Towards a unified description of quantum Hall effects

Ajit C. Balram
cb.ajit@gmail.com

The Institute of Mathematical Sciences (IMSc), Chennai

Plan of the tutorial

- Part 0: Fractional quantum Hall effect (FQHE) in the Lowest Landau level (LLL): composite fermion (CF) theory
- Part 1: New collective mode in FQHE states in the LLL: parton mode

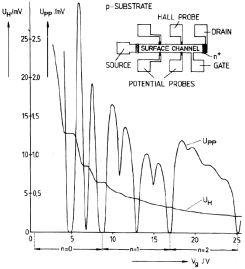
  Ajit C. Balram, Zhao Liu, Andrey Gromov, and Zlatko Papić, Phys. Rev. X **12**, 021008 (2022)
- Part 2: Exotic FQHE states and a parton description of them

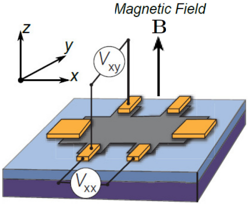
  Ajit C. Balram, SciPost Phys. **10**, 083 (2021)
- Conclusion and outlook

Disclaimer

- Wave function description of quantum Hall effect (QHE).
- Single-component systems.
- Continuum description.
- Focus on states stabilized in GaAs.
- Minimal description.

Experimental discovery of the integer QHE

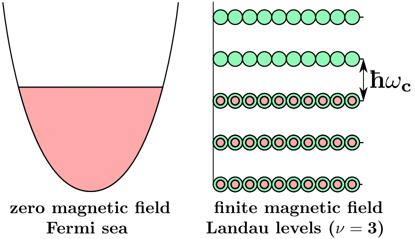




K. v. Klitzing, G. Dorda, and M. Pepper, Phys. Rev. Lett. **45**, 494 (1980)

- Plateau in Hall resistance $R_{xy} = h/(ne^2)$ where n is an integer
- forms the standard of resistance: Klitzing constant $R_{\mathrm{K}} = h/e^2 = 25812.8074593045 \cdots\ \Omega$

IQHE arises from the formation of Landau levels (LLs)



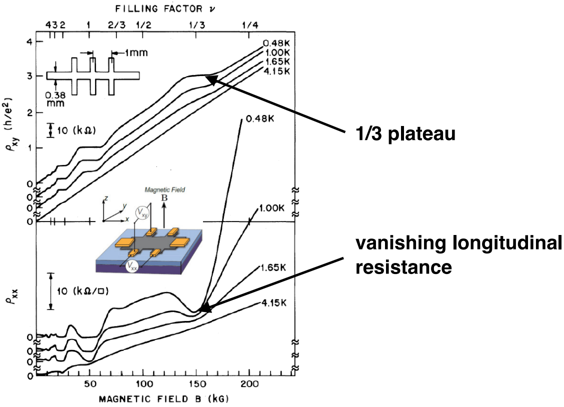
- Excitation gap is set by the cyclotron energy $\rightarrow \hbar\omega_c = \hbar\frac{eB}{m_{\text{eff}}}$

$$\Phi_1 = \prod_{i<j}(z_i - z_j) \times \exp\left[-\frac{1}{4\ell^2}\sum_i |z_i|^2\right]$$

$$z = x - iy, \text{ magnetic length } \ell = \sqrt{\frac{\hbar c}{eB}}, \text{ filling } \nu = \frac{\rho\phi_0}{B}, \ \phi_0 = \frac{hc}{e}.$$

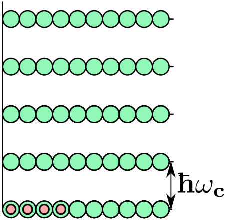
No closed form expression for the wave function of higher LLs.

Plateau at $h/(\frac{1}{3}e^2)$: Quarks???



D. C. Tsui, H. L. Stormer, and A. C. Gossard, Phys. Rev. Lett. **48**, 1559 (1982)

FQHE arises from electron-electron interactions



fractional filling ($\nu = 2/5$)

- Electrons interacting via Coulomb forces:

$$\mathcal{H} = \frac{e^2}{\epsilon}\sum_{i<j}\frac{1}{|r_i - r_j|}$$

- Quantum mechanics $\rightarrow$ *lowest Landau level* constraint for $B \rightarrow \infty$
- Interactions $\rightarrow$ a unique state from the degenerate manifold

Laughlin's ansatz for $\nu = 1/m$

- assumed a Jastrow (pairwise) correlated state.

$$\Psi_{1/m}^{\mathrm{Laughlin}} = \prod_{i<j} (z_i - z_j)^m$$

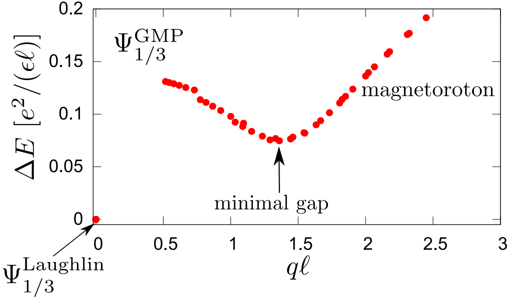
- fermionic wave functions must be antisymmetric, hence m is odd integer
- fluid with fractionally charged particles obeying fractional braid statistics

R. B. Laughlin, Phys. Rev. Lett. **50**, 1395 (1983)

Density-mode ansatz for the neutral excitation

$$\Psi_{\nu}^{\rm GMP} = \bar{\rho}_{\vec{q}}\Psi_{\nu} \text{ [single-mode approximation (SMA)]}$$

$\bar{\rho}_{\vec{q}}$ is the LLL projected density operator

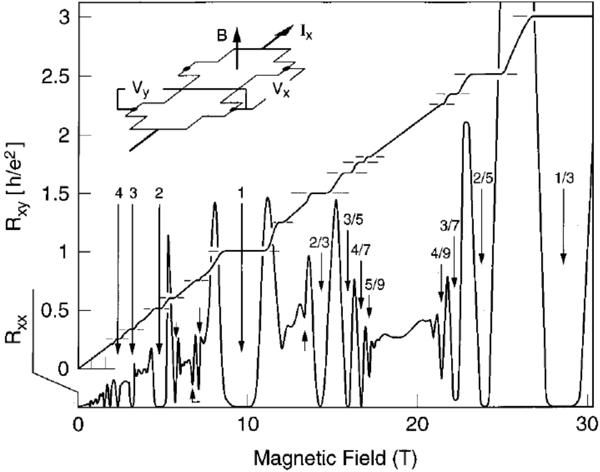


Girvin, MacDonald, and Platzman (GMP), Phys. Rev. Lett. **54**, 581 (1985), Phys. Rev. B **33**, 2481 (1986)

Experimentally observed using light-scattering

A. Pinczuk, B. S. Dennis, L. N. Pfeiffer, and K. West, Phys. Rev. Lett. **70**, 3983 (1993)

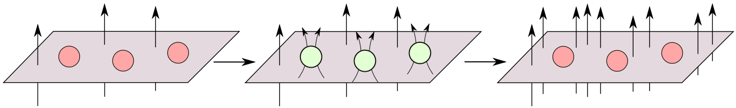
Zoo of fractions in the $\nu = n/(2pn \pm 1)$ sequence



J. P. Eisenstein and H. L. Stormer, Science **248**, 4962, 1510-1516 (1990)

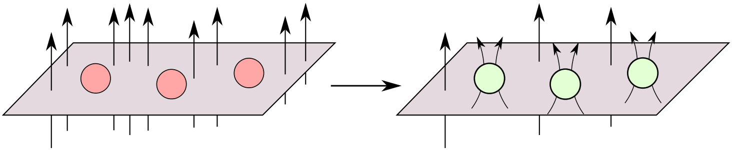
FQHE as IQHE of composite fermions

A composite fermion (CF) is a bound state of an electron and even number of vortices/flux quanta.



