Aalto University
School of Science

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# Transport and superconductivity in ferroelectric metals

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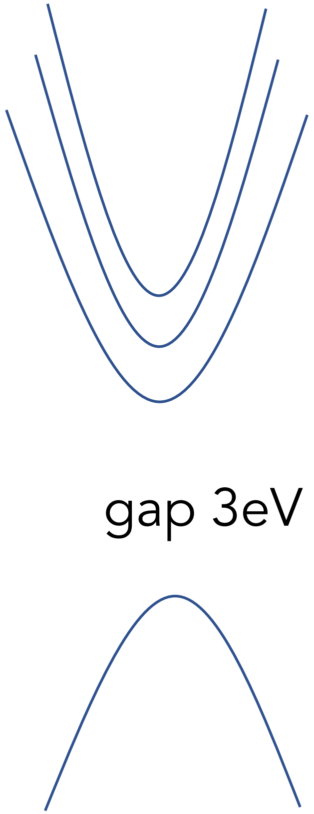
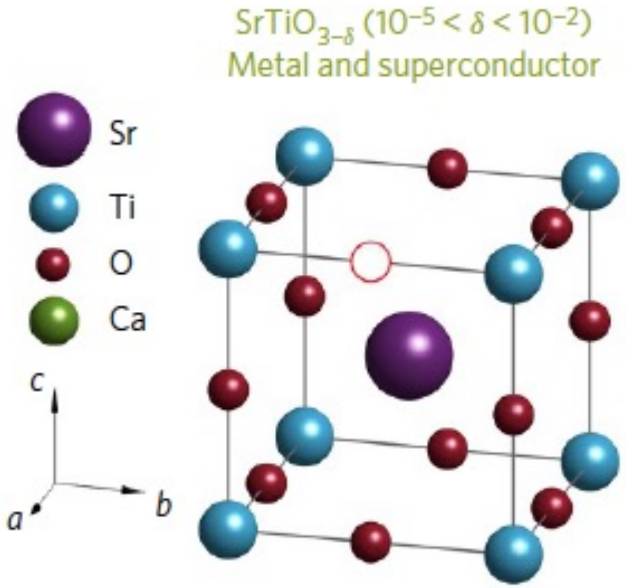
# $SrTiO_3$



gap 3eV




Spherical Fermi surface

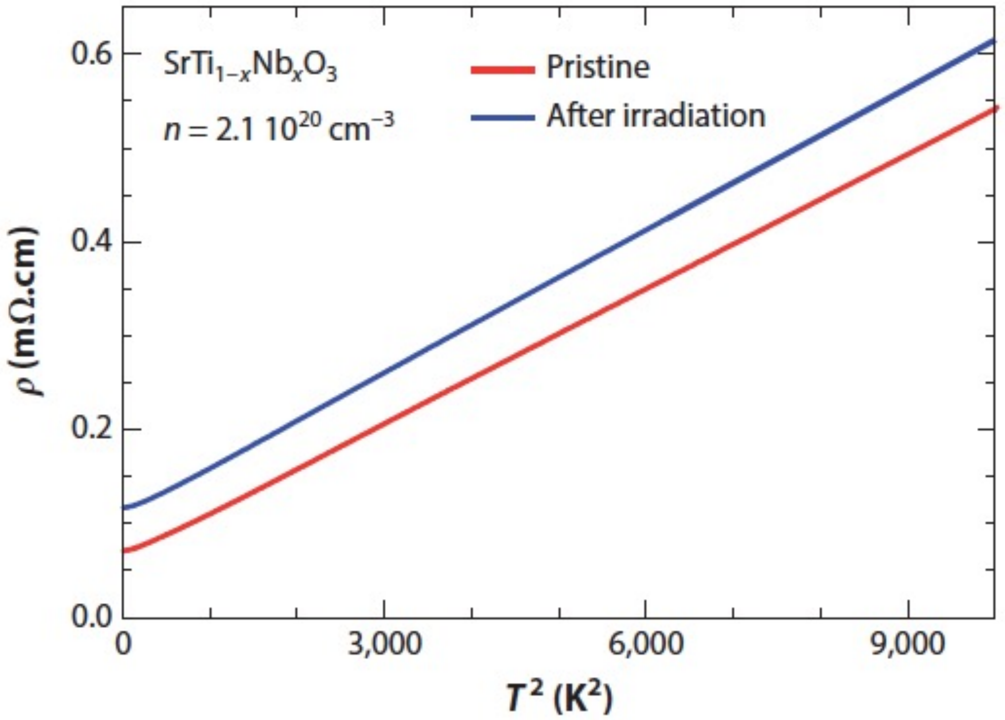
$$E_\mathrm{F} \approx 1\mathrm{meV}, \quad n = 5 \times 10^{17}\ \mathrm{cm}^{-3}$$

