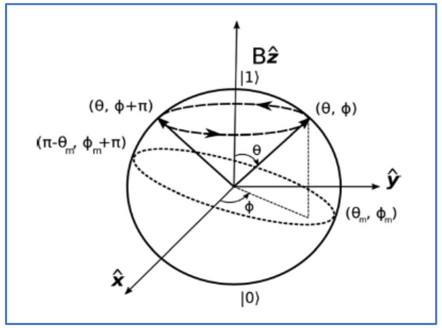
Quantum Time Keeping- "the tick"

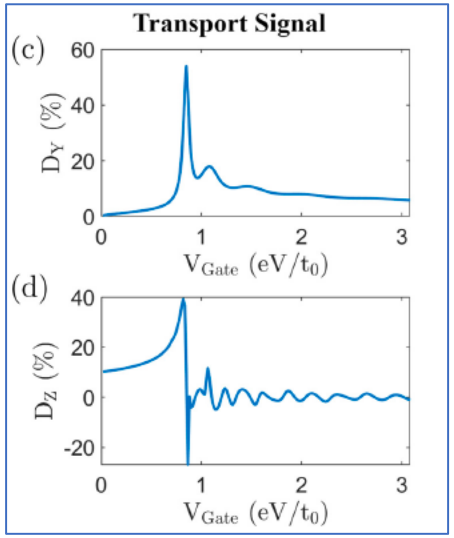
The Larmor clock
Nature (2020)

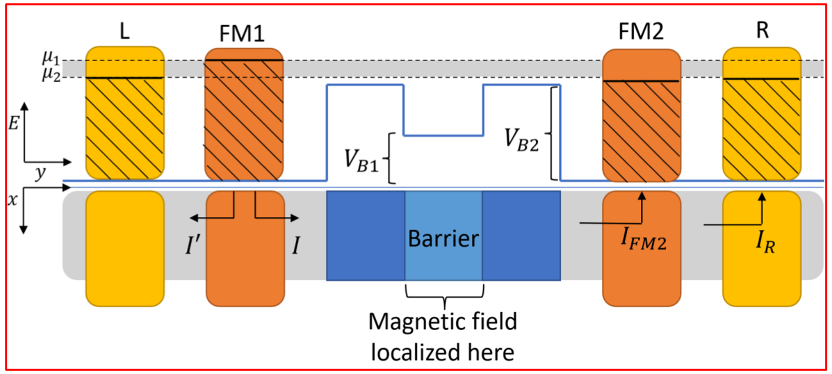
Harnessing
Using Quantum Materials

Phys Rev A, 96, 032339,(2017)









A. Mathew, K. Y. Camsari and B. Muralidharan, Phys Rev B, 105, 144418, (2022)

Device-Circuit-System Co-design

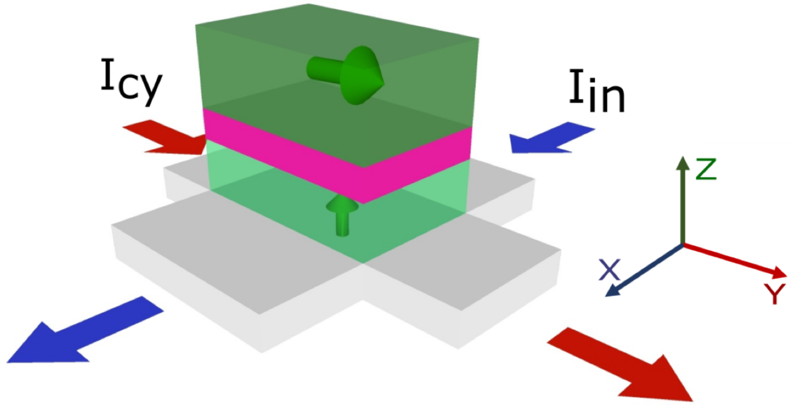
Spintronic, Topological devices

Applications: Non von-Neumann, Neuromorphic Computing

Device Simulation:
Quantum transport

Circuit Design:
Circuit theory

System Design:
Functionality



Incorporating Quantum devices into traditional circuits

ArXiv:2207.14603 (2022).

The "retro-reflection"
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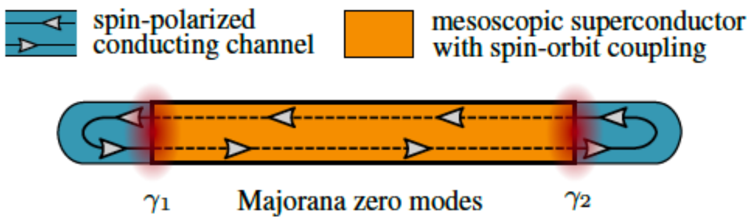
- R Singh (IIT Bombay)

- A.Lahiri, K. Gharavi, J. Baugh and **B. Muralidharan**, Phys. Rev. B, 98, 125417, (2018).
- P. Sriram, S. Kalantre, K. Gharavi, J. Baugh and **B. Muralidharan**, Phys. Rev B, 100, 155431, (2019)
- N. Leumer, M. Grifoni **B. Muralidharan** and M. Marganska, Phys Rev B, 103, 165432, *(2021).*
- C. Duse, P Sriram, K. Gharavi, J. Baugh and **B. Muralidharan,** JPCM, 33, 365301, (2021)
- A. Kejriwal and **B. Muralidharan,** Phys Rev B (Letter), 105, L 161403, (2022) [Editors' suggestion]
- R. Singh and **B. Muralidharan**, ArXiv:2203.08413, (2022)



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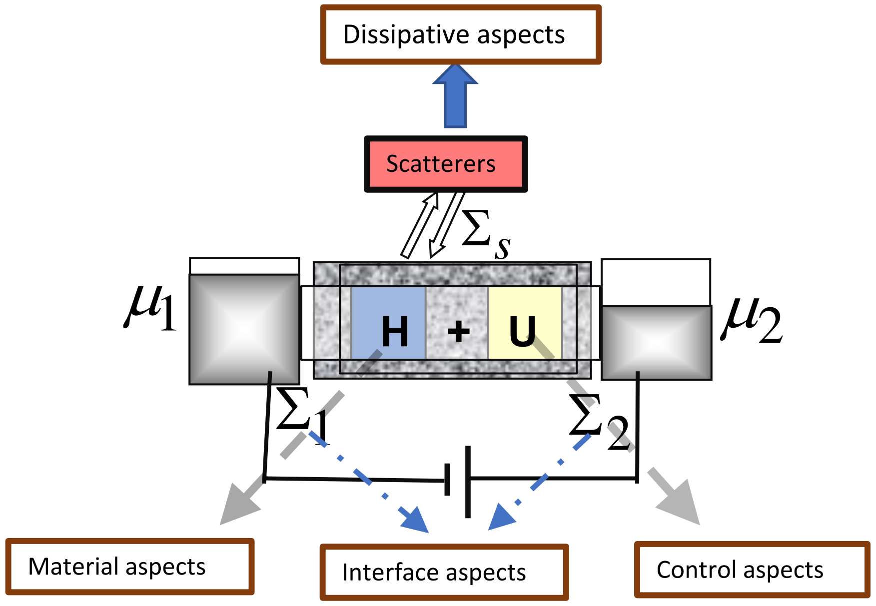

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- Niraj Kumar
- Udit Kumar
- Amal Matthew
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