J. K. Jain, Composite Fermions, Cambridge University Press (2007)

FQHE as IQHE of composite fermions

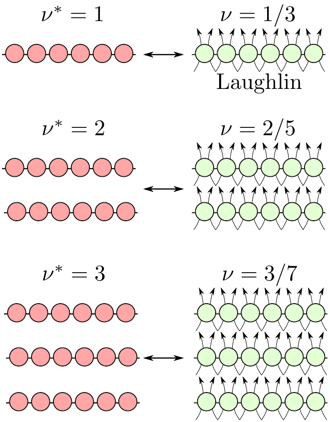


$$B^* = B - 2p\rho\phi_0, \quad \phi_0 = hc/e$$

$$\nu = \frac{\rho\phi_0}{B}, \quad \nu^* = \frac{\rho\phi_0}{|B^*|}, \quad \nu = \frac{\nu^*}{2p\nu^* \pm 1}$$

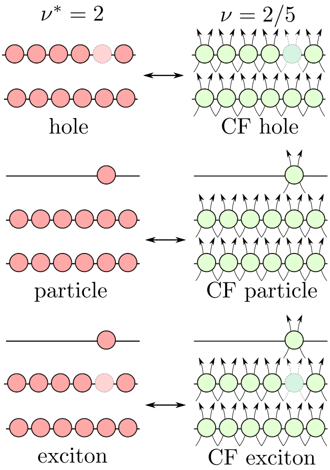
J. K. Jain, Composite Fermions, Cambridge University Press (2007)

FQHE ground states are analogous to IQHE ones



J. K. Jain, Composite Fermions, Cambridge University Press (2007)

FQHE excited states are analogous to IQHE ones



J. K. Jain, Composite Fermions, Cambridge University Press (2007)

FQHE wave functions can be built from IQHE ones

- Jain wave functions at $\nu = n/(2pn \pm 1)$:

$$\Psi^{\mathrm{CF}}_{\nu=\frac{n}{2pn\pm1}} = \mathcal{P}_{\mathrm{LLL}}\left(\Phi_{\pm n}\prod_{i<j}(z_i - z_j)^{2p}\right).$$

(dropped Gaussian factor for ease of notation)

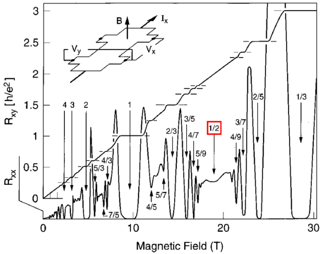
Φ_n wave function of n filled LLs.

$\mathcal{P}_{\mathrm{LLL}}$ implements lowest Landau level projection.

- no adjustable parameters in these wave functions
- wave functions can be evaluated for large system sizes

J. K. Jain, Phys. Rev. Lett. **63**, 199 (1989)

Mystery of the $\nu = 1/2$ state

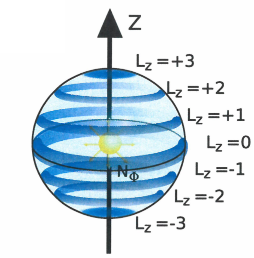


- composite fermions absorb all of the magnetic flux: $B^* = 0$

Halperin, Lee and Read, Phys. Rev. B **47**, 7312 (1993)

- In zero effective magnetic field CFs form a Fermi sea

Spherical geometry



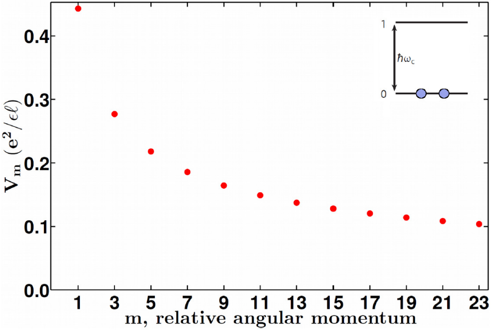
$$l = |Q|, |Q|+1, |Q|+2, \cdots \quad l_n = |Q| + n \quad m = -l, -l+1, \cdots, l-1, l$$

L and its z-component L_z are good quantum numbers

$$N_\phi = 2Q = \nu^{-1} N - \mathcal{S}, \quad \mathcal{S} \to \text{shift, characterizes the state}$$

F. D. M. Haldane, Phys. Rev. Lett. **51**, 605 (1983)

Haldane pseudopotentials parametrize the interaction



V_m: energy of two electrons in a state of relative angular momentum m

fully spin-polarized electrons $\rightarrow$ only odd pseudopotentials relevant

F. D. M. Haldane, Phys. Rev. Lett. **51**, 605 (1983)

Overlaps of CF states with LLL Coulomb ground states

overlaps obtained from direct projected states

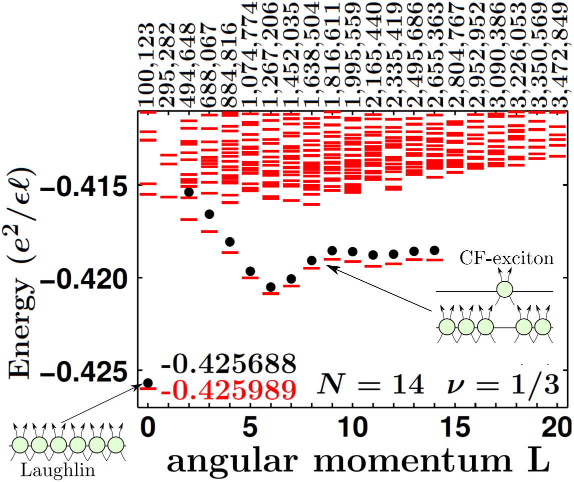
ν	N	Hilbert space dimension	$\vert\langle\Psi^{0\mathrm{LL}}\vert\Psi^{\mathrm{CF}}\rangle\vert$
1/3	15	2×10^9	0.9876 (Laughlin)
1/5	11	4×10^8	0.9413 (Laughlin)
2/5	12	3×10^5	0.9971
3/7	12	6×10^4	0.9988
2/9	10	1×10^7	0.9744