Suppressed Coulomb interaction $\epsilon \approx 2 \times 10^4$

*Fermi surface of the most dilute superconductor.*
Behnia group PRX 2013

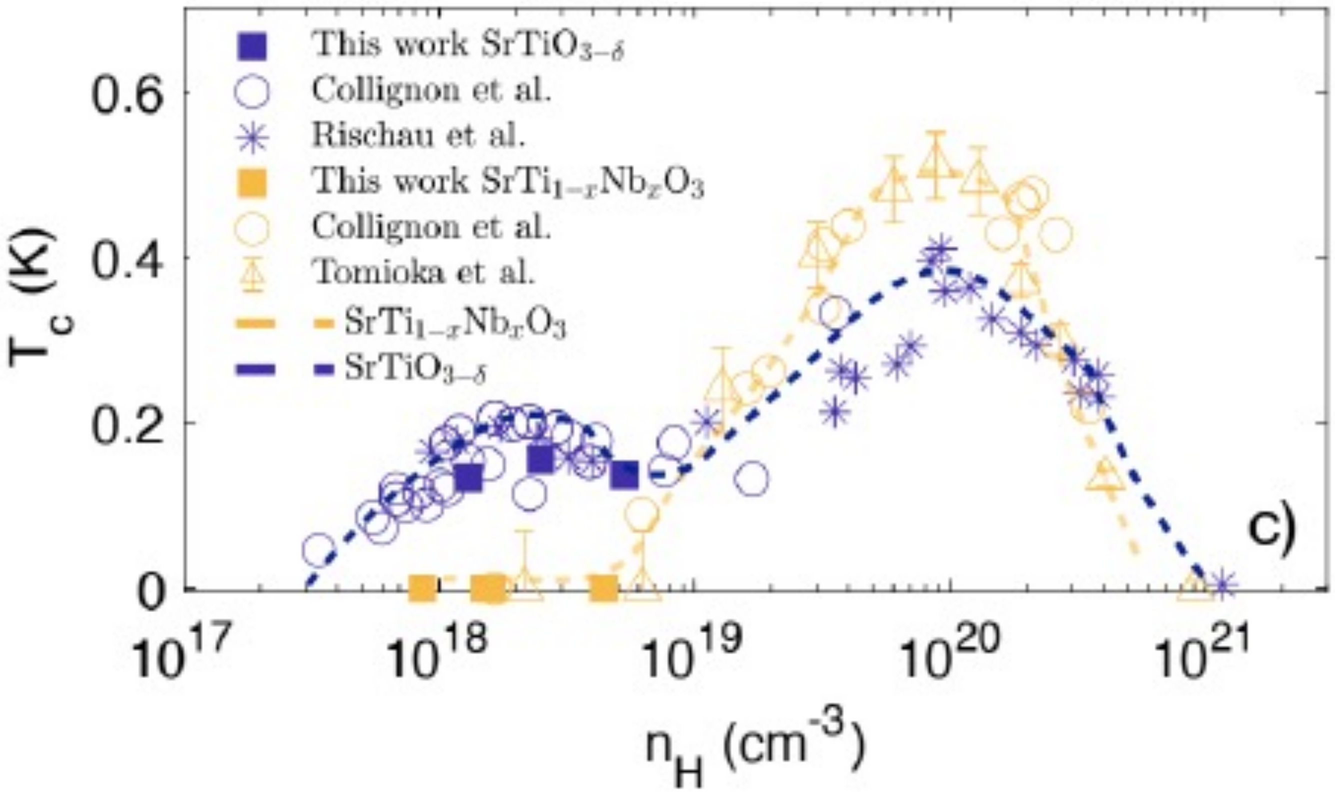
Rischau NatPhys 13, 643 (2017).

$T^2$ dependence of resistivity




Review: Collignon et al. (2019)

Superconductivity

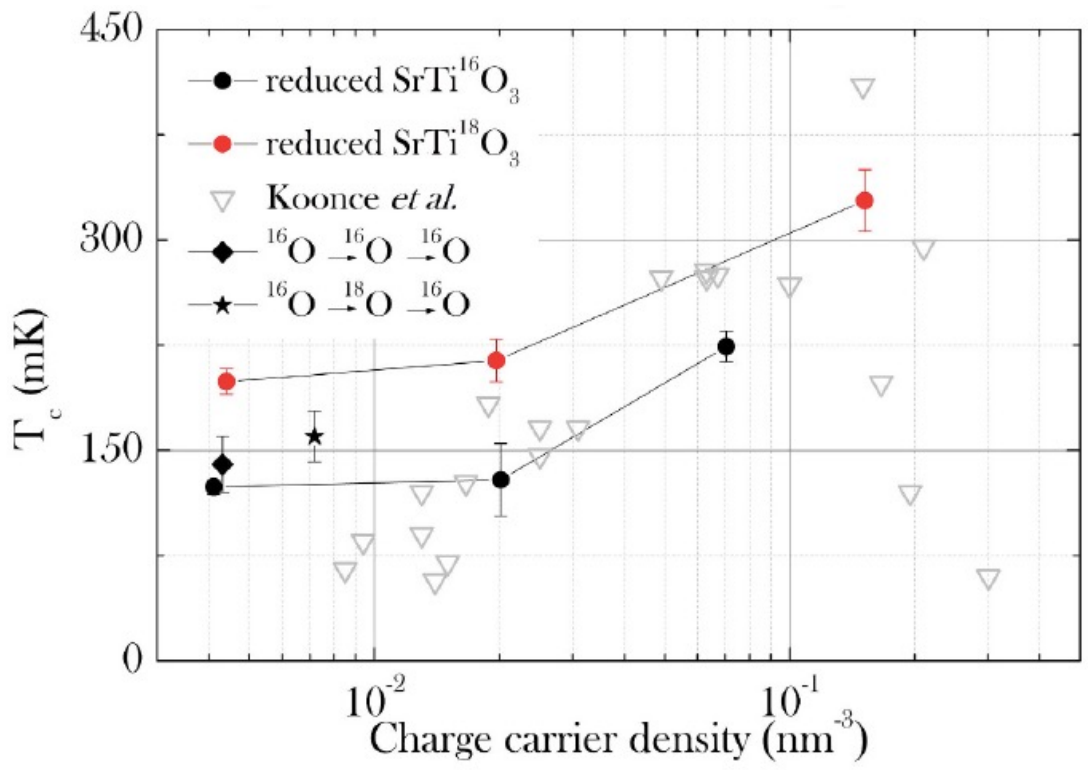



Fauque et al., 2208.09831

Isotope effect within BCS model: $T_c \propto \omega_D \exp\left(-\frac{1}{\lambda_{\text{eff}}\nu}\right), \quad \omega_D \propto 1/\sqrt{M}$