$|\Psi^{0\mathrm{LL}}\rangle$ is obtained by brute-force exact diagonalization

Ajit C. Balram and A. Wójs, Phys. Rev. Research **2**, 032035(R) (2020)
B. Yang and Ajit C. Balram, New J. Phys. **23**, 013001 (2021)

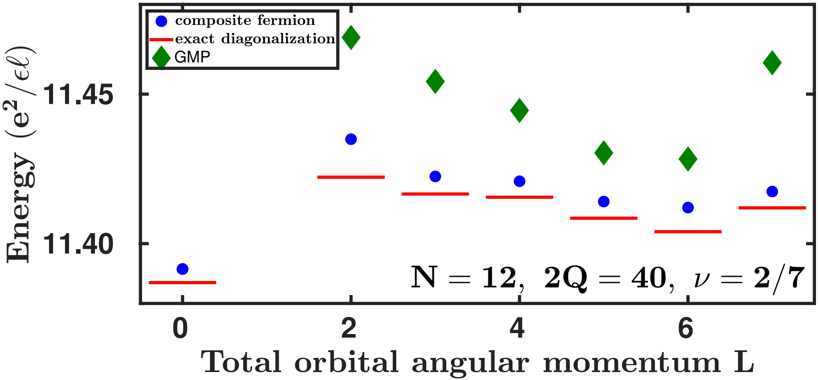
Ajit C. Balram, SciPost Phys. **10**, 083 (2021)

Magnetoroton merges with continuum for $q \to 0$ at $\nu = 1/3$



Ajit C. Balram, Arkadiusz Wójs, and Jainendra K. Jain, Phys. Rev. B **88**, (2013)

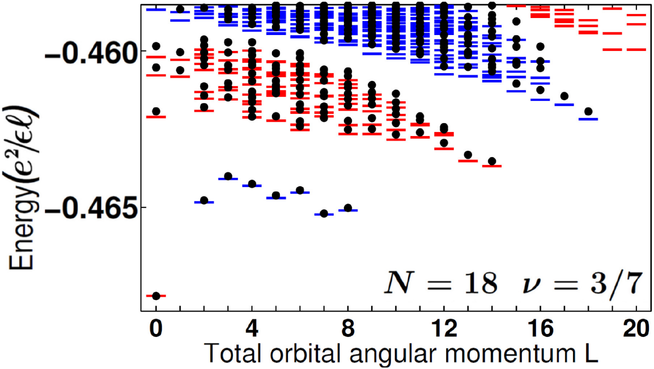
CF-exciton (CFE) is more accurate than the GMP



At $\nu = 1/(2p+1)$, in the $q \to 0$ limit, CFE and GMP are identical

R. K. Kamilla, X. G. Wu, and J. K. Jain, Phys. Rev. B **54**, 4873 (1996)

CF theory is extremely accurate in the lowest Landau level



dashes are obtained by brute-force exact diagonalization
$\sim 10^6$ states at each total orbital angular momentum L

Ajit C. Balram, A. Wójs and J. K. Jain, Phys. Rev. B **88**, 205312 (2013)

Questions?

A puzzle in the $n/(4n \pm 1)$ secondary Jain states

- Present composite fermion field theories for the secondary Jain state states violate the so-called "Haldane bound," which places a lower bound on the coefficient of q^4 in the static structure factor ($\langle \bar{\rho}\bar{\rho} \rangle$) set by the shift.

  requires lowest Landau level (LLL) projection

  H. Goldman and E. Fradkin, Phys. Rev. B **98**, 165137 (2018).

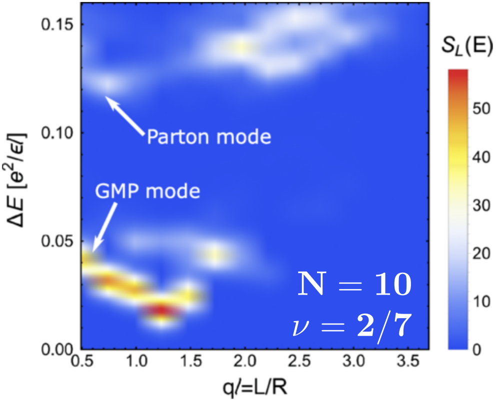
- Nguyen and Son proposed that, in addition to the GMP mode, there exist additional, higher energy collective modes, which resolves this contradiction.

  Dung Xuan Nguyen and Dam Thanh Son, Phys. Rev. Research **3**, 033217 (2021)

- We give a microscopic justification and construct a trial wave function to capture this additional "parton" mode.

Parton mode is seen in the dynamical structure factor

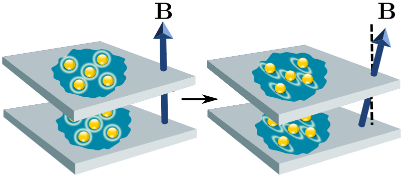
$$S_L(E) = \sum_n |\langle E_n|\bar{\rho}_L|0\rangle|^2 \delta(E_n - E_0 - E)$$



Ajit C. Balram *et al.*, Phys. Rev. X **12**, 021008 (2022)

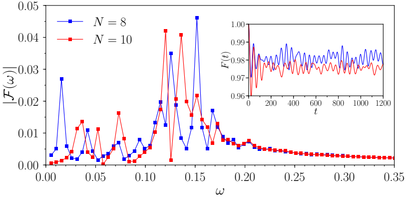
see also Nguyen *et al.*, Phys. Rev. Lett. **128** 246402 (2022)

Parton mode is observed in quench dynamics



Zhao Liu, Andrey Gromov, and Zlatko Papić, Phys. Rev. B **98**, 155140 (2018)

Zhao Liu, Ajit C. Balram, Zlatko Papić, and Andrey Gromov, Phys. Rev. Lett. **126**, 076604 (2021)



Ajit C. Balram *et al.*, Phys. Rev. X **12**, 021008 (2022)

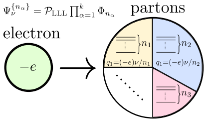
Parton states: product of fermionic states

- break each electron into fictitious partons, place partons into IQH (or any) fermionic states, fuse the partons back to recover the electron

$$\Psi_{\nu}^{\{n_\alpha\}} = \mathcal{P}_{\rm LLL} \prod_{\alpha=1}^{k} \Phi_{n_\alpha}(\{z_i\})$$

- k is odd for fermions

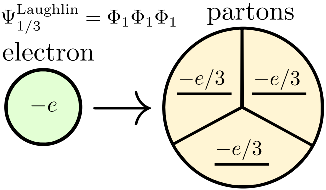
$$q_\alpha = (-e)\frac{\nu}{n_\alpha}, \quad \nu^{-1} = \sum_{\alpha=1}^{k} n_\alpha^{-1}, \quad \mathcal{S} = \sum_{\alpha=1}^{k} n_\alpha$$



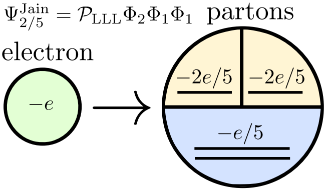
J. K. Jain, Phys. Rev. B **40**, 8079 (1989)

Laughlin and Jain states are parton states

- Laughlin state is a "$111\cdots$" parton state $\left[\Phi_1 \equiv \prod_{i<j}(z_i - z_j)\right]$



- Jain/CF states are "$n11\cdots$" parton states



J. K. Jain, Phys. Rev. B **40**, 8079 (1989)

secondary Jain $\sim$ primary Jain $\times$ bosonic Laughlin

- for $n, p \geq 2$

$$\Psi^{\rm Jain}_{n/(2pn\pm1)} = \mathcal{P}_{\rm LLL}\Phi_1^{2p}\Phi_{\pm n} \sim \mathcal{P}_{\rm LLL}\Phi_1^{2}\Phi_{\pm n} \times \Phi_1^{2(p-1)}$$
$$\sim \Psi^{\rm Jain}_{n/(2n\pm1)} \times \Psi^{\rm Laughlin}_{1/[2(p-1)]}$$

The two collective modes are

- CF-exciton/GMP mode of the primary Jain state
$\Psi^{\rm parton}_{n/(2pn\pm1)} = \Psi^{\rm CFE}_{n/(2n\pm1)} \times \Psi^{\rm Laughlin}_{1/[2(p-1)]}$
- CF-exciton/GMP mode of the bosonic Laughlin state
$\Psi^{\rm parton}_{n/(2pn\pm1)} = \Psi^{\rm Jain}_{n/(2n\pm1)} \times \Psi^{\rm CFE}_{1/[2(p-1)]}$

Ajit C. Balram *et al.*, Phys. Rev. X **12**, 021008 (2022)

Factorization hardly changes the microscopic wave function

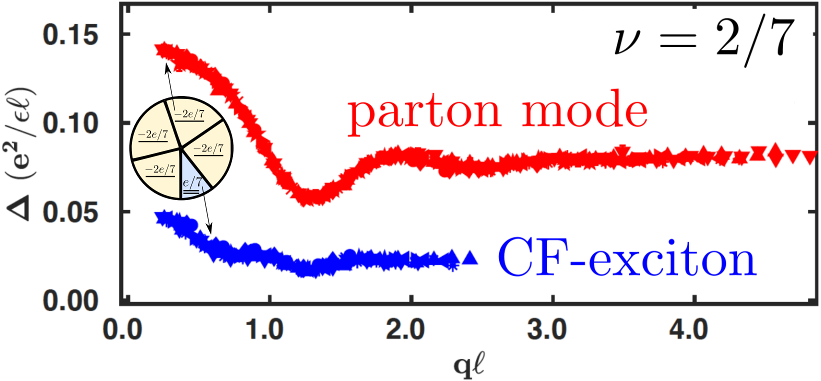
overlaps between different versions of the projected Jain states

ν	N	dimension	$\vert\langle\Psi^{Jain}_{n/(4n\pm1)}\vert\Psi^{Jain}_{n/(2n\pm1)}\times\Psi^{Laughlin}_{1/2}\rangle\vert$
2/7	8	5×10^4	0.9925
2/9	8	1×10^5	0.9999

Different versions of the Laughlin wave function have unit overlap with each other since there is no projection involved in the Laughlin states.

Ajit C. Balram and J. K. Jain, Phys. Rev. B **93**, 235152 (2016)

Trial wave function for the parton mode is accurate

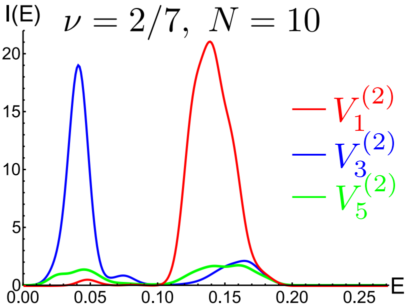


Ajit C. Balram *et al.*, Phys. Rev. X **12**, 021008 (2022)

Clustering properties of the two modes are different

$\Psi_{2/7}^{\rm parton-mode} = \Psi_{2/3}^{\rm Jain} \times \Psi_{1/2}^{\rm CFE} \propto r$ high-energy

$\Psi_{2/7}^{\rm CFE} = \Psi_{2/3}^{\rm CFE} \times \Psi_{1/2}^{\rm Laughlin} \propto r^3$ low-energy $\because \langle V_1 \rangle_{\Psi_{2/7}^{\rm CFE}} = 0$



States also have different chiralities

Ajit C. Balram *et al.*, Phys. Rev. X **12**, 021008 (2022)

Parton mode is absent in primary Jain states

- Numerics show only one mode in the primary Jain states.
- All Laughlin fractions, in particular $\nu = 1/3$, can support only one mode since the state is made up of only one kind of parton, as each parton fills one LL.
- By particle-hole conjugation in the LLL, $\nu = 2/3$ also hosts only one mode.
- The Jain wave function for $2/5$ is analogous to that at $2/3$ and thus since $2/3$ has only one mode $2/5$ is expected to have only one mode.
- Projection removes the parton-like mode $\mathcal{P}_{\rm LLL}\Phi_{\pm n}\Phi_1\Phi_1^{exciton}$.
- A different argument based on geometric fluctuation of conformal Hilbert spaces.

Yuzhu Wang and Bo Yang, arXiv:2201.00020

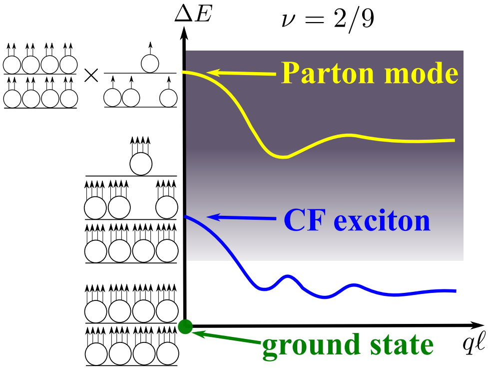
Ajit C. Balram *et al.*, Phys. Rev. X **12**, 021008 (2022)

Outlook

- Very high-energy excitations in the Jain states that are not described by composite fermions but lend themselves to a description in terms of partons.
- How do we experimentally probe these excitations that lie deep in the continuum?
- What is the nature of these excitations in other exotic, in particular non-Abelian, states?

Summary of Part 1

Partons are “real” quasiparticles of quantum Hall fluids and like quarks in QCD, become observable at high energies.