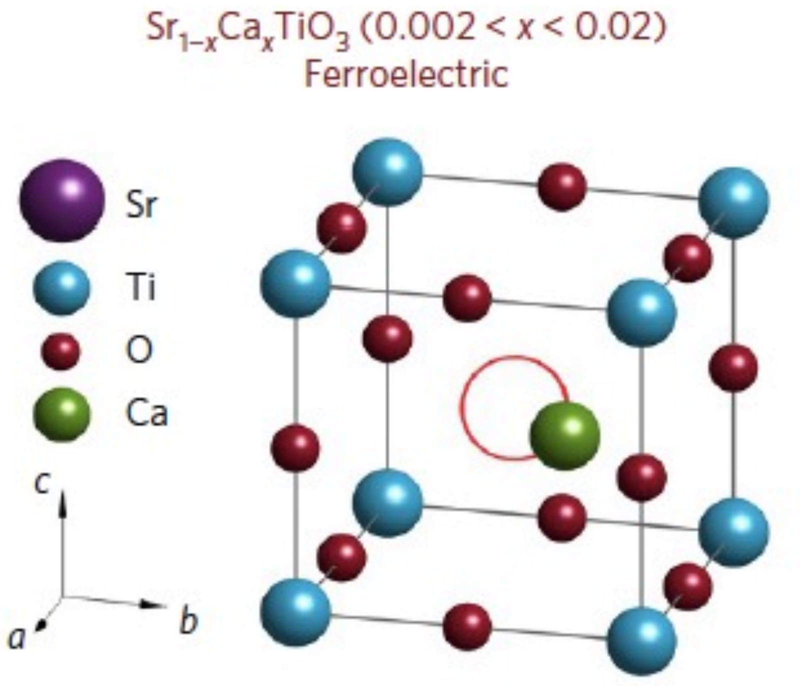
Although in STO:

Substitution of oxygen atoms $^{16}$O by their heavy isotope $^{18}$O resulted in increase of $T_c$ (factor ~ 1.5).




Stucky et al., Sci. Rep. 6, 37582 (2016). Tomioka et al., Nat. Commun. 10, 738 (2019).

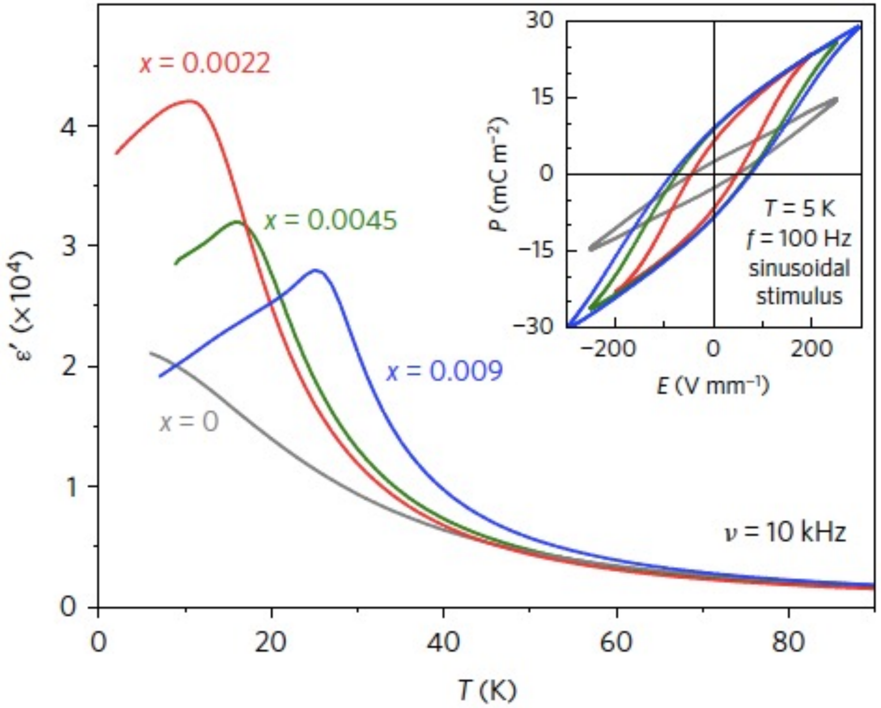
# Ferroelectric STO

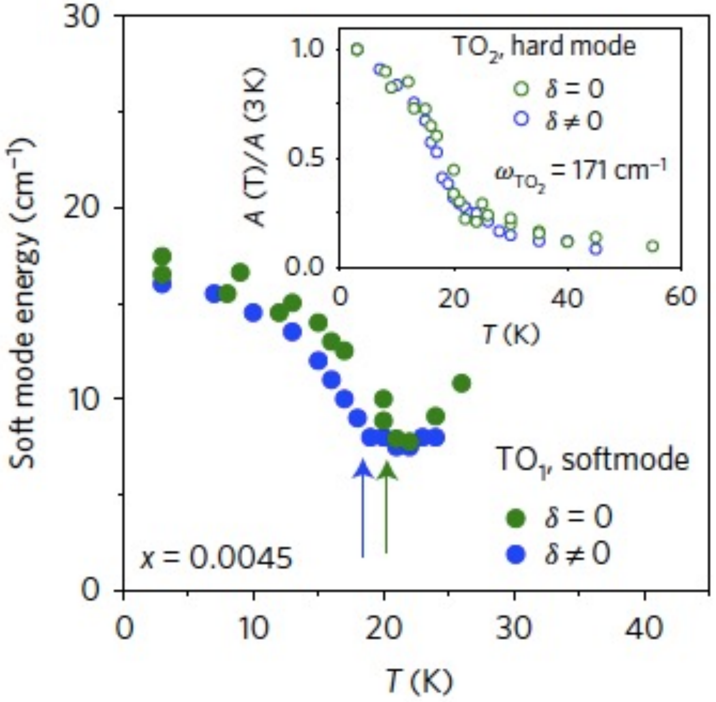



Rischau NatPhys 13, 643 (2017).