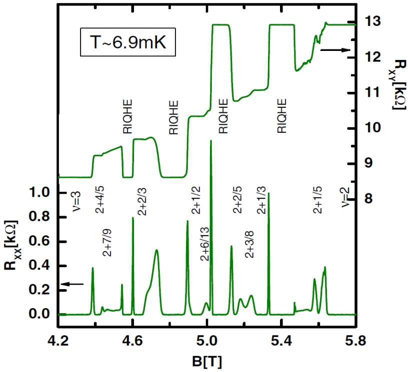


Ajit C. Balram, Zhao Liu, Andrey Gromov, and Zlatko Papić, Phys. Rev. X **12**, 021008 (2022)

Questions?

Onward to Part 2

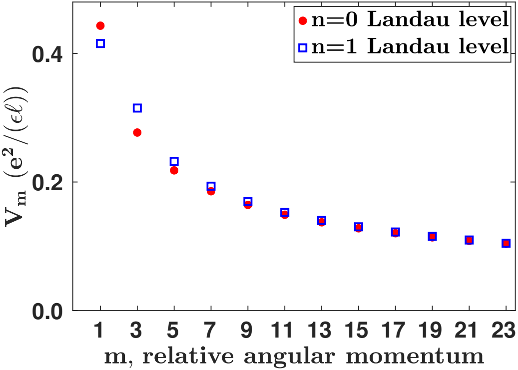
FQH states in the second Landau level



- appearance of *even* denominator fractions
- 6/13 appears "out of order"

A. Kumar *et al.* Phys. Rev. Lett. **105**, 246808 (2010)

Landau levels differ in their Haldane pseudopotentials



V_m energy of two electrons in a state of relative angular momentum m

stronger repulsion at shortest approach in the LLL

F. D. M. Haldane, Phys. Rev. Lett. **51**, 605 (1983)

^{4}CF states in the LLL and SLL are similar

FQHE at $\nu = 2 + 1/5$ and $2 + 4/5$

N	$2l$	$\lvert\langle\Psi^{\rm Laughlin}_{1/5}\vert\Psi^{\rm LLL}_{1/5}\rangle\rvert$	$\lvert\langle\Psi^{\rm Laughlin}_{1/5}\vert\Psi^{\rm SLL}_{1/5}\rangle\rvert$	$\lvert\langle\Psi^{\rm SLL}_{1/5}\vert\Psi^{\rm LLL}_{1/5}\rangle\rvert$
11	50	0.9413	0.9509	0.9993

FQHE at $\nu = 2 + 5/7$ and $\nu = 2 + 2/7$

N	$2l$	$\lvert\langle\Psi^{\rm Jain}_{2/7}\vert\Psi^{\rm LLL}_{2/7}\rangle\rvert$	$\lvert\langle\Psi^{\rm Jain}_{2/7}\vert\Psi^{\rm SLL}_{2/7}\rangle\rvert$	$\lvert\langle\Psi^{\rm SLL}_{2/7}\vert\Psi^{\rm LLL}_{2/7}\rangle\rvert$
8	26	0.9989	0.9762	0.9819

FQHE at $\nu = 2 + 7/9$ and $\nu = 2 + 2/9$

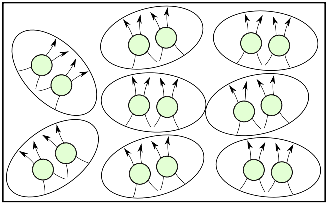
N	$2l$	$\lvert\langle\Psi^{\rm Jain}_{2/9}\vert\Psi^{\rm LLL}_{2/9}\rangle\rvert$	$\lvert\langle\Psi^{\rm Jain}_{2/9}\vert\Psi^{\rm SLL}_{2/9}\rangle\rvert$	$\lvert\langle\Psi^{\rm SLL}_{2/9}\vert\Psi^{\rm LLL}_{2/9}\rangle\rvert$
10	39	0.9744	0.9766	0.9977

Ajit C. Balram, SciPost Phys. **10**, 083 (2021)

Candidate states for $\nu = 5/2$: Pfaffian

$$\Psi^{\rm MR}_{\nu=1/2} = {\rm Pf}\left[\frac{1}{z_i - z_j}\right] \prod_{i<j}(z_i - z_j)^2$$

G. Moore and N. Read, Nucl. Phys. B **360**, 362 (1991)



p-wave paired state of composite fermions

N. Read and Dmitry Green, Phys. Rev. B **61**, 10267 (2000)

N	Hilbert space dimension	$\lvert\langle\Psi^{\rm 1LL}\lvert\Psi^{\rm MR}_{\nu=1/2}\rangle\rvert$
20	4×10^8	0.6736

Ajit C. Balram and A. Wójs, Phys. Rev. Research **2**, 032035(R) (2020)

Candidate states for $\nu = 5/2$: anti-Pfaffian

- anti-Pfaffian is the particle-hole conjugate of Pfaffian

$$\Psi^{\rm aPf}_{\nu=1/2} = \mathcal{P}_{ph}\left(\mathrm{Pf}\left[\frac{1}{z_i - z_j}\right]\prod_{i<j}(z_i - z_j)^2\right)$$

M. Levin *et al.*, Phys. Rev. Lett. **99**, 236806 (2007), S. S. Lee *et al.*, Phys. Rev. Lett. **99**, 236807 (2007)

- construction extremely difficult to implement numerically
- recent numerics suggest anti-Pfaffian is favored in the presence of LL mixing

E. H. Rezayi, Phys. Rev. Lett. **119**, 026801 (2017)

Candidate states for $\nu = 5/2$: PH-Pfaffian

$$\Psi^{\mathrm{PH-Pf,TJ}}_{\nu=1/2} = \mathcal{P}_{\mathrm{LLL}} \left(\left\{ \mathrm{Pf}\left[\frac{1}{z_i - z_j} \right] \prod_{i<j} (z_i - z_j) \right\}^* \prod_{i<j} (z_i - z_j)^3 \right)$$

Th. Jolicoeur, Phys. Rev. Lett. **99**, 036805 (2007)

$$\Psi^{\mathrm{PH-Pf,ZF}}_{\nu=1/2} = \mathcal{P}_{\mathrm{LLL}} \left(\left\{ \mathrm{Pf}\left[\frac{1}{z_i - z_j} \right] \right\}^* \prod_{i<j} (z_i - z_j)^2 \right)$$

P. T. Zucker and D. E. Feldman, Phys. Rev. Lett. **117**, 096802 (2016)

- state is particle-hole symmetric to a good extent

  Ajit C. Balram, Maissam Barkeshli, and Mark. S. Rudner, Phys. Rev. B **98**, 035127 (2018)

  R. V. Mishmash *et. al.*, Phys. Rev. B **98**, 081107(R) (2018)

  Edward H. Rezayi, Kiryl Pakrouski, and F. D. M. Haldane, Phys. Rev. B **104**, L081407 (2021)