# Ferroelectric STO

$Sr_{1-x}Ca_xTiO_3$



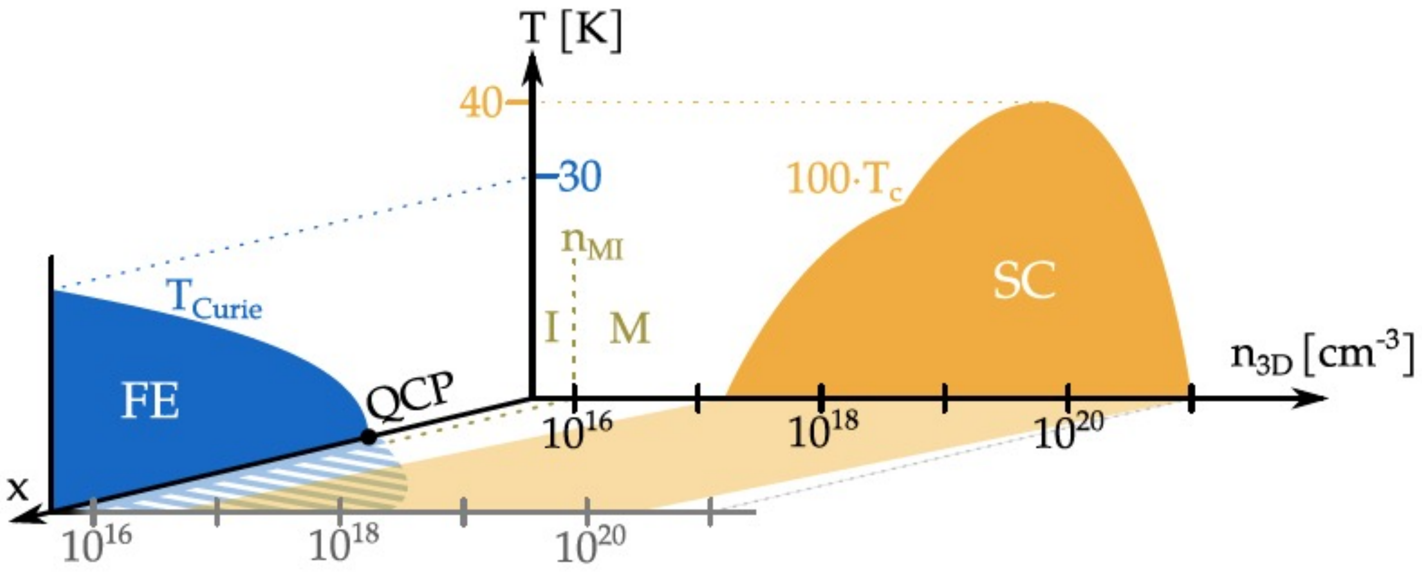

Soft transverse optical phonons $\omega_{\mathbf{q}} = \sqrt{\omega_{\mathrm{TO}}^2 + s^2 q^2}$

$\omega_{\mathrm{TO}} \approx 1.5\mathrm{meV}$

$s = 6.6\ \mathrm{km/s}$

Rischau et al., NatPhys 13, 643 (2017).

Summary:




Scheerer et al., Condens. Matter 5, 60 (2020).

Electrons with spherical FS

$E_{\mathrm{F}} \approx 1\mathrm{meV}, \quad n = 5 \times 10^{17}\ \mathrm{cm}^{-3}$

Suppressed Coulomb interaction $\epsilon \approx 2 \times 10^4$

Soft TO phonons

$\omega_{\mathrm{TO}} \approx 1.5\mathrm{meV}$

$\omega_D \approx 100\mathrm{meV}$

# Electron interaction with two TO phonons

$$\boxed{H_{\mathrm{e-ph}} = g\int_{\mathbf{r}} \mathbf{P}(x)\cdot\mathbf{P}(x)\psi^{\dagger}(x)\psi(x)}$$

Compare with Frohlich type

$$\propto [\mathrm{div}\cdot\mathbf{P}(x)]\,\psi^{\dagger}(x)\psi(x)$$

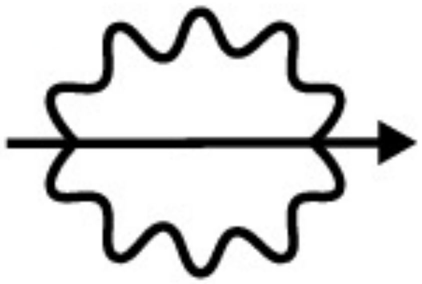
$$\mathbf{P}(x) = \sum_{a;\mathbf{q}} \frac{\mathbf{e}_{a;\mathbf{q}}\Omega_0}{\sqrt{4\pi V\omega_{\mathbf{q}}}}\left[b_{a;\mathbf{q}}(t)e^{i\mathbf{q}\mathbf{r}} + b^{\dagger}_{a;\mathbf{q}}(t)e^{-i\mathbf{q}\mathbf{r}}\right]$$