- consistent with recent thermal Hall measurements

  M. Banerjee *et. al.*, Nature **559**, 205-210 (2018)

Read-Rezayi states: clustering of electrons

$$\Psi^{\mathrm{RRk}}_{\nu=\frac{k}{k+2}} = \prod_{i<j}(z_i - z_j)\, \mathbb{S}\left[\prod_{l=1}^{k}\left(\prod_{i_l<j_l}(z_{i_l} - z_{j_l})^2\right)\right]$$

- $k = 1 \implies \nu = 1/3$: same as Laughlin
- $k = 2 \implies \nu = 1/2$: same as Pfaffian follows from Cauchy identity
- $k = 3 \implies \nu = 3/5$: particle-hole conjugate (aRR3) leading candidate for 12/5 FQHE

  E. H. Rezayi and N. Read, Phys. Rev. B **79**, 075306 (2009)
- $k = 4 \implies \nu = 2/3$: competitive with Laughlin but not consistent with thermal Hall measurements at 7/3

  M. R. Peterson *et al.*, Phys. Rev. B **92**, 035103 (2015)
- 3/8 and 6/13 are not part of this sequence

N. Read and E. H. Rezayi, Phys. Rev. B **59**, 8084 (1999)

Bonderson-Slingerland states

Pfaffian times bosonic Jain

$$\Psi^{\rm BS}_{\nu=\frac{n}{(2p+1)n\pm 1}} = \mathcal{P}_{\rm LLL}{\rm Pf}\left[\frac{1}{z_i - z_j}\right]\left(\Phi_{\pm n}\prod_{i<j}(z_i - z_j)^{2p+1}\right).$$

- $n=1$, $p=0$ and $+ \implies \nu=1/2$: same as the Pfaffian
- $n=2$, $p=0$ and $+ \implies \nu=2/3$: different from Jain 2/3 second LL Coulomb ground state not uniform at this shift
- $n=2$, $p=1$ and $- \implies \nu=2/5$: different from Jain 2/5 and aRR3: competitive with aRR3

  P. Bonderson *et al.*, Phys. Rev. Lett. **108**, 036806 (2012)
- $n=3$, $p=1$ and $- \implies \nu=3/8$: feasible in the second LL

  J. A. Hutasoit *et al.*, Phys. Rev. B **95**, 125302 (2017)
- 6/13 is not part of this sequence

Parsa Bonderson and J. K. Slingerland, Phys. Rev. B **78**, 125323 (2008)

Summary of candidate states in the second Landau level

- 1/2: likely a *p*-wave paired state of CFs
- 1/3 and 2/3: likely analogous to the LLL states

  Ajit C. Balram *et al.*, Phys. Rev. Lett. **110**, 186801 (2013)
- 2/5: aRR3 or a Bonderson-Slingerland state

  N. Read and E. H. Rezayi, Phys. Rev. B **59**, 8084 (1999)

  Parsa Bonderson and J. K. Slingerland, Phys. Rev. B **78**, 125323 (2008)
- 3/8: Bonderson-Slingerland state

  J. A. Hutasoit *et al.*, Phys. Rev. B **95**, 125302 (2017)
- 6/13: Levin-Halperin state

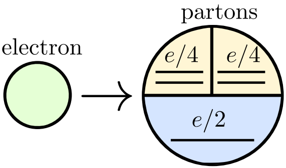
  M. Levin and B. I. Halperin, Phys. Rev. B **79**, 205301 (2009)

Can we find a unified description of the second LL FQHE?

Yes.
In terms of "parton" states.

An example of a non-composite fermion state

$$\Psi^{221}_{1/2} = \mathcal{P}_{\rm LLL}\Phi_2\Phi_2\Phi_1$$



- non-Abelian state different from Pfaffian and anti-Pfaffian

  X.-G. Wen, Phys. Rev. Lett. **66**, 802 (1991)

- can be interpreted as an f-wave paired state of CFs

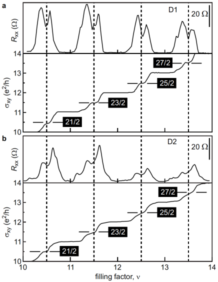
  Ajit C. Balram, Maissam Barkeshli, and Mark. S. Rudner, Phys. Rev. B **98**, 035127 (2018)

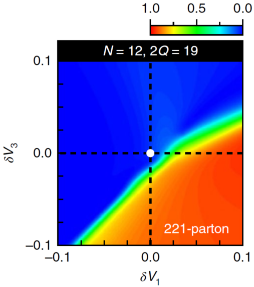
- not a good variational state for $\nu = 5/2$

J. K. Jain, Phys. Rev. B **40**, 8079 (1989)

221 parton state possibly realized in $n = 3$ LL of graphene

$$\Psi^{221}_{1/2} = \mathcal{P}_{\rm LLL}\Phi_2\Phi_2\Phi_1$$





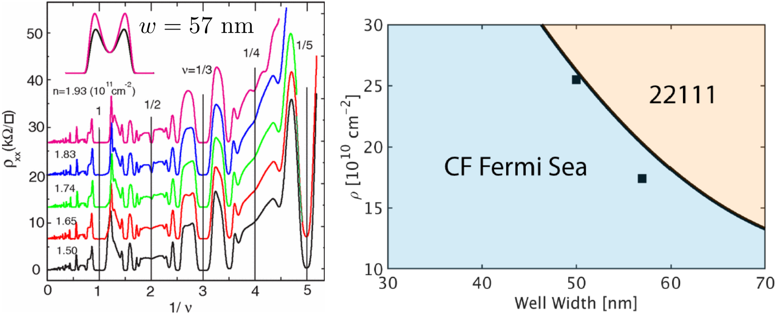
Y. Kim *et al.*, Nature Physics **15**, 154-158 (2019)

Proposal to realize the state in multilayer graphene

Y.-H. Wu, T. Shi and J. K. Jain, Nano Lett. 17 (8), 4643 (2017)

2^21^3 realized in quarter-filled LLL of wide quantum wells

$$\Psi^{2^21^3}_{1/4} = \mathcal{P}_{\rm LLL}\Phi_2^2\Phi_1^3 \sim \frac{[\Psi^{\rm Jain}_{2/5}]^2}{\Phi_1}$$



J. Shabani, T. Gokmen, and M. Shayegan, Phys. Rev. Lett. **103**, 046805 (2009)

W. N. Faugno *et al.*, Phys. Rev. Lett. **123**, 016802 (2019)

Parton sequence for the second Landau level

A parton sequence (with its hole-conjugate states) captures almost all the observed FQH states in the second LL

$$\Psi^{[\bar{2}1]^p \bar{n}1^2}_{\frac{2n}{(p+4)n-2}} = \mathcal{P}_{\rm LLL}[\Phi_{-2}\Phi_1]^p \; \Phi_{-n}\Phi_1^2 \sim \left[\frac{\Psi^{\rm Jain}_{2/3}}{\Phi_1}\right]^p \Psi^{\rm Jain}_{n/(2n-1)}$$