Displacement vector of TO phonons

$$\omega_{\mathbf{q}} = \sqrt{\omega_{\mathrm{TO}}^2 + s^2q^2} \qquad \Omega_0 = 194.4\ \mathrm{meV}$$

Ngai PRL1974; Epifanov, Levanyuk, and Levanyuk Ferroelectrics 1981, 1982;
Marel, Barantani, and Rischau PRR2019;
Kumar, Yudson, and Maslov PRL2021; Nazaryan and Feigel'man PRB2021;

Transport scattering rate



$$\frac{1}{\tau} = \frac{T^2}{E_0}$$

$$E_0 \equiv \frac{16\pi^3 s^4}{m g^2 \Omega_0^4}$$

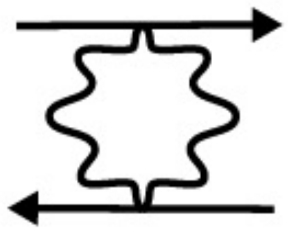
$$E_0 \approx 209K \quad \text{at} \quad n \approx 4 \times 10^{17}\text{cm}^{-3}$$

The corresponding resistivity scales as

$$\rho = \frac{m^*}{ne^2}\frac{T^2}{E_0}$$

Epifanov, Levanyuk, and Levanyuk Feroelectrics1981, 1982
Kumar, Yudson, and Maslov PRL2021, Nazaryan and Feigel'man PRB2021

Two-phonon interaction in the Cooper channel



$\omega_D \gg \mu$ Gorkov and Melik-Barkhudarov, JETP 40, 1452 (1961).

$$\Gamma(\mathbf{q}) = V(\mathbf{q}) - \int_{\mathbf{p}} V(\mathbf{q}-\mathbf{p}) \frac{\tanh\left(\frac{\xi_{\mathbf{p}}}{2T}\right)}{2\xi_{\mathbf{p}}} \Gamma(\mathbf{p})$$

$$\boxed{T_c = \mu \exp\left(-\frac{1}{\lambda_2 \nu}\right), \quad \text{where} \quad \lambda_2 = \left(\frac{g\Omega_0^2}{2\pi}\right)^2 \frac{1}{2s^3} \ln\left[\frac{s q_0}{\max(T_{\mathrm{BG}}, \omega_{\mathrm{TO}})}\right]}$$

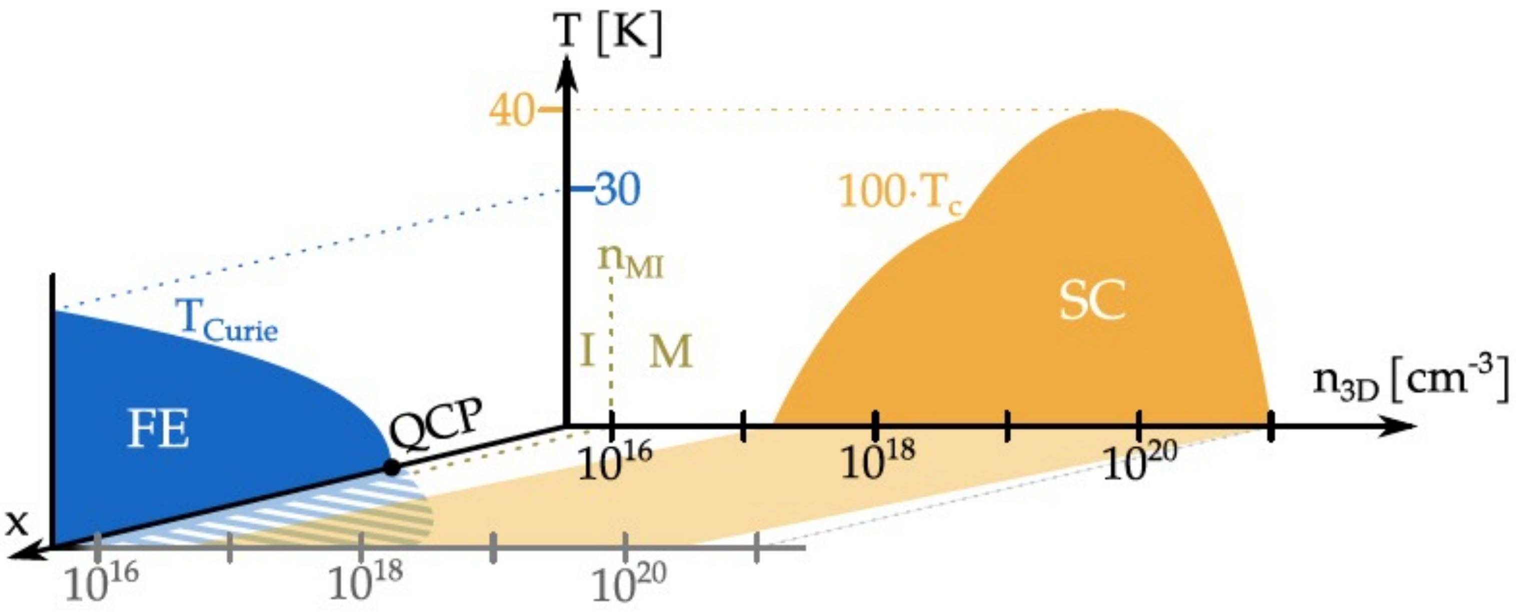
Explains isotope effect due $\omega_{\mathrm{TO}} \to 0$

Ngai PRL1974; Kiseliov and Feigelman PRB2021; Volkov, Chandra, and Coleman NatCom2022.