- $p = 1$: primary sequence $n = 1, 2, 3, \cdots \rightarrow 2/3, 1/2, 6/13, \cdots$
- $p = 2$: secondary sequence $n = 1, 2, 3, \cdots \rightarrow 1/2, 2/5, 3/8, \cdots$
- Predicts FQHE at $\nu = 2 + 4/9$ and $2 + 4/11$

Ajit C. Balram, Maissam Barkeshli, and Mark. S. Rudner, Phys. Rev. B **98**, 035127 (2018) and Phys. Rev. B **99**, 241108 (2019)

Ajit C. Balram *et al.* Phys. Rev. Lett. **121**, 186601 (2018)

The "$\bar{n}\bar{2}1^3$" ansatz

$$\Psi^{\bar{n}\bar{2}1^3}_{\nu=2n/(5n-2)} = \mathcal{P}_{\rm LLL}[\Phi^*_n][\Phi^*_2]\Phi_1^3 \sim \frac{\Psi^{\rm Jain}_{n/(2n-1)}\Psi^{\rm Jain}_{2/3}}{\Phi_1}$$

- $n = 1 \implies \nu = 2/3$: standard composite fermion state
- $n = 2 \implies \nu = 1/2$: parton state in the anti-Pfaffian phase
- $n = 3 \implies \nu = 6/13$: a new candidate state

Ajit C. Balram, Maissam Barkeshli, and Mark. S. Rudner, Phys. Rev. B **98**, 035127 (2018)

Ajit C. Balram *et al.* Phys. Rev. Lett. **121**, 186601 (2018)

The "$\bar{2}^2 1^3$" ansatz $\sim$ anti-Pfaffian

$$\Psi^{\bar{2}^2 1^3}_{\nu=1/2} = \mathcal{P}_{\rm LLL}[\Phi_2^*][\Phi_2^*]\Phi_1^3 \sim \frac{[\Psi^{\rm Jain}_{2/3}]^2}{\Phi_1}$$

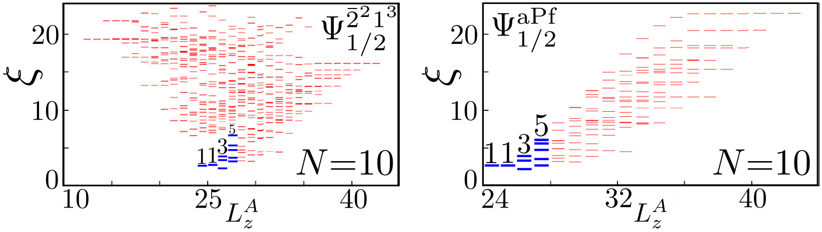
- state occurs at a shift $\mathcal{S} = -1$: same as the anti-Pfaffian shift
- better than anti-Pfaffian for second LL Coulomb

N	$2l$	$\lvert\langle\Psi^{\rm 1LL}_{1/2}\lvert\Psi^{\bar{2}^2 1^3}_{1/2}\rangle\rvert^2$	$\lvert\langle\Psi^{\bar{2}^2 1^3}_{1/2}\lvert\Psi^{\rm aPf}_{1/2}\rangle\rvert^2$	$\lvert\langle\Psi^{\rm 1LL}_{1/2}\lvert\Psi^{\rm aPf}_{1/2}\rangle\rvert^2$
8	17	0.862	0.908	0.702
10	21	0.774	0.881	0.671
12	25	0.614	0.861	0.481

Ajit C. Balram, Maissam Barkeshli, and Mark. S. Rudner, Phys. Rev. B **98**, 035127 (2018)

Ajit C. Balram, Phys. Rev. B **105**, L121406 (2022)

Entanglement spectrum of the $\bar{2}^2 1^3$ state



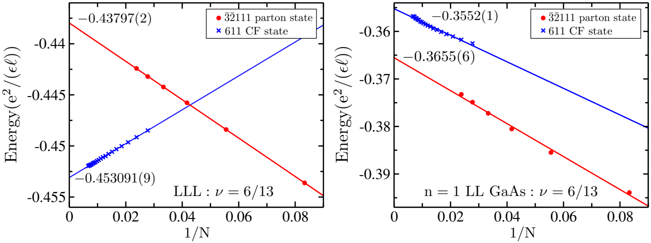
counting: $1, 1, 3, 5, \cdots$

Ajit C. Balram, Maissam Barkeshli, and Mark. S. Rudner, Phys. Rev. B **98**, 035127 (2018)

"$\bar{3}\bar{2}1^3$" is topologically different from the 6/13 Jain state

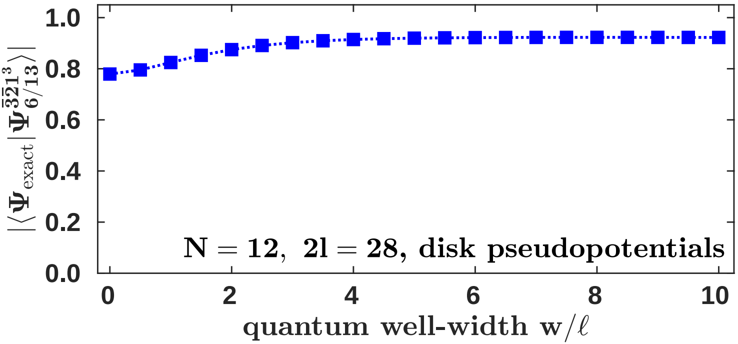
$$\Psi^{\bar{3}\bar{2}1^3}_{\nu=6/13} = \mathcal{P}_{\rm LLL}[\Phi_3^*][\Phi_2^*]\Phi_1^3 \sim \frac{[\Psi^{\rm Jain}_{3/5}][\Psi^{\rm Jain}_{2/3}]}{\Phi_1}$$

- occurs at $\mathcal{S}=-2$: topologically different from 6/13 Jain state
- different thermal Hall conductance from the 6/13 Jain state
- energetically better than the 6/13 Jain state in the second LL



Ajit C. Balram *et al.* Phys. Rev. Lett. **121**, 186601 (2018)

The "$\bar{3}\bar{2}1^3$" ansatz gives a good description of $2 + 6/13$



- a good overlap with the second LL Coulomb ground state
- likely in the same universality class as the Levin-Halperin state

Ajit C. Balram, SciPost Phys. **10**, 083 (2021)

The "$[\bar{2}]^k 1^{k+1}$" ansatz $\sim$ anti-Read-Rezayi

$$\Psi^{[\bar{2}]^k 1^{k+1}}_{\nu=2/(k+2)} = \mathcal{P}_{\rm LLL}[\Phi_2^*]^k \Phi_1^{k+1} \sim \frac{[\Psi^{\rm Jain}_{2/3}]^k}{\Phi_1^{k-1}}$$

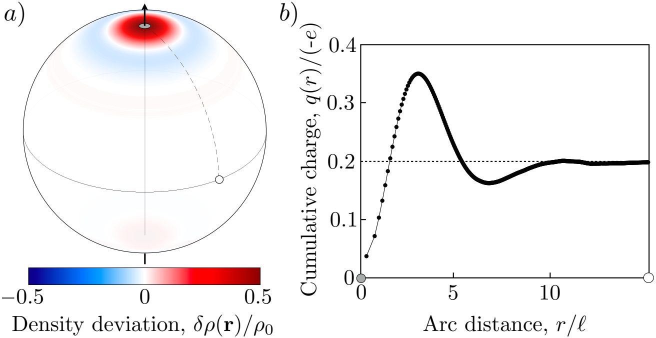
- $k=1 \implies \nu=2/3$: same as the composite fermion state
- $k=2 \implies \nu=1/2$: parton state in the anti-Pfaffian phase
- $k=3 \implies \nu=2/5$: this state lies in the same phase as the particle-hole conjugate of the Read-Rezayi $k=3$ (aRR3) state

Ajit C. Balram, Maissam Barkeshli, and Mark. S. Rudner, Phys. Rev. B **98**, 035127 (2018)

Ajit C. Balram, Maissam Barkeshli, and Mark. S. Rudner, Phys. Rev. B **99**, 241108 (2019)

Anyon with charge $(-e)/5$

$$\Psi_{2/5}^{2-\text{quasiparticles}} = \mathcal{P}_{\text{LLL}}[\Phi_2^{2-\text{holes}}]^*[\Phi_2^2]^*\Phi_1^4,$$



Density profile for $N = 80$ electrons

Ajit C. Balram, Maissam Barkeshli, and Mark. S. Rudner, Phys. Rev. B **99**, 241108 (2019)

The "$\bar{n}\bar{2}^2 1^4$" ansatz

$$\Psi^{\bar{n}\bar{2}^2 1^4}_{\nu=n/(3n-1)} = \mathcal{P}_{\rm LLL}[\Phi_n^*][\Phi_2^*]^2\Phi_1^4 \sim \frac{\Psi^{\rm Jain}_{n/(2n-1)}[\Psi^{\rm Jain}_{2/3}]^2}{\Phi_1^2}$$

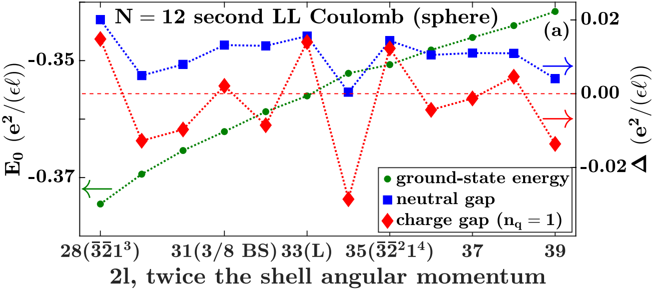
- $n=1 \implies \nu=1/2$: parton state in the anti-Pfaffian phase
- $n=2 \implies \nu=2/5$: same phase as the particle-hole conjugate of the Read-Rezayi $k=3$ state
- $n=3 \implies \nu=3/8$: a new candidate state different from the Bonderson-Slingerland state

Ajit C. Balram, Maissam Barkeshli, and Mark. S. Rudner, Phys. Rev. B **99**, 241108 (2019)

The "$\bar{3}\bar{2}^2 1^4$" ansatz

$$\Psi^{\bar{3}\bar{2}^2 1^4}_{\nu=n/(3n-1)} = \mathcal{P}_{\rm LLL}[\Phi^*_3][\Phi^*_2]^2\Phi_1^4 \sim \frac{\Psi^{\rm Jain}_{3/5}[\Psi^{\rm Jain}_{2/3}]^2}{\Phi_1^2}$$

- variationally the Bonderson-Slingerland state is better but exact diagonalization favors the $\bar{3}\bar{2}^2 1^4$ shift



- thermal Hall conductance can tell the two states apart

Ajit C. Balram, SciPost Phys. **10**, 083 (2021)

States outside Jain sequence observed in the LLL

- $4/11$ and $5/13$ well-established in GaAs

  W. Pan *et al.*, Phys. Rev. B **89**, 241302 (2014)

  N. Samkharadze *et al.*, Phys. Rev. B **91**, 081109 (2015)

  $$\Psi_{4/11}^{4\bar{2}1^3} = \mathcal{P}_{\rm LLL}\Phi_4[\Phi_2]^*\Phi_1^3 \sim \frac{\Psi_{4/9}^{\rm Jain}\Psi_{2/3}^{\rm Jain}}{\Phi_1}$$

  Ajit C. Balram, Phys. Rev. B **103**, 155103 (2021)

- $4/11$ and $4/13$ observed in monolayer graphene

  Manohar Kumar, Antti Laitinen, and Pertti Hakonen, Nat. Comm. **9**, 2776 (2018)

  $$\Psi_{4/13}^{\bar{4}21^3} = \mathcal{P}_{\rm LLL}[\Phi_4]^*\Phi_2\Phi_1^3 \sim \frac{\Psi_{4/7}^{\rm Jain}\Psi_{2/5}^{\rm Jain}}{\Phi_1}$$

  Rakesh K. Dora and Ajit C. Balram, Phys. Rev. B **105**, L241403 (2022)

Consistent with description in terms of FQHE of CFs in their SΛL

S. Mukherjee, S. S. Mandal, *et al.*, Phys. Rev. Lett. **112**, 016801 (2014)

S. Mukherjee and S. S. Mandal, *et al.*, Phys. Rev. B **92**, 235302 (2015)

What makes our parton states special?

- Composite fermion ($n11\cdots$ parton) states capture the most prominent LLL plateaus
  $\rightarrow$ placing partons into $\nu = 1$ states, i.e., $\Phi_1 = \prod_{i<j}(z_i - z_j)$ builds good correlations in the many-body state
- Simplest generalization $\rightarrow$ $nm11\cdots$ where $m = 2$ or $m = -2$
- Comes down to energetics: for the second LL interaction our sequence of parton states appear most plausible
- Open problem: for a given interaction which parton state(s) is likely to be stabilized

Outlook

- Parton theory can potentially also capture delicate states observed in the LLL that are *not* part of the Jain sequence.

  Rakesh K. Dora and Ajit C. Balram, Phys. Rev. B **105**, L241403 (2022)

  Ajit C. Balram, Phys. Rev. B **103**, 155103 (2021)

  Ajit C. Balram and A. Wójs, Phys. Rev. Research **3**, 033087 (2021)

- Very high-energy excitations in the Jain states that are not described by composite fermions but lend themselves to a description in terms of partons.

  Ajit C. Balram, Zhao Liu, Andrey Gromov, and Zlatko Papić, arXiv:2111.10395

Almost all fractional quantum Hall states can be described as products of integer quantum Hall states.

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- Koyena Bose (IMSc, Chennai)

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