Another set of experiments put the two-phonon mechanism to the test.

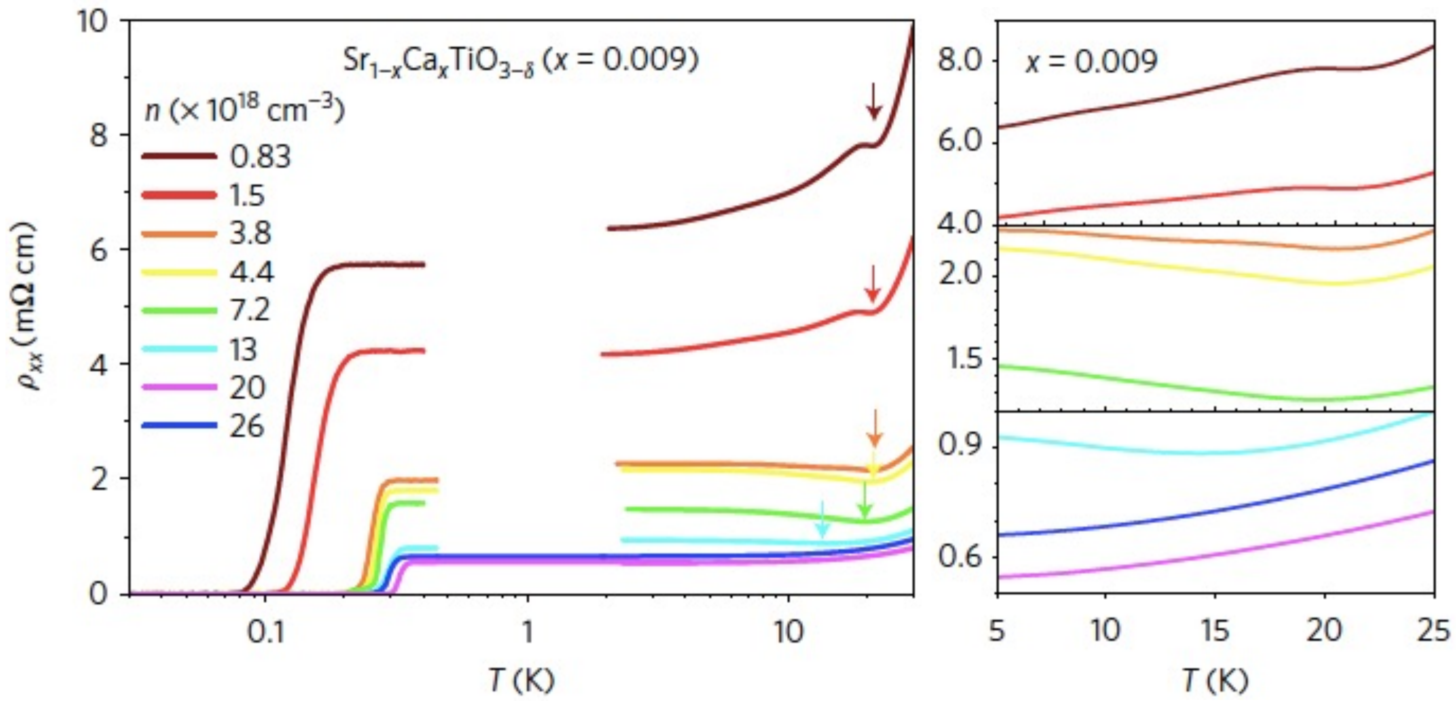
New experimental signatures upon making STO ferroelectric.

# Ferroelectricity and superconductivity in STO




Scheerer et al., Condens. Matter 5, 60 (2020).

$$Sr_{1-x}Ca_xTiO_{3-\delta}$$




Rischau et al., NatPhys 13, 643 (2017).

Superconductivity and ferroelectricity-related resistivity anomaly in metallic STO.

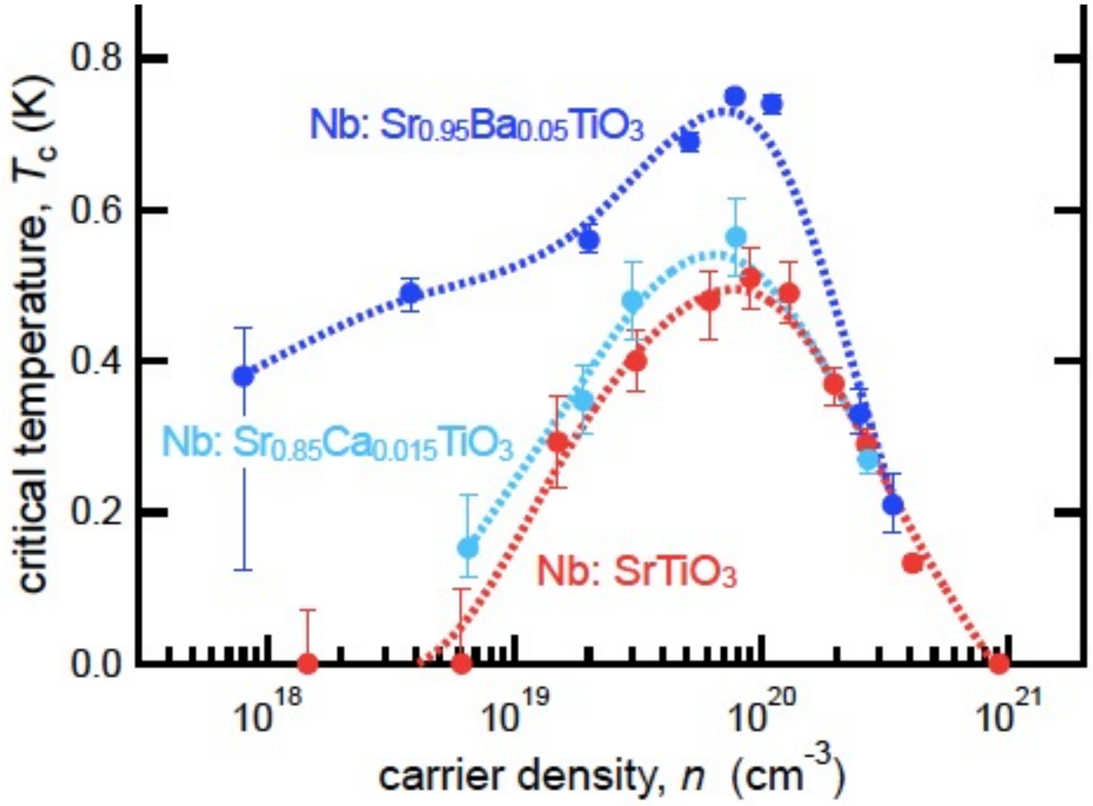
$T_{FE}$ in metallic phase coincides with that in insulating phase.

Resistivity anomaly at $T_{FE}$ confirms that metallic and ferroelectric phases coexist.

Experiments show that ferroelectricity enhances $T_c$

Tomioka et al., arxiv: 2203.16208:

*"Surprisingly, as the system goes deeper into the polar region where the ferroelectric fluctuations must be suppressed, the superconductivity is more enhanced"*



We propose the following electron-phonon interaction in the ferroelectric state:

$$\boxed{H_{\mathrm{e-ph}} = g \int_{\mathbf{r}} [\mathbf{P}_0 + \mathbf{P}(x)]^2 \psi^\dagger(x)\psi(x)}$$

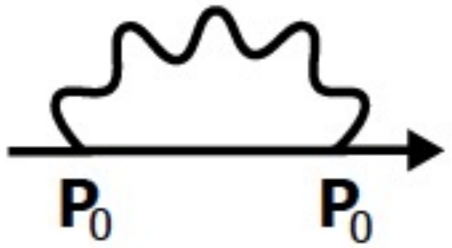
Where $\mathbf{P}_0$ is the vector of spontaneous ferroelectric polarization.

$$P_0 \propto \sqrt{1 - \frac{T}{T_{\mathrm{FE}}}}$$

The TO phonon gap in the ferroelectric state is $\omega_{\mathrm{TO}} \propto P_0$

The electron-phonon interction potential is anisotropic:

$$V_1(\mathbf{q},\omega_n) = -\frac{2}{\pi}\frac{(gP_0\Omega_0)^2}{\omega_n^2+\omega_{\mathbf{q}}^2}\sin^2(\phi_{\mathbf{q},\mathbf{P}_0})$$




$$\omega_{\mathbf{q}} = \sqrt{\omega_{\mathrm{TO}}^2 + s^2q^2}$$

To compare with the acoustic phonons:

$$U(\mathbf{q},\omega_n) = -g_{\mathrm{eff}}^2\frac{s^2q^2}{\omega_n^2+s^2q^2}$$

# Result: electron life-time due to interaction with one TO phonon

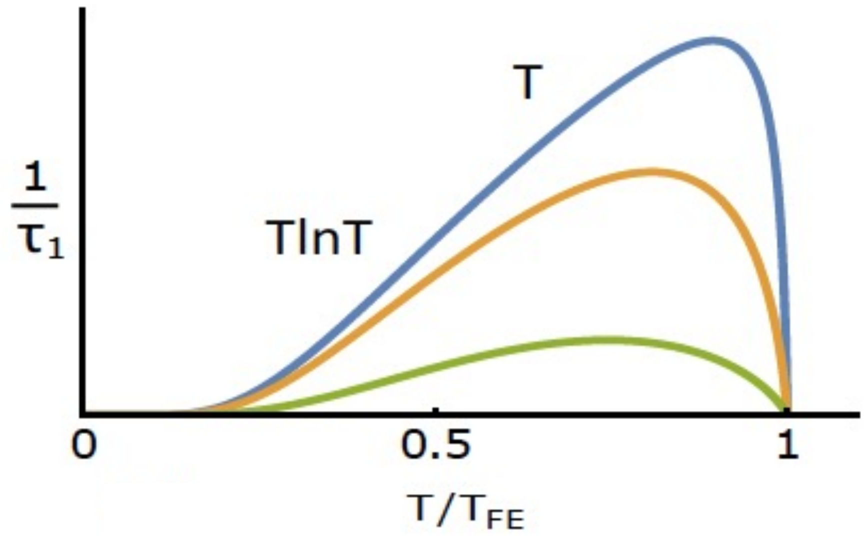
$$\frac{1}{\tau_{1\mathbf{k}}} = 8\pi^2\left[1+\cos^2(\phi_{\mathbf{k}\mathbf{P}_0})\right]\left(gP_0\frac{\Omega_0}{T_{\rm BG}}\right)^2$$
$$\times\,\nu T\ln\left|\tanh\left(\frac{\sqrt{T_{\rm BG}^2+\omega_{\rm TO}^2}}{2T}\right)\coth\left(\frac{\omega_{\rm TO}}{2T}\right)\right|$$

$$n = 5\times 10^{17}\ \mathrm{cm}^{-3}$$

$$T_{\rm BG} = 2sk_{\rm F}$$
$$\nu = mk_{\rm F}/2\pi^2$$

$$\omega_{\rm TO}\propto P_0\sim\sqrt{1-T/T_{\rm FE}}$$

$$\frac{1}{\tau_{1\rm k}}\propto T,\ \text{ at }\ T_{\rm FE}>T\gtrsim T_{\rm BG},\omega_{\rm TO}$$
$$\frac{1}{\tau_{1\rm k}}\propto T\ln\left|\frac{T}{\omega_{\rm TO}}\right|,\ \text{ at }\ T_{\rm FE},T_{\rm BG}>T\gtrsim\omega_{\rm TO}$$



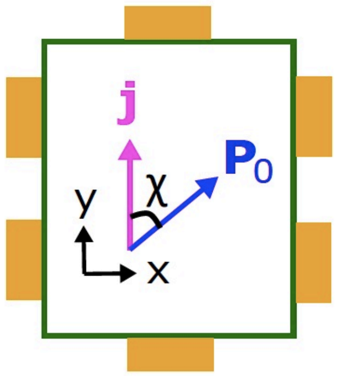

The amplitude decreases with the increase of $T_{\rm BG}/\omega_{\rm TO}(0)$

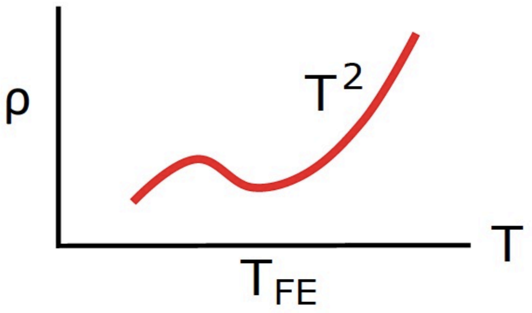
# Result: anisotropy of electric current

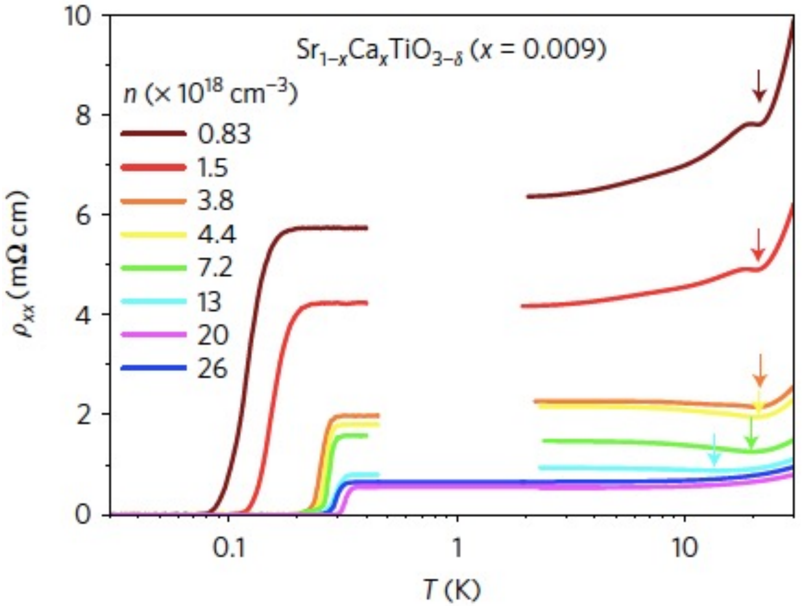
$$\boxed{\mathbf{j} = \sigma\left(1 - \frac{\tau}{5\tau_1}\right)\mathbf{E} - \sigma\frac{2\tau}{5\tau_1}\frac{1}{P_0^2}(\mathbf{E}\cdot\mathbf{P}_0)\mathbf{P}_0}$$

$$\sigma = e^2\nu D$$

$$D = v_F^2\tau/3$$



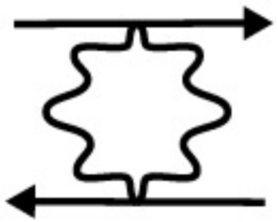




Rischau et al., NatPhys (2017).

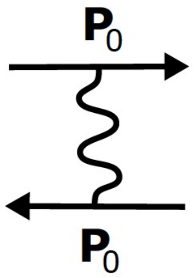
# Result: enhancement of superconducting $T_c$ in ferroelectric metal

$$T_c \propto \mu \exp\left\{-\frac{1}{(\lambda_1+\lambda_2)\,\nu}\right\}$$

$$\lambda_2 = \left(\frac{g\Omega_0^2}{2\pi}\right)^2 \frac{1}{2s^3} \ln\left[\frac{sq_0}{\max(T_{\rm BG},\omega_{\rm TO})}\right]$$



$$\boxed{\lambda_1 = \frac{4}{3\pi}\left(\frac{g\Omega_0 P_0}{T_{\rm BG}}\right)^2 \ln\left(\frac{\omega_{\rm TO}^2+T_{\rm BG}^2}{\omega_{\rm TO}^2}\right)}$$




One-phonon adds to two-phonon contribution and increases $T_c$.

## Main results

Electron interaction with one TO phonon in ferroelectric metal.

One-phonon mechanism results in anisotropic electric transport.

Enhancement of superconducting transition temperature.

V. Zyuzin and A. Zyuzin, arxiv: 2201.03091

$$\varepsilon_p(\omega, q) = \varepsilon_\infty \prod_{j=1}^{3} \frac{\omega_{L,j}^2 - \omega^2}{\omega_{T,j}^2 - \omega^2}\,.$$

$$\omega_{T,1}^2(q, T, E, n) = \omega_0^2 + (c_T q)^2 + (\gamma_T T)^2 + (\gamma_E E)^2 + \gamma_n n$$

$$\varepsilon_0(T) \propto (T - T_0)^{-1}$$